

IMPORTANT: Always take that case into consideration where we will have only 1 element in the array.

BINARY SEARCH 1

- Any Searching problem has 2 components, namely target and search space.
- If I have to search for a word in newspaper, then newspaper is search space and word will be my target.

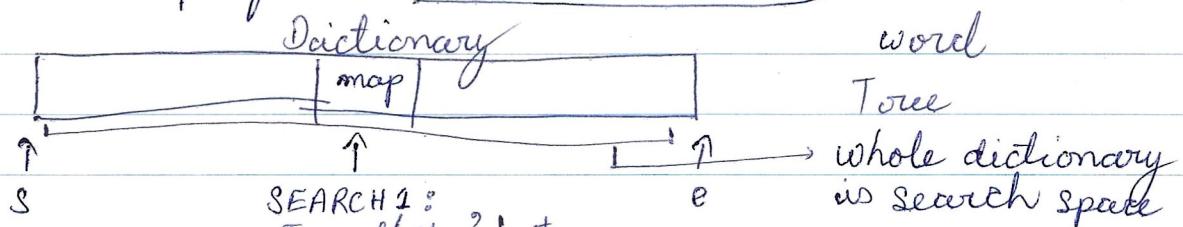
- Word in newspaper
- Word in dictionary
- Name in Contact List

Search Space	Target
Word	Newspaper
Dictionary	Word
Contact List	Name

- Is there any difference between finding a word in newspaper and dictionary?

Newspaper - Words are unsorted
Dictionary - Words are sorted.

- Newspaper - To search a word in newspaper, one will do linear search. $O(N)$
- Dictionary - To search a word in dictionary, one performs random search.



RANDOM SEARCH

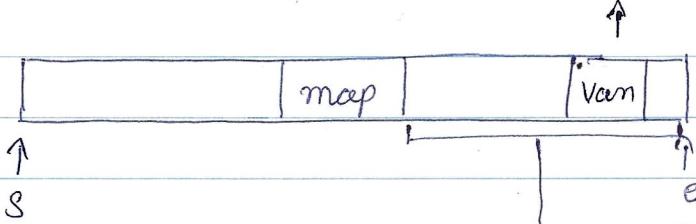
Found 'Map' but
'Tree' lies after map.

So, we will search the
search space i.e. in
right of word Map.

BINARY SEARCH 1

Found 'Van' when looked further

RANDOM
SEARCH



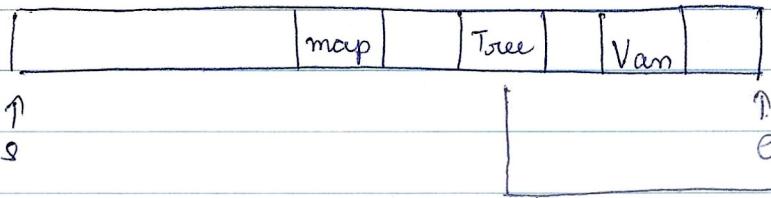
Only this part of dictionary is new search space

RANDOM
SEARCH



This time search space shrunk even more.

RANDOM
SEARCH

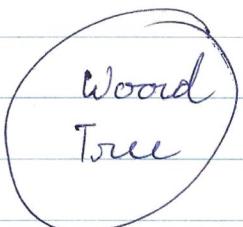
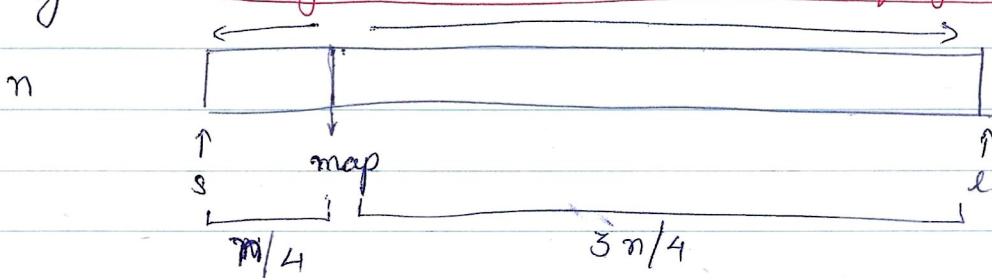


We found our word this time

Does random search saves time? Yes

Is it a best way to search? No

Why No? Why RANDOM SEARCH is not good?



Suppose, we searched for word tree but firstly we found 'map'. Now if 'tree' lies on left of map then we will search $n/4$ search space or $1/4^{\text{th}}$ of search space and if it lies on right then we need to search $3n/4$ search space

BINARY SEARCH 1

or $\frac{3}{4}$ th of the search space. By randomly picking a word for search, search space is not getting evenly distributed which is a drawback of random search. This is called unbalanced Split.

NOTE : Because of the problem of unbalanced split, binary search is preferred which provides up to balanced split or splits of equal size in form of search space.

