Game Playing

Question 1

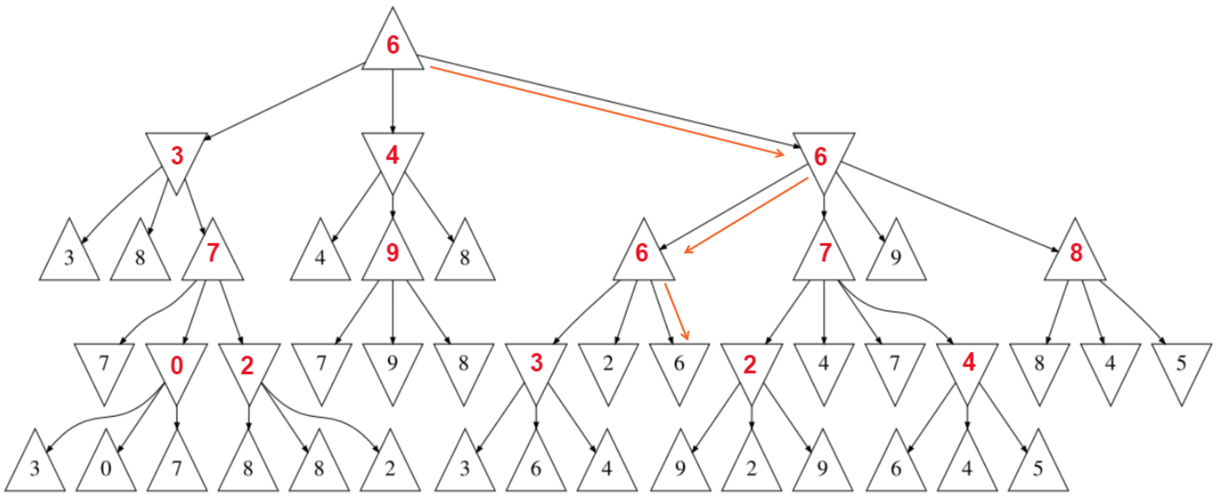

Figure 1: Minmax Search

Question 2

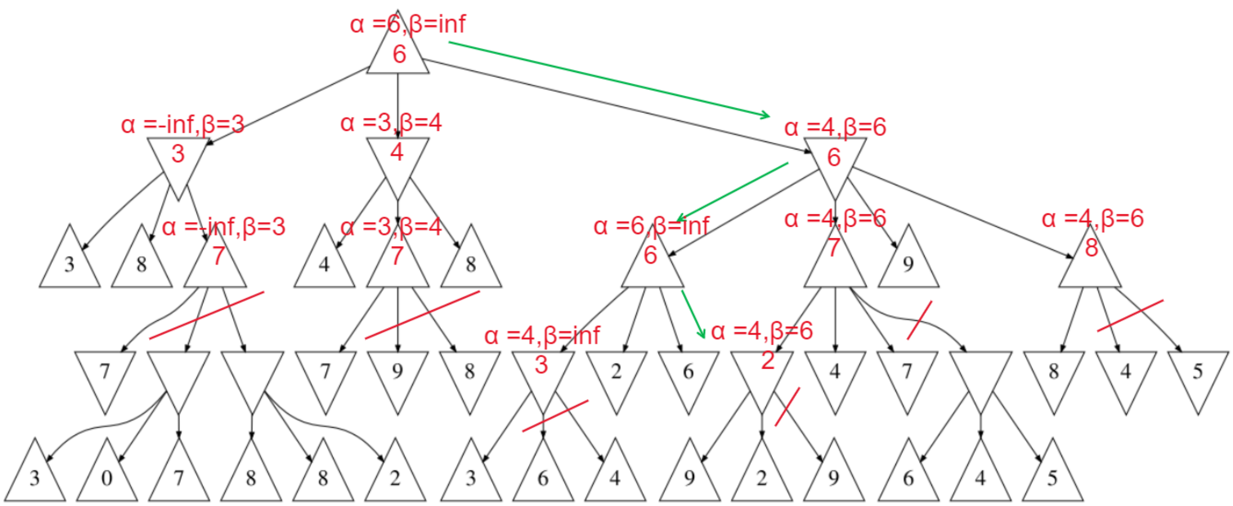

Figure 2: Minmax Search

Question 3

Depth-first search is a search algorithm that explores the deepest node in the search tree first before backtracking to explore shallower nodes. This can result in the search getting stuck in a deep branch of the game tree and missing out on potential better moves that are available at shallower depths.

