

Name :

Roll No. :

Invigilator's Signature :

CS/B.TECH(OLD)/CSE,IT/SEM-3/M-301/2011-12

2011

MATHEMATICS

Time Allotted : 3 Hours

Full Marks : 70

The figures in the margin indicate full marks.

*Candidates are required to give their answers in their own words
as far as practicable.*

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

- i) If the R.V. X has *p.d.f.* $f(x) = \frac{1}{2}x$, $0 \leq x \leq 2$, find the mean value of X .

a) $\frac{4}{3}$

b) $\frac{7}{8}$

c) $\frac{3}{4}$

d) $\frac{2}{3}$.

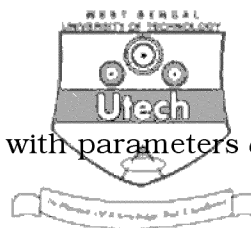
- ii) If $x = 4y + 5$ and $y = kx + 4$ be two regression equations of x on y and y on x respectively, then the value of k lies in the interval

a) $[4, 5]$

b) $[0, 4]$

c) $[0, 5]$

d) none of these.



- iii) The variable of a uniform distribution with parameters a and b is

- a) $\frac{a+b}{2}$ b) $\frac{a-b}{2}$
 c) $\frac{(b-a)^2}{12}$ d) $\frac{(b+a)^2}{12}$.

- iv) Probability of type II Error in the case of testing hypothesis, is (notations have their usual meanings)

- a) Probability of rejection of $H_0(\theta \neq \theta_0)$ on the hypothesis that H_0 is true
 b) Probability of acceptance of $H_0(\theta = \theta_0)$ on the hypothesis that $(\theta \neq \theta_0)$
 c) Level of significance of the test
 d) None of these.

- v) In Tchebycheff's Inequality, where X is an R.V. having mean m and S.D. σ , then for any $\epsilon > 0, P(|X - m| \geq \epsilon) \leq$

- a) σ/ϵ b) σ^2/ϵ
 c) σ^2/ϵ^2 d) σ/ϵ^2

- vi) Under SRSWOR total possible samples of size two from the Population $\{1, 2, 3\}$ are

- a) $\{1, 1\}, \{2, 2\}$ b) $\{1, 1\}, \{2, 2\}, \{3, 3\}$
 c) $\{1, 2\}, \{1, 3\}, \{2, 3\}$ d) $\{2, 2\}, \{3, 3\}$

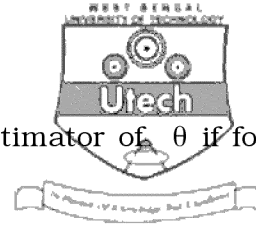
- a) $\frac{12}{25}$ b) $\frac{6}{25}$
- c) $\frac{18}{25}$ d) none of these.

- $$f(x)=\begin{cases} 1 & \text{for } 0 \leq x \leq 1 \\ 0 & \text{otherwise} \end{cases}$$

a) $\frac{1}{2}$ b) $\frac{1}{3}$
c) $\frac{1}{4}$ d) 1.

- a) $e^{\lambda t}$ b) $e^{(e^t+1)}$
c) $e^{\lambda(e^t-1)}$ d) none of these.

- a) $\frac{10}{9}S^2$ b) S^2
- c) $\frac{9}{10}S^2$ d) $\frac{11}{9}S^2$



xi) A statistic t_n is called a consistent estimator of θ if for arbitrary $\varepsilon > 0$

a) $\lim_{n \rightarrow \infty} \{ |t_n - \theta| < \varepsilon \} = 0$

b) $\lim_{n \rightarrow \infty} \{ |t_n - \theta| > \varepsilon \} = 0$

c) $\lim_{n \rightarrow \infty} \{ |t_n - \theta| = \varepsilon \} = 0$

d) none of these.

xii) If $-4 < t < 4$ be a region of acceptance for a statistical test of a hypothesis then the critical region of the test is given by

a) $t < -4$

b) $t > 4$

c) $-4 < t < 4$

d) none of these.

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. The distribution function $F(x)$ of a random variable x is defined as follows :

$$F(x) = A, \quad -\infty < x < -1$$

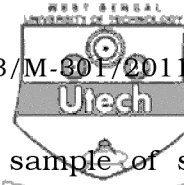
$$= B, \quad -1 \leq x < 0$$

$$= C, \quad 0 \leq x < 2$$

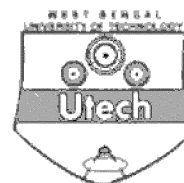
$$= D, \quad 2 \leq x < \infty$$

where A, B, C, D are constants. Determine the value of

$$A, B, C, D; \text{ given } P(X=0) = \frac{1}{6} \text{ and } P(X>1) = \frac{2}{3}.$$