Example of Binary Search 10:15

Example 1

Search 23

0	1	2	3	4	5	6	7	8	9
2	5	8	12	16	23	38	56	72	91
S=0			M=4			E=9			16 < 23

0	1	2	3	4	5	6	7	8	9
2	5	8	12	16	23	38	56	72	91
S=5				M=7		E=9			56 > 23

0	1	2	3	4	5	6	7	8	9
2	5	8	12	16	23	38	56	72	91
S=5				E=6					E = M - 1

Return 5 return M

Example 2

Search 15

0	1	2	3	4	5	6
3	5	8	10	15	20	25
S=0			M=3		E=6	10 < 15

Binary Sl(3)

BINARY SEARCH 1

Search 15

0	1	2	3	4	5	6
3	5	8	10	15	20	25

S = 4 M = 5 E = 6

20 > 15

S = M + 1

0	1	2	3	4	5	6
3	5	8	10	15	20	25

S = 4

15 = 15

E = 4

E = M - 1

Return 4 ~~return M~~ M = 4

Example 3

Search 14

0	1	2	3	4	5	6
3	5	8	10	15	20	25

S = 0

M = 3

E = 6

10 < 14

S = M + 1

0	1	2	3	4	5	6
3	5	8	10	15	20	25

~~S = 0~~

S = 4 M = 5 E = 6

20 > 14

E = M - 1

0	1	2	3	4	5	6
3	5	8	10	15	20	25

S = 4

E = 4

M = 4

15 > 14

E = M - 1

0	1	2	3	4	5	6
3	5	8	10	15	20	25

E = 3 S = 4

S > E, terminate the loop and return
 -1. Element doesn't exist

Return -1

Binary S(4)

BINARY SEARCH 1

How mid is determined in binary search?

Simplest way:

$$M = \frac{(S + E)}{2}$$

But when S and E are very high values, we could face the issue of overflow and could lead to -ve values for indices of M . In this case we could use the formula below.

Search 15

$$M = S + (E - S)/2$$

0	1	2	3	4	5	6
3	5	8	10	15	20	25

$S=0$ $M=3$ ~~$E=6$~~

$$M = (0+6)/2 = 3$$

$$M = 0 + ((6-0)/2)$$

$$= 0 + (6/2) = 3$$

$10 < 15$

0	1	2	3	4	5	6
3	5	8	10	15	20	25

$S=4$ $M=5$ $E=6$

$$S = M + 1 = 3 + 1 = 4$$

$$M = (4+6)/2 = 5$$

$$M = 4 + ((6-4)/2)$$

$$= 4 + (2/2)$$

$$= 4 + 1 = 5$$

0	1	2	3	4	5	6
3	5	8	10	15	20	25

$$S=4$$

$$E=4$$

$$M=4$$

$20 > 15$

$$E = M - 1 = 5 - 1 = 4$$

$$M = (4+4)/2 = 4$$

$$M = 4 + ((4-4)/2)$$

$$= 4$$

Return 4;

15 = 15 Binary S(5)

BINARY SEARCH 1

Pseudocode

```
int binarySearch (int arr[], int T) {
```

```
    int s = 0;
```

```
    int e = arr.length - 1;
```

```
    while (s <= e) {
```

Loop will stop if $s > e$

```
        mid = (s + e) / 2;      mid = s + (e - s) / 2
```

$O(\log n)$

```
        if (arr[mid] == T) { return mid; }
```

```
        else if (arr[mid] > T) { e = mid - 1; }
```

```
        else { s = mid + 1; }
```

```
}
```

```
return -1;
```

```
}
```

Time Complexity - $O(\log n)$

Space Complexity - $O(1)$

Note: As we know, whenever we have a loop in an answer, we can also solve a problem with the help of recursive calls. But we will not use recursion to solve any binary search problems because it increases space complexity to $O(\log n)$ from $O(1)$ in simple loop approach.

BINARY SEARCH 1

Binary Search on an array with duplicates

Suppose we have following array

	0	1	2	3	4	5	6	7	8	9	10	11	12	S E M Ans
	1	1	2	3	3	3	3	3	4	5	8	10	10	0 12 6 6
	↑		↑								↑			Target 3

$S=0$

$M=6$

$E=12$

Finding first occurrence

* Here we found the target at 6th index but we want to return 3^{index} as the answer which has first occurrence of 3 in the array.

	0	1	2	3	4	5	6	7	8	9	10	11	12	S E M Ans
	1	1	2	3	3	3	3	3	4	5	8	10	10	0 5 2 6
	$S=0$	$M=2$		$E=5$										

	0	1	2	3	4	5	6	7	8	9	10	11	12	S E M Ans
	1	1	2	3	3	3	3	3	4	5	8	10	10	3 5 4 4
	$S=3$	$M=4$	$E=5$											

	0	1	2	3	4	5	6	7	8	9	10	11	12	S E M Ans
	1	1	2	3	3	3	3	3	4	5	8	10	10	3 3 3 3
	$S=3$		$E=3$											
	$M=3$													

	0	1	2	3	4	5	6	7	8	9	10	11	12	S E M Ans
	1	1	2	3	3	3	3	3	4	5	8	10	10	3 2
	$E=2$	$S=3$												$S > E$, Terminate
	$S > E$, Terminate													

BINARY SEARCH I

Pseudocode - Binary search on an array with duplicates. Finding first occurrence

```
int binarySearch (int arr[], int T) {
```

```
    int s = 0;
```

```
    int e = arr.length - 1;
```

```
    int ans = -1;
```

```
    while (s <= e) {
```

```
        mid = (s + e) / 2;
```

```
        if (arr[mid] == T) { ans = mid; e = mid - 1; }
```

```
        else if (arr[mid] > T) { e = mid - 1; }
```

```
        else { s = mid + 1; }
```

```
}
```

```
return ans;
```

```
}
```

BINARY SEARCH 1

Pseudocode - Binary search on an array with duplicates *Finding last occurrence*

```
int binarySearch (int arr[], int T) {
```

```
    int s = 0;
```

```
    int e = arr.length - 1;
```

```
    int ans = -1;
```

```
    while (s <= e) {
```

```
        mid = (s + e) / 2;
```

```
        if (arr[mid] == T) { ans = mid; s = mid + 1; }
```

```
        else if (arr[mid] > T) { e = mid - 1; }
```

```
        else { s = mid + 1 }
```

```
}
```

Conditions for when to apply Binary Search in a Data Structure:

- ① The data structure must be sorted
- ② Access to any element of the data structure takes constant time

IMPORTANT NOTE: It is not necessary that we will ~~be~~ always be provided with a number to find in sorted array. Sometimes, there could be certain other conditions that are available in question. We need to use those conditions to find the answer. NEXT QUESTION is one such problem which doesn't have a specified no. to find but there are some conditions.

