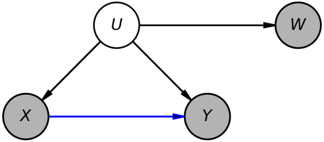
Problem Statement

Identification: Can the causal effect of X on Y be uniquely estimated given the distribution and model structure?

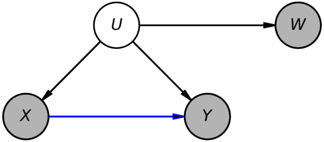


$$pr\{y|do(x)\} = \sum_u pr(y|x,u)pr(u)$$

Case 1: Single proxy variable with known error mechanism

Required: $pr(w|u)$

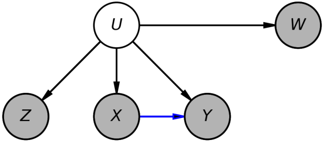
Use matrix adjustment to estimate $pr(y|x,u)$ and $pr(u)$.

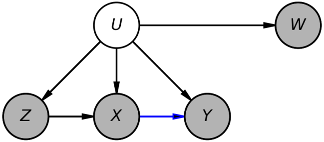


Case 2: Multiple proxy variables.

Required: One proxy variable should only be connected to U.

1. Choose one of the proxy variables as the "indicator variable", W.
2. Use eigen value decomposition to estimate $pr(w|u)$.
3. Follow the method of first case.



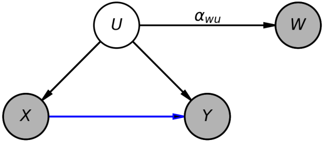


Case 3: Linear SEM

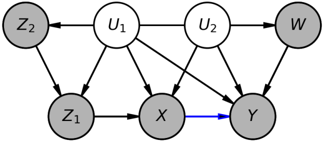
Estimate: $\beta_{xy.u} = \frac{\sigma_{xy.u}}{\sigma_{xx.u}}$

Required: The contribution of U's variance to W's variance: $\alpha_{wu}^2 \sigma_{uu} = \sigma_{ww} - \sigma_{\epsilon_w, \epsilon_w}$

1. If single proxy variable, needs $\alpha_{wu}^2 \sigma_{uu}$.
2. If multiple proxy variables, $\alpha_{wu}^2 \sigma_{uu}$ can be computed.



Case 4: Linear SEM with instrumental variables



Estimate: $\beta_{xy.u} = \frac{\sigma_{yz.u}}{\sigma_{xz.u}}$

THEOREM 2. *Suppose that:*

(i) *a nonempty set* $\{Z_1, Z_2\}$ *of distinct variables satisfies one of the following conditions:* (i-a) *both* Z_1 *and* Z_2 *are conditional instrumental variables given a univariate* U *relative to* (X, Y), (i-b) Z_1 *is a conditional instrumental variable given* U *relative to* (X, Y), *and* $Z_2 = X$ *and* U *satisfies the back door criterion relative to* (X, Y), (i-c) Z_1 *is a conditional instrumental variable given* U *relative to* (X, Y), *and* U *d-separates* Z_2 *from* $\{X, Y\}$;

(ii) U *d-separates* $\{Z_1, Z_2\}$ *from an observed variable* W.

Then the total effect τ_{yx} *of* X *on* Y *is identifiable and is given by the formula* (11).

Kuroki, M., & Pearl, J. (2014). Measurement bias and effect restoration in causal inference