CS 580 HW 1

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Q1

T(n) = $\theta(n^{log_5 7}\text{lgn})$. Using the notation of the master method, a = 7, b = 5 so f(n) = $\theta(n^{log_5 7})$ putting us in case 2 of the master method. Thus, $n^{log_5 7}$lgn is an asymptotic upper and lower bound on T(n).

Q2

Claim: T(n) $\leq$ 45/8 cn

Proof: By induction on n

Base case:

T(n) = c $\leq$ 45/8 cn for n < 5

Induction Step: Assume the statement is true for n' < n.

We must show the statement is true for n.

T(n) =

$$
\begin{aligned}
T(\left\lfloor \frac{2n}{9} \right\rfloor) + 3T(\left\lfloor \frac{n}{5} \right\rfloor) + cn \\
\leq a\left\lfloor \frac{2n}{9} \right\rfloor + 3a\left\lfloor \frac{n}{5} \right\rfloor + cn \\
\leq a2n/9 + 3an/5 + cn \\
= an(2/9 + 3/5) + cn \\
= an(37/45) + cn = \\
37/8cn + cn \\
= 45/8cn \\
= an
\end{aligned} \tag{1}
$$

where the second step follows by the induction hypothesis. Thus, by the property of mathematical induction, the statement holds.

Q3

(a) Base case: a = 1

$$\begin{aligned} H(3)H(n-1) - H(1)H(n-2) - H(2)H(n-3) \\ = 2H(n-1) - H(n-3) \\ = H(n) \end{aligned} \tag{2}$$

so the base case holds.

Induction step: Suppose true for a' < a < n - 2

We want to show the claim holds for a:

$$\begin{aligned} H(n) = H(a+2)H(n-a) - H(a)H(n-a-1) - H(a+1)H(n-a-2) \\ = [2H(a+1) - H(a-1)]H(n-a) - H(a)H(n-a-1) - H(a+1)H(n-a-2) \\ = H(a+1)[2H(n-a) - H(n-a-2)] - H(a-1)H(n-a) - H(a)H(n-a-1) \\ = H(a+1)H(n-a) - H(a-1)H(n-a) - H(a)H(n-a-1) \\ = H(n) \end{aligned} \tag{3}$$

where the last step follows with the induction hypothesis applied with a - 1. Thus, by the principle of mathematical induction the claim holds.

(b) Using a = k-1:

$$\begin{aligned} H(2k) = H(k+1)H(2k-(k-1)) - H(k-1)H(2k-(k-1)-1) - H(k)H(2k-(k-1)-2) \\ = H(k+1)H(k+1) - H(k-1)H(k) - H(k)H(k-1) \\ = H(k+1)H(k+1) - 2H(k)H(k-1) \end{aligned} \tag{4}$$

$$\begin{aligned} H(2k-1) = H(k+1)H(2k-1-(k-1) - H(k-1)H(2k-1-(k-1)-1) \\ -H(k-1+1)H(2k-1-(k-1)-2) \\ = H(k+1)H(k) - H(k-1)H(k-1) - H(k)H(k-2) \\ = H(k+1)H(k) - H(k-1)H(k-1) - H(k)[2H(k) - H(k+1)] \\ = 2H(k+1)H(k) - 2H(k)H(k) - H(k-1)H(k-1) \end{aligned} \tag{5}$$

(c) Using a = k-1:

$$\begin{aligned} H(2k+1) = H(k-1+2)H(2k+1-(k-1)) - H(k-1)H(2k+1-(k-1)-1) \\ -H(k-1+1)H(2k+1-(k-1)-2) \\ = H(k+1)H(k+2) - H(k-1)H(k+1) - H(k)H(k) \\ = H(k+1)[2H(k+1) - H(k-1)] - H(k-1)H(k+1) - H(k)H(k) \\ = 2H(k+1)H(k+1) - 2H(k+1)H(k-1) - H(k)H(k) \end{aligned} \tag{6}$$

$$
\begin{aligned}
H(2k-2) = H(k-1+2)H(2k-2-(k-1)) - H(k-1)H(2k-2-(k-1)-1) \\
-H(k-1+1)H(2k-2-(k-1)-2) \\
= H(k+1)H(k-1) - H(k-1)H(k-2) - H(k)H(k-3) \\
= H(k+1)H(k-1) - H(k-1)[2H(k) - H(k+1)] - H(k)[2H(k-1) - H(k)] \\
= 2H(k+1)H(k-1) - 4H(k)H(k-1) + H(k)H(k)
\end{aligned}
\tag{7}
$$

(d)