BINARY SEARCH 1

Ques 1 There is a sorted array which is K rotated. We need to find the pivot element. (2) We need to find out the index of any element in array of unique elements *check question 7 too*

K Rotated means

1	2	3	4	5	6	7
---	---	---	---	---	---	---

4	5	6	7	1	2	3
---	---	---	---	---	---	---

Kth rotated array

Either of them could be pivot element

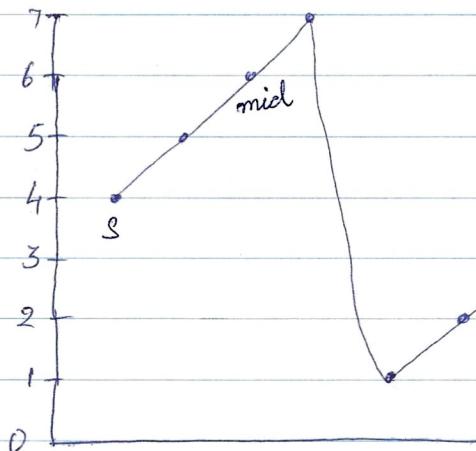
Ans Approaches

- (1) Linear Search $O(n)$
- (2) Binary Search $O(\log n)$

Binary Search

- (1) Pivot element could be found through binary search. $O(\log n)$
- (2) Once we know the pivot element we could find the index of certain element, again by binary search. $O(2 \log n)$
This element will be provided in the question.

Case 1

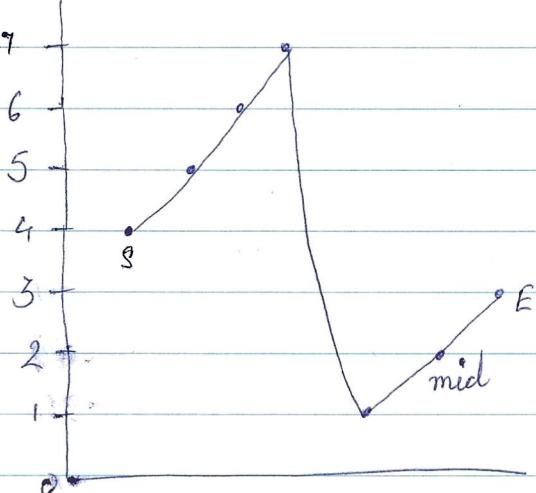


- if $(\text{mid} > s) \quad s = \text{mid} + 1;$

*- if ($\text{arr}[\text{mid}] > \text{arr}[s]$)
 $s = \text{mid} + 1;$*

BINARY SEARCH 1

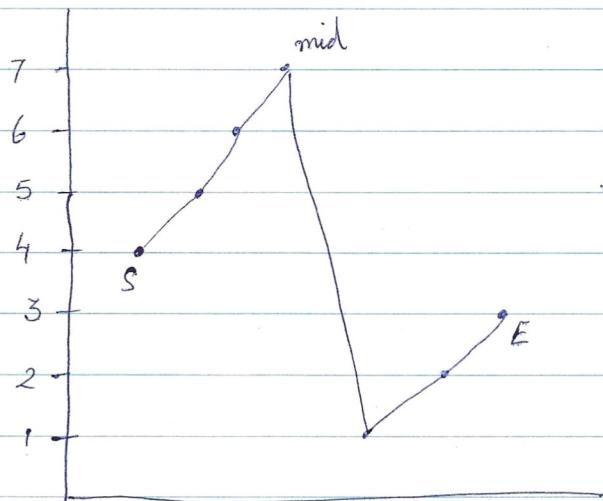
Case 2



~~if ($\text{mid} < s$) $E = \text{mid} - 1$~~

if ($\text{arr}[\text{mid}] < \text{arr}[s]$)
 $E = \text{mid} - 1$

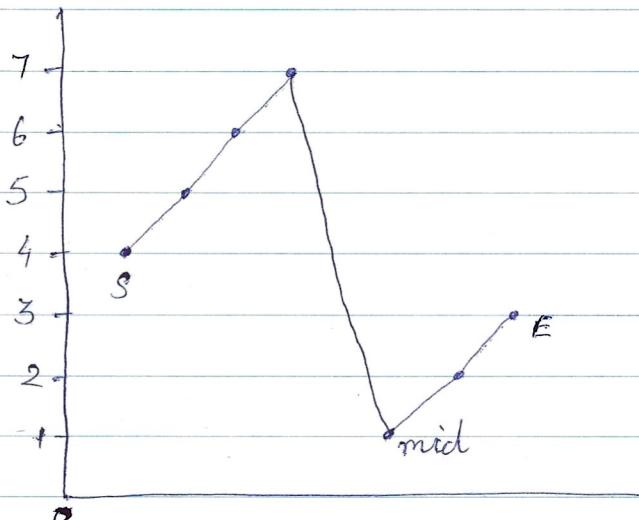
Case 3



~~if ($\text{mid} > \text{mid} + 1$) return $\text{mid} + 1$~~

if ($\text{arr}[\text{mid}] > \text{arr}[\text{mid} + 1]$)
return $\text{arr}[\text{mid} + 1]$

Case 4



~~if ($\text{mid} < \text{mid} - 1$) return mid~~

if ($\text{arr}[\text{mid}] < \text{arr}[\text{mid} - 1]$)
return $\text{arr}[\text{mid}]$

We can use above 4 cases to define the binary search conditions to find out the pivot point.

