

Implement the following problems using Python to get the results.

**A. Finding Roots of the Equations:** Present each iteration result in a table

1. Implement the **Bisection Method** to approximate the root of the equation  $x^2 = \sin x$  by taking the initial guesses  $a = 0.5$  and  $b = 1.0$ . (Algorithm:  $x_0 = \frac{a+b}{2}$ )
2. Implement the **Newton-Raphson's Method** to approximate the root of the equation  $e^x = 4x$  by taking the initial guess  $x_0 = 1.0$  (Algorithm:  $x_1 = x_0 - \frac{f(x_0)}{f'(x_0)}$ )

**B. Interpolation**

3. Construct the finite difference table of the function  $f(x) = e^x$  on the interval  $-1 \leq x \leq 1$  by dividing the interval by equally spaced points of step-size 0.1
4. Estimate the value of the function  $f(0.21)$  and  $f(0.29)$  applying Newton's forward and backward interpolation polynomials using following table:

x	0.20	0.22	0.24	0.26	0.28	0.30
y	1.6596	1.6698	1.6804	1.6912	1.7024	1.7139

5. Estimate  $y(2)$  from the following data using Lagrange's formula

x	0	1	3	4	5
y	0	1	81	256	625

**C. Numerical Integration**

6. Implement Trapezoidal-rule to approximate the definite integral  $I = \int_0^{\pi} \frac{\sin x}{e^x} dx$  by taking 20-equal divisions of the interval  $[0, \pi]$ . (Algorithm:  $I = \frac{h}{2} [y_0 + 2(y_1 + y_2 + \dots + y_{n-1}) + y_n]$ )
7. Implement Simpson's 1/3-rule to approximate the definite integral  $I = \int_{-4}^4 \frac{1}{2\pi} e^{-\frac{x^2}{2}} dx$  by taking 50-equal divisions of the interval  $[-4, 4]$ . (Algorithm:  $I = \frac{h}{3} [y_0 + 4(y_1 + y_3 + \dots + y_{n-1}) + 2(y_2 + y_4 + \dots + y_{n-2}) + y_n]$ )

**D. Numerical Differential Equations:** Present the solution in a table and graph it.

8. Implement **Euler's method** to approximate the solution  $y(x)$  of the differential equation  $\frac{dy}{dx} = x^2 + x, y(0) = 1$  on the interval  $[0, 2]$  by dividing it into 20- equal sub-intervals. [Algorithm:  $y_{i+1} = y_i + hf(x_i, y_i)$ ]

9. Implement **Runge-Kutta 2<sup>nd</sup> order method** to approximate the solution  $y(x)$  of the differential equation  $\frac{dy}{dx} = x^2 + x, y(0) = 1$  on the interval  $[0, 2]$  by dividing it into 10-equal sub-intervals. **[Algorithm:  $y_{i+1} = y_i + \frac{1}{2}(k_1 + k_2)$ , where  $k_1 = hf(x_i, y_i)$  and  $k_2 = hf(x_i + h, y_i + k_1)$  ]**
10. Implement the Boundary valued second order differential equation  $y'' - 64y' + 10 = 0$  by using **finite different method** with boundary conditions  $y(0) = y(1) = 0$  and taking the step size  $h = 0.1$ .

## E. Curve Fitting

11. Using least square method to fit the straight line  $y = a_0 + a_1x$  to the following data:

x	1	2	3	4	5	6
y	2.4	3.1	3.5	4.2	5.0	6.0

Using this fit estimate the value of  $y$  at  $x = 2.5$

12. Using least square method to fit the curve  $y = ae^{bx}$  to the following data:

x	2	4	6	8	10
y	4.077	11.084	30.128	81.897	222.62

Using this fit estimate the value of  $y$  at  $x = 9$