Despite using a depth-first exploration strategy, Minmax search is still able to give an optimal solution because it evaluates all possible moves in the game tree and selects the move with the highest score for the maximizing player and the lowest score for the minimizing player. By doing so, it guarantees that the selected move will lead to the best possible outcome for the player, given the opponent's optimal moves

Propositional Logic

Question 4

P	Q	R	$P \vee Q$	$\neg Q \vee R$	$(P \vee Q) \wedge (\neg Q \vee R)$	$P \vee R$	$[(P \vee Q) \wedge (\neg Q \vee R)] \rightarrow (P \vee R)$
T	T	T	T	T	T	T	T
T	T	F	T	F	F	T	T
T	F	T	T	T	T	T	T
T	F	F	T	T	T	T	T
F	T	T	T	T	T	T	T
F	T	F	T	F	F	F	T
F	F	T	F	T	F	T	T
F	F	F	F	T	F	F	T

Question 5

1. $(P \wedge \neg Q) \vee R$
   $= (P \vee R) \wedge (\neg Q \vee R)$ [distributive law]

2. $(P \wedge \neg Q) \vee (P \wedge R) \vee S$
   $=(P \wedge (\neg Q \vee R)) \vee S$ [distributive law (backward)]
   $=(P \vee S) \wedge ((\neg Q \vee R) \vee S)$ [distributive law]
   $= (P \vee S) \wedge (\neg Q \vee R \vee S)$

3. $(P \wedge Q) \rightarrow \neg(R \rightarrow S)$
   $= (P \wedge Q) \rightarrow \neg(\neg R \vee S)$ [implication]
   $=(P \wedge Q) \rightarrow (R \wedge \neg S)$ [De Morgan's Law]
   $= \neg(P \wedge Q) \vee (R \wedge \neg S)$ [implication]
   $= (\neg P \vee \neg Q) \vee (R \wedge \neg S)$ [De Morgan's Law]
   $= \neg P \vee \neg Q \vee (R \wedge \neg S)$

Question 6

1. $P \vee Q \vee R$

2. $S \rightarrow \neg Q = \neg S \vee \neg Q$ [implication]

3. $(R \rightarrow P) \wedge \neg P = (\neg R \vee P) \wedge \neg P$ [implication] $=(\neg R \wedge \neg P) \vee (P \wedge \neg P)$ [distributive] $= \neg R \wedge \neg P$ (negation)

Negated Conclusion: $\neg(\neg S \wedge \neg R) = \neg(\neg S) \vee \neg(\neg R)$ [De Morgan's Law] $= S \vee R$ [double negation]

C1: $P \vee Q \vee R$
C2: $\neg S \vee \neg Q$
C3: $\neg R$
C4: $\neg P$
C5: $S \vee R$

Question 7

C6: $P \vee Q$ (1,3)
C7: Q (4,6)
C8: S (3,5)
C9: $\neg Q$ (2,8)
C10: $False$ (7,10)

First Order Logic (Basics)

Question 8

1. $\forall x \neg(\exists y P(x,y) \wedge \neg Q(x))$
   $= \forall x(\forall y \neg P(x,y) \vee Q(x))$ [De Morgan's law]
   $= \forall x \forall y(\neg P(x,y) \vee Q(x))$

2. $\neg \forall x(\neg P(x) \vee \neg(\forall y Q(x,y)))$
   $= \neg \forall x(\neg P(x) \vee (\exists y \neg Q(x,y)))$
   $= \exists x(P(x) \wedge \neg(\exists y \neg Q(x,y)))$ [De Morgan's law]
   $= \exists x(P(x) \wedge (\forall y Q(x,y)))$
   $= \exists x \forall y(P(x) \wedge Q(x,y))$

3. $\neg \forall x((\forall y Q(x,y)) \rightarrow \neg P(x))$
   $= \neg \forall x(\neg(\forall y Q(x,y)) \vee \neg P(x))$ [Implication Law]
   $= \neg \forall x((\exists y \neg Q(x,y)) \vee \neg P(x))$
   $= \exists x(\neg(\exists y \neg Q(x,y)) \wedge P(x))$ [De Morgan's law]
   $= \exists x((\forall y Q(x,y)) \wedge P(x))$
   $= \exists x(\forall y(Q(x,y) \wedge P(x)))$
   $= \exists x \forall y(Q(x,y) \wedge P(x))$

Question 9

1. $\exists x P(x)$
   $= P(a)$

2. $\forall x \exists y P(x,y)$
   $= \forall x P(x, f(x))$

3. $\exists x \exists y \forall z P(x,y) \wedge Q(y,z)$
   $= \exists y \forall z P(a,y) \wedge Q(y,z)$
   $= \forall z P(a,b) \wedge Q(b,z)$

4. $\forall x \exists y \exists z P(x,y,z) \wedge Q(y,z)$
   $= \forall x \exists z P(x, f(x), z) \wedge Q(f(x), z)$
   $= \forall x P(x, f(x), g(x)) \wedge Q(f(x), g(x))$

5. $\forall x \forall y \exists z P(x,y) \wedge Q(x,y,z)$
   $= \forall x \forall y P(x,y) \wedge Q(x,y,f(x,y))$