IMPORTANT: If the target is not present in the array and we want to find the potential index where it could be placed then in a general binary search algorithm start variable in the end bears the potential index where it could be placed.

BINARY SEARCH 1

Question 2 Find the expected index of a number in sorted array if it doesn't exist else return the index, if it is present. TC should be $O(\log n)$

Approaches

① Linear Search $TC - O(N)$, $SC - O(1)$

② Binary Search $TC - O(\log n)$, $SC - O(1)$

Corner Cases 1

~~arr = 1, 2, 3, 4, 6, 7, 8~~

target = 0

0 1 2 3 4 5 6
1, 2, 3, 4, 6, 7, 8
 \downarrow \downarrow \downarrow
 $S=0$ $M=3$ $E=6$
 $4 > 0$

$S=0 \ M=1 \ E=2$

$2 > 0$

$S=0$
 $E=0$
 $M=0$
 $1 > 0$

$E=-1, S=0$ (Terminate)

Answer = S

Corner Case 2

~~arr = 1, 2, 3, 4, 6, 7, 8~~

target = 9

0 1 2 3 4 5 6
1, 2, 3, 4, 6, 7, 8
 \downarrow \downarrow \downarrow
 $S=0$ $M=3$ $E=6$
 $9 > 4$
 $S=4 \ M=5 \ E=6$
 $9 > 7$

$S=6$

$E=6$

$M=6$

$9 > 8$

$S = 6 + 1 = 7$

$E = 6$ (Terminate)

Answer = S

IMPORTANT: Try to plot a graph for the problem to understand the ask of the question and solve it.

BINARY SEARCH 1

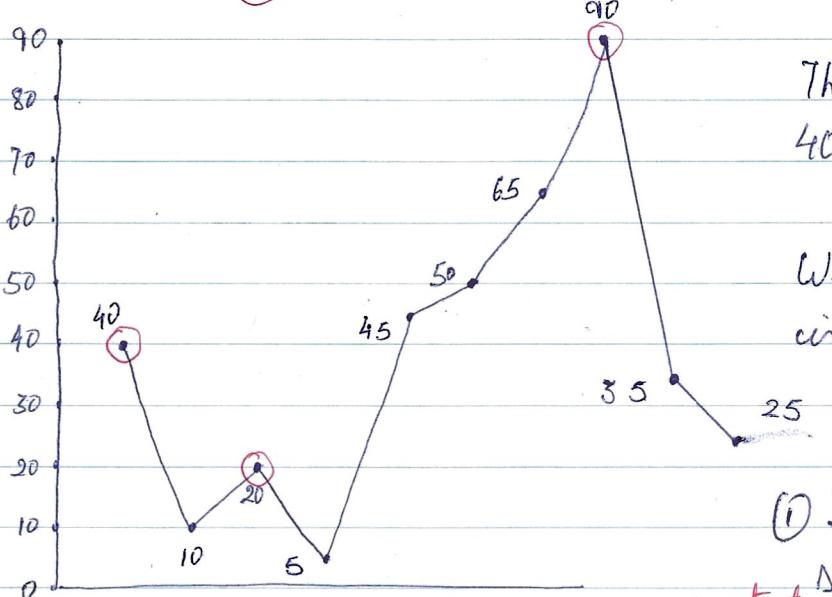
Question 3 Given an array, we need to find the peak element in the array. A peak element is an array element that is greater than its neighbours. There could be duplicates in the array. There could be multiple peak elements in an array. In that case, we will can return only one of them.

Approaches

- ① Linear Search $TC = O(n)$, $SC = O(1)$.
- ② Binary Search $TC = O(\log n)$, $SC = O(1)$

Binary Search → Intuition Building.

$$Arr = [40, 10, 20, 5, 45, 50, 65, 90, 35, 25]$$



There are 3 peak elements.
40, 20, 90

We need to write algorithm in a way that flow should move toward peak element

① If we are at 10 then we should move toward 40 because $40 > 10$

End = ~~start~~ 40 because $40 > 10$

$start = mid + 1$

② If we are at 50 then we should move toward 65 because $65 > 50$

corner case 1

③ If we reach at 0th index 40 then we can return answer as 40

corner case 2

④ In the same way if have 46 as the last element after 25 then that could be returned as a peak element.

BINARY SEARCH 1

Question 4

Given two sorted arrays arr1 and arr2 of size. Return the median of two sorted arrays.

Approaches

(1) Two pointer method to sort the array while merging them.

(2) Thereafter find the median in even and odd case differently.

(3) $TC = O(M+N)$, $SC = O(M+N)$ [A new array will be created]

(2) Two pointer method

(a) We will first find the total no. of elements that merged array could have but we will not create one.

(b) Check even or odd length.

(c) We will maintain the variables for both even length or odd length to record the indices value.

(d) Iterate over both the arrays as we did in approach

(1)

(e) If counter matches the index needed to calculate median, we will store the $arr[i]$ in a variable.

(f) In the end, we will calculate the median and return it.

$TC = O(M+N)$, $SC = O(1)$

(3) Binary Search.