3. Show that if S^2 be sample variance of a sample of size n drawn from a population with mean μ and S.D. σ then $E(S^2) = ((n-1)/n)\sigma^2$ (Where population size is infinite or the sample is drawn with replacement).
4. If the weekly wage of 10,000 workers in a factory follows normal distribution with mean and S.D. Rs. 70 and Rs. 5 respectively, find the expected number of workers whose weekly wages are (i) between Rs. 66 and Rs. 72, (ii) less than Rs. 66 and (iii) More than Rs. 72. [Given that $\frac{1}{\sqrt{2\pi}} \int_0^z e^{-t^2/2} dt = 0.1554$ and 0.2881 according as $z = 0.4$ and $z = 0.8$].
5. Ten individuals are chosen at random from a normal $((m, \sigma))$ population and their heights in inches are found to be 63, 66, 63, 67, 68, 69, 70, 71, 72, 71. On the basis of the above data, obtain 95% confidence interval for the parameter m when σ is unknown. [Given $P(\chi^2 > 2.7) = 0.975$ and $P(t > 2.262) = 0.025$ for 9 d.o.f]
6. A random sample with observations 65, 71, 64, 71, 70, 69, 64, 63, 67, 68 is drawn from a normal population with S.D. $\sqrt{7.056}$. Test the hypothesis that the population mean is 69 at 1% level of significance. [Given that $P(0 < z < 2.58) = 0.495$].



GROUP – C

(Long Answer Type Questions)

Answer any *three* of the following. $3 \times 15 = 45$

7. a) A random variable X has a density function $f(x)$ is given by $f(x) = e^{-x}$, $x \geq 0$

$= 0$, elsewhere.

Show that Techebycheff's inequality gives

$$P(|X-1| \geq 2) \leq \frac{1}{4} \text{ and show that actual probability is } e^{-3}.$$

6

- b) If $x_1, x_2, x_3, x_4, x_5, x_6$ be an independent simple random sample from a normal population with unknown variable σ^2 , Find k so that $k[(x_1 - x_2)^2 + (x_3 - x_4)^2 + (x_5 - x_6)^2]$ is an unbiased estimator.

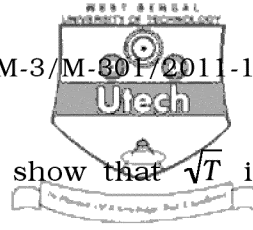
5

- c) If X is uniformly distributed in $(-1, 1)$, find the density of $|X|$.

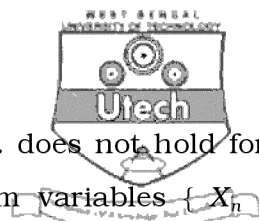
4

8. a) Show by Techebycheff's inequality that if a die is thrown 3,600 times, the probability of number of sixes lies between 550 and 650 is at least $\frac{4}{5}$.

- b) If X_n is mutually independent and identically distributed random variable with mean m and finite variance σ^2 , and $S_n = X_1 + X_2 + \dots + X_n$, then prove that WLLN (Weak Law of Large Numbers) holds for the sequence $\{S_n\}$.



- c) If T is an unbiased estimator of θ , show that \sqrt{T} is biased estimate of $\sqrt{\theta}$.
9. a) Find the regression coefficient of y on x , of x on y and the correlation coefficient between x and y from the following values :
 $\sum xy = 1500, \bar{x} = 15, \bar{y} = 12, \sigma_x = 6.4, \sigma_y = 9$ and number of observations is 10 where the notation have their usual meanings.
- b) Find the moment generating function of Normal Distribution $N(\mu, \sigma)$ with parameters μ and σ and from it determine its mean and variance. 5 + 10
10. a) X follows $Bi(3, p)$ i.e. Binomial distribution with parameters 3 and p where $0 < p < 1$.
 $H_0 : p = \frac{1}{4}, H_1 : p = \frac{3}{4}$
 A test rejects H_0 if $X \geq 2$. Find the Type I and Type II errors of this test.
- b) A company claims that its light bulbs are superior to those of a competitor on the basis of a study which showed that a sample of 40 of its bulbs had an average life time of 628 hours of continuous use with a standard deviation of 27 hours, while a sample of 30 bulbs made by the competitor had an average life time of 619 hours of continuous use with a standard deviation of 25 hours. Check at 5% level of significance whether this claim is justified. 8 + 7



11. a) Show that the weak law of large Nos. does not hold for the sequence of independent random variables $\{X_n\}$ with the distribution given as $P(X_n = \pm n) = \frac{1}{2}$.
- b) For the variables x and y the equations of the regression lines are $4x - 5y + 33 = 0$ and $20x - 9y = 107$. Identify the regression line of y on x and that of x on y . Find the standard deviation of y .
- c) The probability of a missile hitting a target is $\frac{1}{4}$. How many missiles must be sent so that the probability of hitting the Target at least once is greater than $\frac{2}{3}$?

5 + 5 + 5
