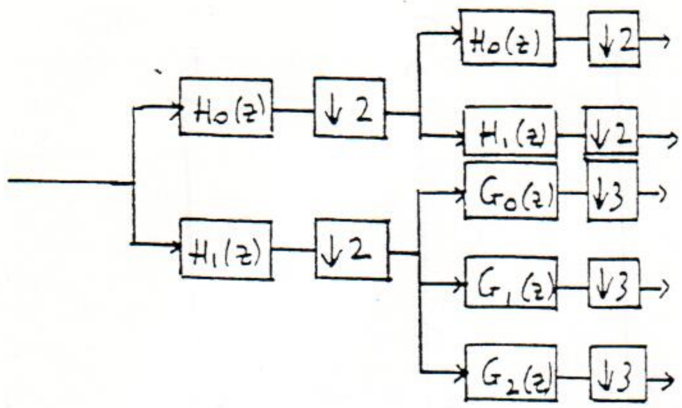
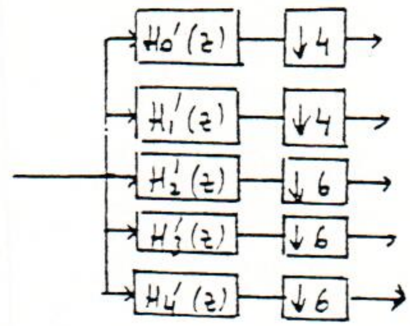
Problem 3 (25pt)

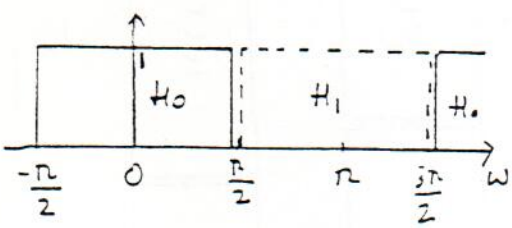
Consider the following multirate systems:

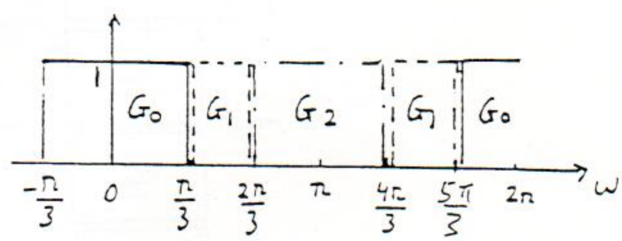




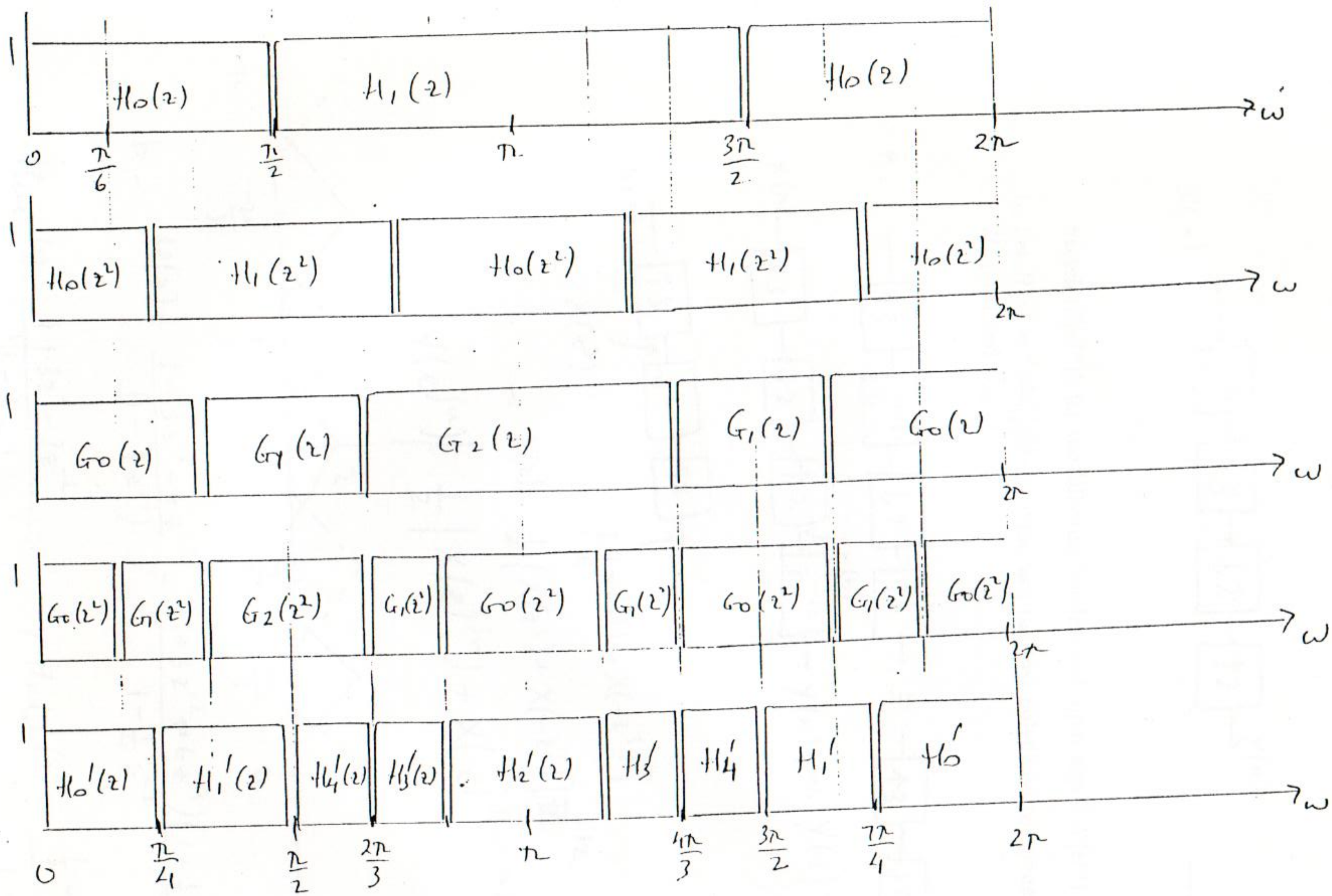
a. Find $H_k'(z)$ in terms of $H_k(z)$ and $G_k(z)$?

b. Sketch the ideal responses of $H_k'(z)$ if $H_k(z)$ and $G_k(z)$ are the following ideal filters:



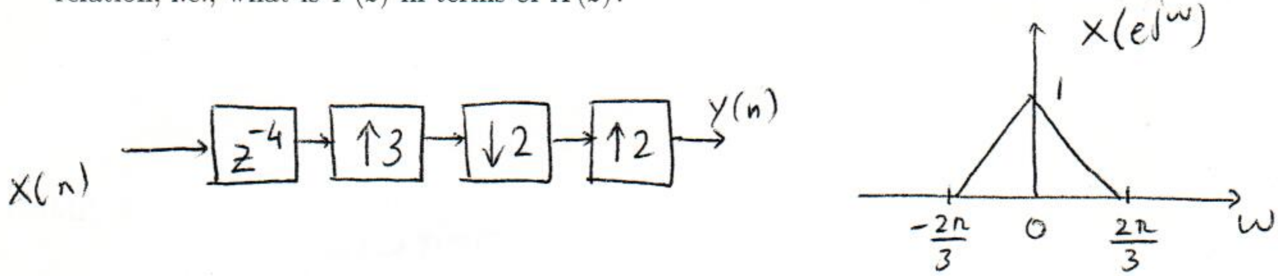


a -
$$\begin{cases} H_0'(z) = H_0(z)\, H_0(z^2) \\ H_1'(z) = H_0(z)\, H_1(z^2) \\ H_2'(z) = H_1(z)\, G_0(z^2) \\ H_3'(z) = H_1(z)\, G_1(z^2) \\ H_4'(z) = H_1(z)\, G_2(z^2) \end{cases}$$