Median even length -

$$\frac{(n/2 - 1)^{\text{th}} \text{ ele} + (n/2)^{\text{th}} \text{ ele}}{2}$$

Median odd length - $(\text{Math.floor}(n/2) + 1)^{\text{th}} \text{ ele}$

$$\text{arr1} = [7, 12, | 14, 15] \rightarrow 4$$

$$\text{arr2} = [1, 2, 3, | 4, 9, 11] \rightarrow 6$$

$$S=0$$

$$E=4$$

we can take minimum 0 and maximum 4 elements from arr1

$$\text{medianPos} = \frac{10}{2} = 5$$

EVEN

$$\text{mid1} = \frac{S+E}{2} = \frac{0+4}{2} = 2$$

$$\text{mid2} = 5 - 2 = 3$$

$$L1 = 12 \quad R1 = 14$$

$$L2 = 3 \quad R2 = 4$$

$$\boxed{\begin{array}{l} L1 < R2 \text{ } \checkmark \\ \text{---} \\ E = \text{mid1} - 1 \\ = 2 - 1 = 1 \\ \text{condition} \end{array}}$$

$$\text{arr1} = [| 7, 12, 14, 15]$$

$$\text{arr2} = [1, 2, 3, 4, | 9, 11]$$

$$S=0, E=1 \text{ This is condition mid1} = \frac{0+1}{2} = \frac{1}{2} = 0$$

$$\text{After min and max condition } \text{mid2} = 5 - 0 = 5$$

$$L1 = \text{int-min} \quad R1 = 7$$

$$L2 = 9 \quad R2 = 11$$

$$\boxed{\begin{array}{l} \min \\ L1 < R2 \text{ } \checkmark \\ L2 < R1 \text{ } \times \\ S = \text{mid1} + 1 \\ = 0 + 1 = 1 \\ \text{condition} \end{array}}$$

$$\text{arr1} = [7, | 12, 14, 15]$$

$$\text{arr2} = [1, 2, 3, 4, | 9, 11]$$

$$S=1 \quad E=1$$

$$\text{mid1} = \frac{1+1}{2} = 1$$

$$\text{mid2} = 5 - 1 = 4$$

$$L1 = 7 \quad R1 = 12$$

$$L2 = 4 \quad R2 = 9$$

$$\boxed{\begin{array}{l} L1 < R2 \text{ } \checkmark \\ L2 < R1 \text{ } \checkmark \\ \text{condition} \end{array}}$$

Satisfied

$$\text{(Even) Median} = \frac{\text{Math.max}(L1, L2) + \text{Math.min}(R1, R2)}{2}$$



(P1) 12 marks

NOTE: Out of 2 sorted arrays provided use the smaller array to run binary search on.

IMPORTANT: Always try to decrease the search space.

BINARY SEARCH 1

$$\text{arr 1} = [7, 12, 14, 15, 16] \rightarrow 5 \quad \text{arr 2} = [1, 2, 3, 4, 9, 11] \rightarrow 6$$

odd

$$5+6=11 \quad \text{medianPos} = \frac{11+1}{2} = \frac{12}{2} = 6$$
$$S = 0 \quad E = 5 \quad \text{mid 1} = \frac{S+E}{2} = \frac{0+5}{2} = \frac{5}{2} = 2.5$$
$$\text{mid 2} = 6 - 2 = 4$$

$$L1 = 12 \quad R1 = 14$$

$$L2 = 4 \quad R2 = 9$$

$$L1 < R2 \quad \text{X}$$

$$E = \text{mid 1} - 1 = 2 - 1 = 1$$

condition

$$\text{arr 1} = [7, 12, 14, 15, 16]$$
$$\text{arr 2} = [1, 2, 3, 4, 9, 11]$$

$$\text{mid 1} = \frac{0+1}{2} = 0.5 = 0$$

$$\text{mid 2} = 6 - 0 = 6$$

$$S = 0 \quad E = 1$$

$$L1 = \text{int_min} \quad R1 = 7$$

$$L2 = 11 \quad R2 = \text{int_max}$$

$$L1 < R2 \quad \text{C}$$

$$L2 < R1 \quad \text{X}$$

$$S = \text{mid 1} + 1 = 0 + 1 = 1$$

Condition

$$\text{arr 1} = [7, 12, 14, 15, 16]$$
$$\text{arr 2} = [1, 2, 3, 4, 9, 11]$$
$$S = 1 \quad E = 1$$

$$\text{mid 1} = \frac{1+1}{2} = 1$$

$$\text{mid 2} = 6 - 1 = 5$$

(odd) Median = Max(L1, L2)



$$L1 = 7 \quad R1 = 12$$

$$L2 = 9 \quad R2 = 11$$

$$L1 < R2 \quad \text{C}$$

$$L2 < R1 \quad \text{C}$$

Condition satisfied

BINARY SEARCH 1

Question 2.1.5 Find the number or index of the number which is just greater than the given number.

arr = 1, 3, 5, 5, 5, 6, 6

num = 5

0	1	2	3	4	5	6
1, 3, 5, 5, 5, 6, 6						
S=0	Nid=3	E=6				

STEP 1

$$\text{Mid} = \frac{S+E}{2} = \frac{0+6}{2} = 3$$

arr [3] > 5

arr [mid] > num
3 > 5

$$S = \text{mid} + 1 \\ = 3 + 1 = 4$$

STEP 2

0	1	2	3	4	5	6
1, 3, 5, 5, 5, 6, 6						
S=4					E=6	
					Mid=5	

$$\text{Mid} = \frac{S+E}{2} = \frac{4+6}{2} = 5$$

arr [mid] > num

arr [5] > 5
6 > 5

$$E = \text{mid} - 1 = 5 - 1 = 4$$

STEP 3

0	1	2	3	4	5	6
1, 3, 5, 5, 5, 6, 6						
S=4						
E=4						
Mid=4						

$$\text{Mid} = \frac{S+E}{2} = \frac{4+4}{2} = 4$$

arr [mid] > num

arr [4] > 5
5 > 5

$$S = \text{mid} + 1 = 4 + 1 = 5$$

STEP 4

S > E Terminate

0	1	2	3	4	5	6
1, 3, 5, 5, 5, 6, 6						
					E=4 S=5	

return S;