Question 10

$\forall x[\neg(\exists y P(x,y)) \rightarrow (\exists z(Q(z) \rightarrow R(x,z)))]$
$= \forall x[(\forall y \neg P(x,y)) \rightarrow (\exists z(Q(z) \rightarrow R(x,z)))]$
$= \forall x[(\forall y \neg P(x,y)) \rightarrow (\exists z(\neg Q(z) \vee R(x,z)))]$ [implication]
$= \forall x[\neg(\forall y \neg P(x,y)) \vee (\exists z(\neg Q(z) \vee R(x,z)))]$[De Morgan's rule]
$= \forall x[(\exists y P(x,y)) \vee (\exists z(\neg Q(z) \vee R(x,z)))]$
$= \forall x \exists y \exists z[P(x,y) \vee (\neg Q(z) \vee R(x,z))]$
$= \forall x \exists y \exists z[P(x,y) \vee \neg Q(z) \vee R(x,z)]$ [Associative]
$= \forall x \exists z[P(x,f(x)) \vee \neg Q(z) \vee R(x,z)]$
$= \forall x[P(x,f(x)) \vee \neg Q(g(x)) \vee R(x,g(x))]$

Question 11

1. $P(f(A), y) \wedge Q(f(A))$
2. $P(A, f(z)) \vee Q(A)$
3. $P(x,x) \vee Q(x)$

Question 12

1. Step 0: $\sigma_0 = \{\}$
   $W_0 = \{P(x, f(x)), P(A, f(B))\}$
   $D = \{x, A\}$

   Step 1: Substitution rule: $\{v/t\}$= $\{x/A\}$
   $\sigma_1 = \sigma_0 \circ \{x/A\} = \{x/A\}$
   $W_1 = \{P(A, f(A)), P(A, f(B))\}$
   $D = \{A, B\}$
   These expressions are not unifiable because there is no variable in the distractor set D .

2. Step 0: $\sigma_0 = \{\}$
   $W_0 = \{P(x, A), P(y, y)\}$
   $D = \{x, y\}$
   Step 1: Substitution rule: $\{v/t\} = \{x/y\}$
   $\sigma_1 = \sigma_0 \circ \{x/y\} = \{x/y\}$

$W_1 = \{P(y, A), P(y, y)\}$
$D = \{A, y\}$

Step 2: Substitution rule: $\{v/t\} = \{y/A\}$
$\sigma_2 = \sigma_1 \circ \{y/A\} = \{x/y\} \circ \{y/A\} = \{x/A, y/A\}$
$W_2 = \{P(A, A), P(A, A)\} = \{P(A, A)\}$
The unified expression is P(A, A) and the unifier is $\{x/A, y/A\}$

3. Step 0: $\sigma_0 = \{\}$
   $W_0 = \{P(x, f(x), y), P(A, f(g(w)), g(A))\}$
   $D = \{x, A\}$
   Step 1: Substitution rule: $\{v/t\} = \{x/A\}$
   $\sigma_1 = \sigma_0 \circ \{x/A\} = \{\} \circ \{x/A\} = \{x/A\}$
   $W_1 = \{P(A, f(A), y), P(A, f(g(w)), g(A))\}$
   $D = \{A, g(w)\}$
   These expressions are not unifiable because there is no variable in the distractor set D .

4. Step 0: $\sigma_0 = \{\}$
   $W_0 = \{P(A, f(y), y, B), P(x, f(g(x)), g(A), w)\}$
   $D = \{A, x\}$

   Step 1: Substitution rule: $\{v/t\} = \{x/A\}$
   $\sigma_1 = \sigma_0 \circ \{x/A\} = \{\} \circ \{x/A\} = \{x/A\}$
   $W_1 = \{P(A, f(y), y, B), P(A, f(g(A)), g(A), w)\}$
   $D = \{y, g(A)\}$
   Step 2: Substitution rule: $\{v/t\} = \{y/g(A)\}$
   $\sigma_2 = \sigma_1 \circ \{y/g(A)\} = \{x/A\} \circ \{y/g(A)\} = \{x/A, y/g(A)\}$
   $W_2 = \{P(A, f(g(A)), g(A), B), P(A, f(g(A)), g(A), w)\}$
   $D = \{B, w\}$
   Step 3: Substitution rule: $\{v/t\} = \{w/B\}$
   $\sigma_3 = \sigma_2 \circ \{w/B\} = \{x/A, y/g(A)\} \circ \{w/B\} = \{x/A, y/g(A), w/B\}$
   $W_3 = \{P(A, f(g(A)), g(A), B), P(A, f(g(A)), g(A), B)\} = \{P(A, f(g(A)), g(A), B)\}$
   The unified expression is $\{P(A, f(g(A)), g(A), B)\}$ and the unifier is $\{x/A, y/g(A), w/B\}$