1
$H_0(z)$
$H_1(z)$
$H_0(z)$
0
$\frac{\pi}{6}$
$\frac{\pi}{2}$
π
$\frac{3\pi}{2}$
2π
ω
$H_0(z^2)$
$H_1(z^2)$
$H_0(z^2)$
$H_1(z^2)$
$H_0(z^2)$
$G_0(z)$
$G_1(z)$
$G_2(z)$
$G_1(z)$
$G_0(z)$
$G_0(z^2)$
$G_1(z^2)$
$G_2(z^2)$
$G_1(z^2)$
$G_0(z^2)$
$G_1(z^2)$
$G_0(z^2)$
$G_1(z^2)$
$G_0(z^2)$
$H_0'(z)$
$H_1'(z)$
$H_4'(z)$
$H_3'(z)$
$H_2'(z)$
H_3'
H_4'
H_1'
H_0'
$\frac{\pi}{4}$
$\frac{2\pi}{3}$
$\frac{4\pi}{3}$
$\frac{7\pi}{4}$

Problem 4 (25pt)

a. Consider the multirate system shown below. Find the input-output relation, i.e., what is $Y(z)$ in terms of $X(z)$?



Sketch $|Y(e^{j\omega})|$, for the following bandlimited input signal $X(e^{j\omega})$.

b. Let $H(z) = \frac{1+3z^{-2}-6z^{-3}}{1-0.5z^{-1}}$. What are the two polyphase components $H_{even}(z)$ and $H_{odd}(z)$?

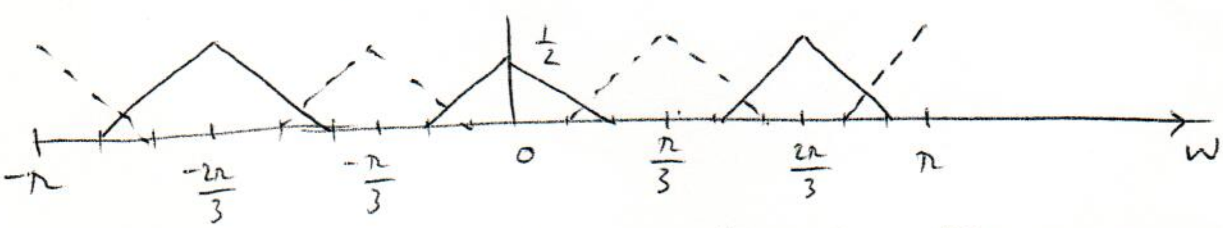
a. $\rightarrow \boxed{\uparrow 3} \rightarrow \boxed{z^{-12}} \rightarrow \boxed{\downarrow 2} \rightarrow \boxed{\uparrow 2} \rightarrow \;\equiv\; \rightarrow \boxed{\uparrow 3} \rightarrow \boxed{\downarrow 2} \rightarrow \boxed{z^{-6}} \rightarrow \boxed{\uparrow 2} \rightarrow$

$x(n) \rightarrow \boxed{\uparrow 3} \rightarrow \boxed{\downarrow 2} \rightarrow \boxed{\uparrow 2} \xrightarrow{u(n)} \boxed{z^{-12}} \rightarrow y(n) \Rightarrow Y(z) = z^{-12}\, U(z)$

$X(z) \rightarrow \boxed{\uparrow 3} \xrightarrow{X(z^3)} \boxed{\downarrow 2} \rightarrow \boxed{\uparrow 2} \rightarrow \frac{1}{2}\left[X(z^3) + X(-z^3)\right]$

$$\Rightarrow Y(z) = \frac{1}{2}\left[X(z^3) + X(-z^3)\right] z^{-12}$$

$$|Y(e^{j\omega})| = \frac{1}{2}\left|X(e^{j3\omega}) + X(e^{j3(\omega-\pi)})\right|$$



b. $$H(z) = \frac{1+3z^{-2}-6z^{-3}}{(1-\frac{1}{2}z^{-1})} = \frac{(1+3z^{-2}-6z^{-3})(1+\frac{1}{2}z^{-1})}{1-\frac{1}{4}z^{-2}} = \frac{1+\frac{1}{2}z^{-1}+3z^{-2}-9z^{-3}-3z^{-4}}{1-\frac{1}{4}z^{-2}}$$

$$H_{even}(z) = \frac{1+3z^{-1}-3z^{-2}}{1-\frac{1}{4}z^{-1}} \; ; \; H_{odd}(z) = \frac{\frac{1}{2}-\frac{9}{2}z^{-1}}{1-\frac{1}{4}z^{-1}}$$