BINARY SEARCH 1

arr = 1, 3, 5, 5, 5, 6, 6

0 1 2 3 4 5 6

1, 3, 5, 5, 5, 6, 6

↓ ↓ ↓

S=0 E=6

Mid = 3

num = 4

STEP 1

$$\text{mid} = \frac{S+E}{2} = \frac{0+6}{2} = 3$$

arr [mid] > num

arr [3] > num

$$5 > 4$$

$$E = \text{mid} - 1 = 3 - 1 = 2$$

STEP 2

$$\text{mid} = \frac{0+2}{2} = 1$$

arr [mid] $\cancel{>} \text{ num}$

arr [1] $\cancel{>} 4$

$$3 \cancel{>} 4$$

$$S = \text{mid} + 1 = 1 + 1 = 2$$

0 1 2 3 4 5 6

1, 3, 5, 5, 5, 6, 6

↓

S=2

E=2

mid = 2

STEP 3

$$\text{mid} = \frac{2+2}{2} = 2$$

arr [mid] > num

arr [2] > 4

$$5 > 4$$

$$E = \text{mid} - 1 = 2 - 1 = 1$$

0 1 2 3 4 5 6

1, 3, 5, 5, 5, 6, 6

↓

E=1

S=2

STEP 4

S > E

2 > 1

Terminate

Return S;

BINARY SEARCH I

Question 6

Given a matrix of integers A of size $N \times M$ in which each row is sorted. Find and return the overall median of matrix A .

$N \times M$ is always odd

Note: No extra memory is allocated

Note: Rows are numbered from top to bottom and columns are numbered from left to right.

Approaches

With extra Space

- ① We can make an extra array to put all the elements of matrix into it. $TC = O(m \times n)$
- ② After that we will sort the array. $TC = m \cdot n \quad SC = O(m \cdot n)$
- ③ We will find out the median from the sorted array.
- ④ $TC = m \cdot n \quad SC = O(m \cdot n)$

Without extra Space Binary Search

- ① Firstly, we will find the median position (index in array)
- ② We will fetch the smallest and the highest no. in the matrix.
- ③ Now, we will run a binary search using smallest and highest no.
- ④ In the binary search, we will make comparison between the mid and median position calculated in step 1. no. of elements smaller than

$$10^9 = 2^{32} - \text{Max range of integer}$$

BINARY SEARCH I

⑤ To calculate the no. of elements smaller than mid, we again need to run a binary search inside the previous binary search loop.

$$\begin{aligned} \text{⑥ TC} &= \log_2(2^{32}) \times N \times \log_2 M \quad SC = O(1) \\ &= 32 \times N \times \log_2 M \end{aligned}$$

Example: $n=3$

$$\text{arr} = \begin{bmatrix} 0 & 1 & 2 \\ [2, 3, 5] & [2, 3, 4] & [1, 7, 9] \end{bmatrix} \quad \begin{array}{l} \min \text{ smallest no} = 1 \\ \max \text{ highest no} = 10^9 \\ \text{median Pos} = m \times n / 2 \end{array}$$

$m=3$

OR

$$\begin{array}{l} \boxed{\min} \text{ smallest no} = 1 \quad \begin{array}{l} \text{smallest no. in} \\ \text{matrix} \end{array} \\ \boxed{\max} \text{ highest no} = 9 \quad \begin{array}{l} \text{highest no. in matrix} \\ \text{median Pos} = m \times n / 2 = 3 \times 3 / 2 = 9 / 2 \\ = 4 \end{array} \end{array}$$

$$\text{STEP 1: } \boxed{\min} = 1, \boxed{\max} = 9, \text{ mid} = 1 + 9 / 2 = 10 / 2 = 5$$

» No. of elements smaller than equal to 5 are 1, 2, 2, 3, 3, 4, 5 which are 7
 Since $7 > \boxed{\max}$, max will be updated to mid - 1
 $\boxed{\max} = \text{mid} - 1 = 5 - 1 = 4$

$$\text{STEP 2: } \min = 1, \max = 4, \text{ mid} = 1 + 4 / 2 = 5 / 2 = 2$$

» No. of elements smaller than equal to 2 are 1, 2, 2 which are 3

Since $3 < \text{median Pos}[4]$, $\min = \max$ will be updated to mid + 1
 $\min = \text{mid} + 1 = 2 + 1 = 3$

BINARY SEARCH 1

STEP 3: $\min = 1 \boxed{3}$, $\max = 4$, $\text{mid} = \frac{3+4}{2} = 7/2 = \boxed{3}$

» No. of elements smaller than equal to 3, 1, 2, 2, 3, 3 which are 5 ^{medianPos}

Since $5 > 4$, max will be updated to mid - 1

$$\max = \text{mid} - 1 = \boxed{3} - 1 = \boxed{2}$$

STEP 4: $\min = \boxed{3}$, $\max = \boxed{2}$

$\min > \max$ TERMINATE

ANSWER = $\boxed{\min = 3}$

Difference of way of calculation of ^{medianPos} mid for question 5 and 6 in case of odd no. of elements in search space in question

» In que 5, we find the no. of elements $\leq \text{medianPos}$

Que 7 There is a sorted array which is K rotated. We need to find out the index of any element in Array of duplicate elements. Check Question 1 too. If element exists return true otherwise false.

