CS 381 HW 8

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Q1

(a) We can transform an instance of element uniqueness to closest pair in linear time as follows:

Given X = $\{v_1, v_2, ...v_n\}$, let P = $\{(v_1, v_1)$, (v_2, v_2) ... $(v_n, v_n)\}$. The set P is then the corresponding instance of closest pair. It's obvious that this is just a linear time transform if we iterate through each point in X and create the corresponding point in P.

(b) Given a solution to the instance P above, we can solve the element uniqueness problem in constant time. It's clear that the min-distance, δ, returned is 0 iff X contains some pair, v_i and v_j such that $v_i = v_j$. Thus, if $\delta = 0 \Rightarrow$ there is some non-unique pair in X, and the corresponding points in P are returned by the solution to closest pair. If $\delta > 0 \Rightarrow$ the elements of X are unique. Once we have the solution to P, checking the δ value is just a constant time check.

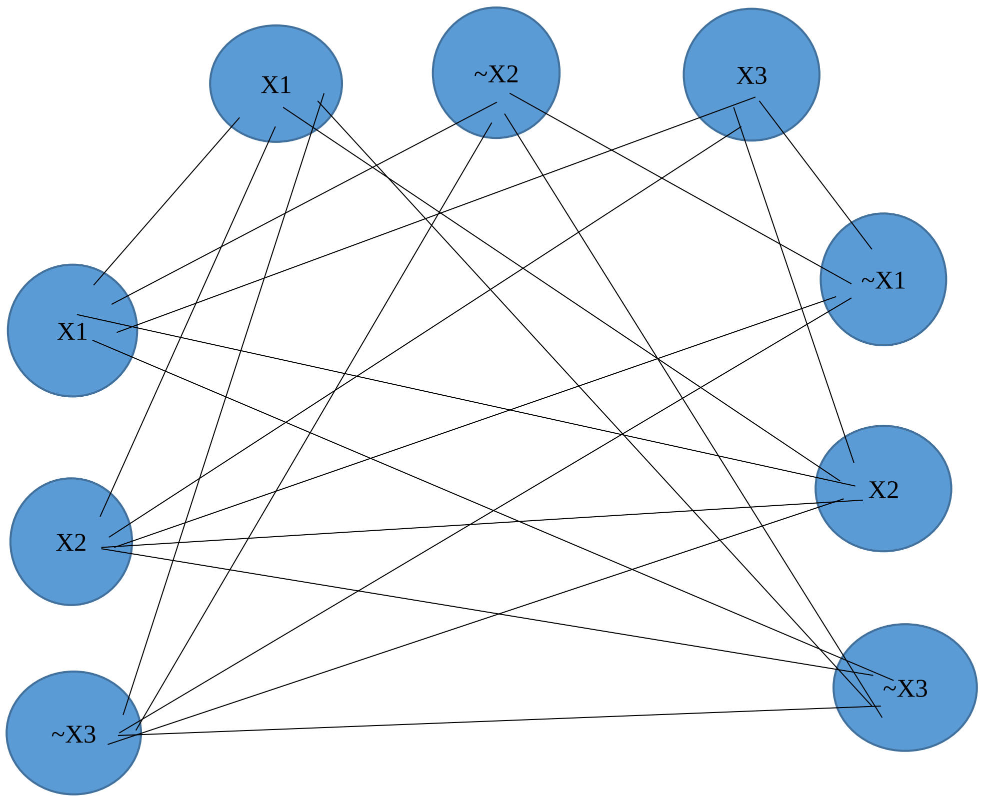
(c) Suppose the time needed to solve closest pair is faster than $\propto$ nlogn. Then, we can convert an instance of element uniqueness to closest pair in O(n) time, solve the closest pair instance in faster than $\propto$ nlogn, and use the solution to solve the element uniqueness instance in constant time. Thus, the total running time needed to solve the element uniqueness problem becomes faster than $\propto$ nlogn. But this contradicts our Ω(nlogn) on the element uniqueness problem. Thus, closest pair cannot be faster than $\propto$ nlogn (i.e closest pair is Ω(nlogn)).

Q2

Graph corresponding to

$(x1 \vee \neg x2 \vee x3) \wedge (x1 \vee x2 \vee \neg x3) \wedge (\neg x1 \vee x2 \vee \neg x3)$

There is a clique in this graph corresponding to X1 from the top triple, X2 from the left triple, and ¬ X3 from the right triple. Thus, the assignment X1 = 1, X2 = 1, and X3 = 0 is a satisfying assignment. (Note: This isn't the only satisfying assignment. X1 = 1, X2 = 1, X3 = 1 is also a satisfying assignment)


X1
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