K rotated means

0	0	1	2	2	5	6
---	---	---	---	---	---	---

2	5	6	0	0	1	2
---	---	---	---	---	---	---

 target = 0

Ans Approaches

① Linear Search - TC - $O(n)$

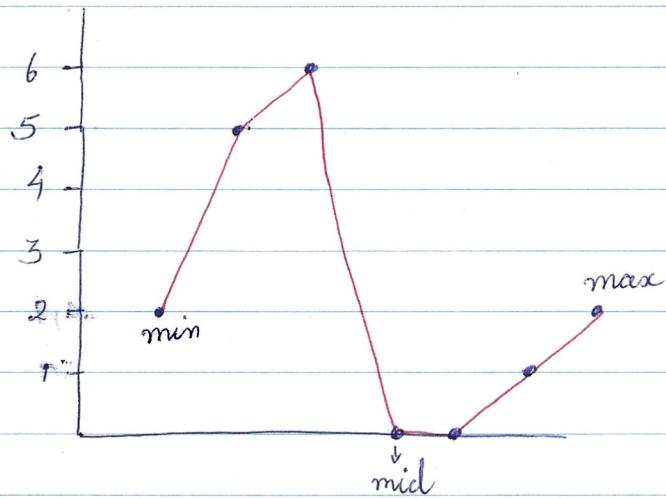
② Binary Search - TC - $O(\log n)$ for the best and average case.
 $O(n/2)$ for the worst case. Here, N = size of the given array.

Reason: In the best and average scenarios, the binary search algorithm is primarily utilized and hence the time complexity is $O(\log N)$.

BINARY SEARCH 1

However, in the worst-case scenario, where all array elements are the same but not the target (e.g. given array = [3, 3, 3, 3, 1, 3, 3], target = 1), we continue to reduce the search space by adjusting the low and high pointers until they intersect. This worst situation incurs a time complexity of $O(N/2)$.

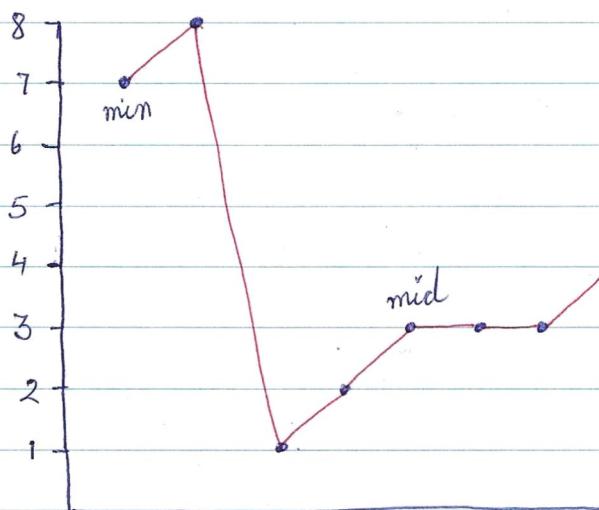
Best and average case



min mid max
0 3 6
[2, 5, 6, 0, 0, 1, 2]
target = 0

Best Case

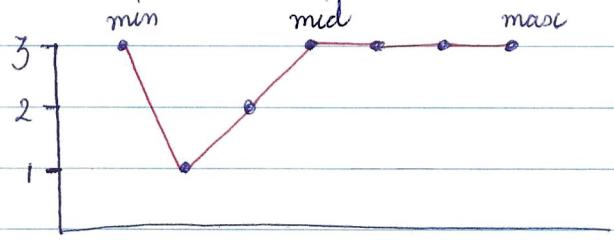
Answer = True
arr[mid] == target



Average Case
min mid max
0 4 9
[7, 8, 1, 2, 3, 3, 3, 4, 5, 6]
target = 3

~~arr[mid] < arr[max]~~
arr[mid] == target
Answer = True.

BINARY SEARCH I



Worst Case

min	mid	max
0	3	6

[3, 1, 2, 3, 3, 3, 3]

Target = 1

$$\text{arr[min]} == \text{arr[mid]} == \text{arr[max]}$$

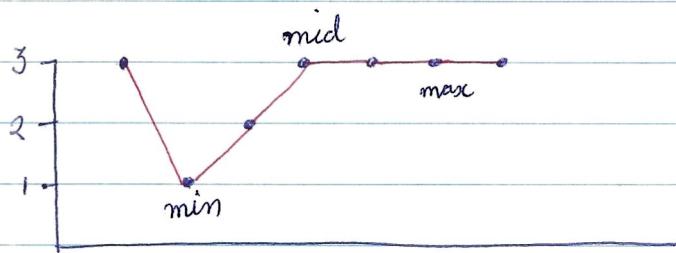
Extra Step

min ++;

max --;

min	mid	max
1	3	5

[3, 1, 2, 3, 3, 3, 3]



$$\text{arr[min]} < \text{arr[mid]}$$

~~target >= arr[min]~~

~~arr[min] <= target <= arr[max]~~

mid max = mid - 1

mid
min max

[3, 1, 2, 3, 3, 3, 3]

$$\text{arr[mid]} == \text{target}$$

return true;



BINARY SEARCH 1

Question 8 Find minimum in Rotated sorted array with duplicate elements.

1	2	3	4	5	6	7
---	---	---	---	---	---	---

Rotated sorted array =

5	6	7	1	2	3	4
---	---	---	---	---	---	---

Rotated sorted array =

4	6	7	1	3	3	4
---	---	---	---	---	---	---

with duplicate elements

Answer Approaches

- ① Linear Search - $O(n)$ - TC
- ② Binary Search - TC - $O(\log n)$ in Average case
 $O(1)$ in best case
 $O(n/2)$ in ~~worst~~ worst case.

Average Case

arr =

0	1	2	3	4	5	6
3	4	4	6	7	1	3

$\rightarrow \min = 0, \max = 6, \text{mid} = 3, \text{answer} = \text{int} \cdot \max$

arr[mid] \geq arr[min] Condition 3

else answer = arr[min];

min = mid + 1;

$3 < \text{int} \cdot \max$

$\rightarrow \min = 4, \max = 6, \text{mid} = 5, \text{answer} = 3$

arr[mid] \leq arr[max] Condition 4 else

answer = arr[mid]

max = mid - 1;

BINARY SEARCH 1

1 < 3

→ min = 4, max = 4, mid = 4, answer = 1

arr[min] == arr[mid] == arr[max] Condition 2

min++;

max--;

answer = min(answer, arr[mid]);

→ min = 5, max = 3,

min > max Terminate.

return answer;

Best Case — Array is not rotated at all

arr = [0, 1, 2, 3, 4];

→ min = 0, max = 4, mid = 2, answer = arr[mid]

arr[min] <= arr[max] Condition 1

answer = arr[min];

break;

return answer;

BINARY SEARCH 1

Worst Case

$$\text{arr} = [\underset{0}{3}, \underset{1}{3}, \underset{2}{3}, \underset{3}{3}, \underset{4}{1}, \underset{5}{3}, \underset{6}{3}]$$

$\rightarrow \min = 0, \max = 6, \text{mid} = 3, \text{answer} = \text{int} \cdot \max$

$$\text{arr}[\min] == \text{arr}[\text{mid}] == \text{arr}[\max]$$

$\min++;$

$\max--;$

$\rightarrow \min = 1, \max = 5, \text{mid} = 3, \text{answer} = \text{int} \cdot \max$

$$\text{arr}[\min] == \text{arr}[\text{mid}] == \text{arr}[\max]$$

$\min++;$

$\max--;$

$\rightarrow \min = 2, \max = 4, \text{mid} = 3, \text{answer} = \text{int} \cdot \max$

$$\text{arr}[\min] <= \text{arr}[\text{mid}], \\ \text{answer} = \text{arr}[\min],$$

$$\text{arr}[\text{mid}] >= \text{arr}[\min]$$

$\text{answer} = \text{arr}[\min];$

$\min = \text{mid} + 1;$

$\rightarrow \min = 4, \max = 4, \text{mid} = 4, \text{answer} = 3$

$$\text{arr}[\min] == \text{arr}[\text{mid}] == \text{arr}[\max]$$

$\text{answer} = \text{arr}[\text{mid}] ; // 1 < 3$

$\min++;$

$\max--;$

$\rightarrow \min = 5, \max = 3 \quad \min > \max, \text{terminate}$

return answer;

BINARY SEARCH 1

Question 9 You are given a sorted array consisting of only integers where every element appears exactly twice, except for one element which appears exactly once.

Return the single element that appears only once.

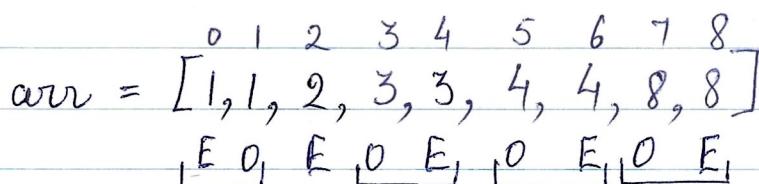
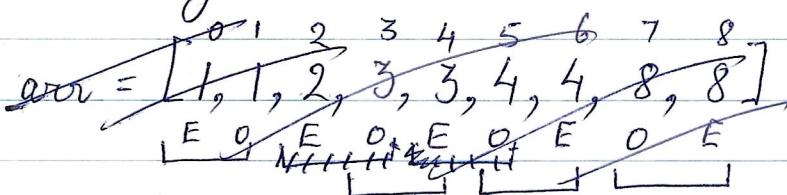
Your solution must run in $O(\log n)$ time and $O(1)$ space.

Eg: $\text{arr} = [1, 1, 2, 3, 3, 4, 4, 8, 8]$

Answer

Approaches

- ① Linear Search - $TC = O(n)$, $SC = O(1)$
- ② Linear Search with hash map - $TC = O(n)$, $SC = O(n)$
- ③ Binary Search :



$$\rightarrow \min = 0, \max = 8, \text{mid} = 4$$

$$\text{arr}[\text{mid}] == \text{arr}[\text{mid}-1] \& \& \text{mid}-1 \% 2 != 0$$

$$\text{max} = \text{mid} - 1$$

BINARY SEARCH 1

→ min = 0, max = 3, mid = 1

arr[mid] == arr[mid - 1] && mid - 1 % 2 == 0

min = mid + 1

→ min = 2, max = 3, mid = 2

arr[mid] != arr[mid - 1] && arr[mid] != arr[mid + 1]

return arr[mid];

arr = [1, 1, 2, 2, 3, 3, 4, 5, 5]
~~0 F 0~~

E 0 E 0 E 0 E 1 0 E

→ min = 0, max = 8, mid = 4

arr[mid] == arr[mid + 1] && mid + 1 % 2 != 0

min = mid + 1

→ min = 5, max = 8, mid = 6

arr[mid] != arr[mid - 1] && arr[mid] != arr[mid + 1]

return arr[mid];