Identification of Wavelets and Filter Bank Coefficients in Musical Instruments

M.S.Sinith, Madhavi Nisha Nair, Niveditha P. Nair, Parvathy S.
sinith@ieee.org, madhavinair88@gmail.com, niveditha024@gmail.com, parvathy_sreekumar2003@yahoo.com
Dept. of Electronics and Communication Engineering
Amrita Vishwa Vidyapeetham, Amritapuri, Kerala, India

Abstract

In this paper, we present a technique to identify the scaling function and the wavelet functions of the wavelets present in musical instruments, violin and flute. We have used the NLMS algorithm to identify the filter bank coefficients of wavelet-like elements, found repeating in musical notes of the instruments. Scaling functions of the standard wavelets are also found out by an iterative manner using NLMS algorithm.

1. Introduction

Wavelets form an integral part of digital signal processing today. Wavelets match a specific signal like a musical note played on an instrument, patterns [1] etc. In widely adopted Fourier representation in signal processing, one can get the features of the signal either in Time Domain or in Frequency Domain, one at a time. Both are extensively used in analysis, design and various applications. However there are many instances in which the localization in time as well as localization in frequency, both are required simultaneously. Short duration signals need to be localized in time and small bandwidth signals localized in frequency. In musical signals, small duration signals or small bandwidth musical pieces are placed at an effective temporal position to give special effects. They need to be captured in time as well as in frequency simultaneously. Fourier representation is not suited for such requirements. Wavelet transform of finite energy can effectively capture the above requirement. .Wavelet transform replaces Fourier transform's sinusoidal waves by a family generated by translations and dilations of a window called wavelet. The idea of multiresolution analysis is studying signals at different scales of resolution. Wavelets are manipulated in two ways. The first one is translation where the position of the wavelet is changed along the time axis. The second one is scaling. Scaling means changing the frequency of the signal. To understand wavelet analysis, two special functions: the (1) wavelet function and (2) the scaling function are used. They have unique expressions:

$$\Psi(t)=\sum_{-\infty}^{\infty} g(n)\Phi(2t-n) \qquad (1)$$

$$\Phi(t)=\sum_{-\infty}^{\infty} h(n)\Phi(2t-n) \qquad (2)$$

where $$g(n)=(-1)^{1-n}h(1-n) \qquad (3)$$

2. Filter bank theory

As studies reveal wavelets can be generated from a set of transfer functions H (z) and G (z). A wavelet can be reconstructed from its approximation and detail coefficients [2]. The wavelet acts as a band pass filter [3]. Most of the wavelet applications are dealt with the coefficients h (n) and g (n) from (1) and (2). These are viewed as quadrature mirror filters whose spectra are mirror images. The filter formed by the mother wavelet acts as constant-Q filters. Q-factor is given by:

Q-factor= centre frequency/bandwidth (4)

This breaks down into the filter bank implementation of discrete wavelet transform. In Fig.2 the filters $g'(n)$ forms the high pass filter segment of the QMF filter [4] that gives the detail coefficients of the wavelet and $h'(n)$ forms the low pass filter. The approximation coefficients obtained at the output of $h'(n)$ is down sampled and passed through another pair of QMF filters and the process of decomposition continues. We can reconstruct the wavelet by using the same QMF filters by giving the approximation and detail coefficients as filter inputs. In this paper, we propose a new algorithm to find out the $h'(n)$ and $g'(n)$ in an iterative manner. Once $h'(n)$ is obtained, $g'(n)$ can be obtained using (3). The $h'(n)$ of the standard wavelets are fed into the normalized LMS algorithm. The role of an adaptive filter is to find the filter coefficients that give minimum error. Our algorithm suggests that by up sampling the estimated signal obtained at output end of the adaptive filter and giving this new signal as input, a new set of filter weights can be obtained that would give minimum error. This result however is accomplished by repeating the above steps iteratively for many times. This set of filter weights help to reconstruct the $h'(n)$ with minimum error. This process was carried out with all the standard wavelets. For all these wavelets, with this set of filter weights, their respective $h'(n)$ were successfully reconstructed. Thus the wavelet and scaling functions of each wavelet for flute and violin is obtained using this algorithm.

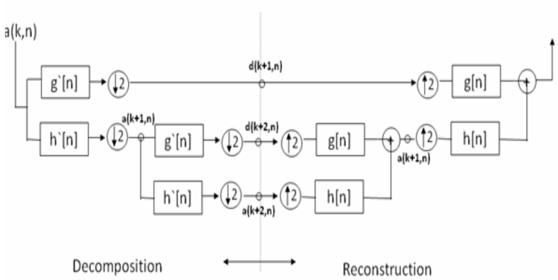

Fig. 1. Concept of DWT as filter bank

3. Least mean square algorithm

In designing an FIR adaptive filter, [5], [6] the goal is to find the weights of filter, wn at time n that minimizes the quadratic equation, i.e. the error of prediction:

$$\xi(n) = E\{|e(n)|^2\} \tag{5}$$

Although the vector that minimizes ξ(n) may be found by setting the derivates of ξ(n) with respect to w*(k) equal to zero. Or the weight-vector update equation used here:

$$w_{n+1} = w_n + \mu E\{e(n)x^*(n)\} \tag{6}$$

A practical limitation with this algorithm is that the expectation E(e(n)x*(n)) is generally unknown. Therefore it must be replaced with an estimate such as the sample mean:

$$\hat{E}\{e(n)x^*(n)\} = 1/L \sum_{l=0}^{L-1} e(n-l)x^*(n-l) \tag{7}$$

Incorporating this estimate into the steepest descent algorithm, the update for w_n becomes:

$$w_{n+1} = w_n + \mu / L \sum_{l=0}^{L-1} e(n-l)x^*(n-l) \tag{8}$$

A special case occurs if we use a one point sample mean (L=1),

$$\hat{E}\{e(n)x^*(n)\} = e(n)x^*(n) \tag{9}$$

In this case, the weight vector update equation assumes a particularly simple form:

$$w_{n+1} = w_n + \mu e(n)x^*(n) \tag{10}$$

This is known as LMS algorithm. The simplicity of the algorithm comes from the fact that the update for the kth coefficient is:

$$w_{n+1}(k) = w_n(k) + \mu e(n)x^*(n-k) \tag{11}$$

An LMS adaptive filter having p+1 coefficients require p+1 multiplication and p+1 addition to update the filter coefficients. In addition, it is necessary to compute the error $e(n) = d(n) - y(n)$. Since w_n is a vector of random variables, the convergence of the LMS algorithm [7] must be considered within a statistical framework. Therefore supposing that $x(n)$ and $d(n)$ are jointly wide sense stationary processes, then the LMS algorithm will converge if the step size satisfies the condition:

$$0 < \mu < \frac{2}{\lambda_{max}} \tag{12}$$

The main disadvantage of the LMS algorithm is that it is very sensitive to the changes in the input signal $x(n)$. As a result, it is very difficult to find an optimum step size or convergence parameter μ, which guarantees convergence of the algorithm, as well as minimizes the time of computation. A suitable alternative is to use the normalized LMS algorithm [8], which normalizes the LMS step size with the power of the input. When the input $x(n)$ becomes too small, the NLMS algorithm can be modified by adding a small positive value ϵ to the power of the input signal.

$$w_{n+1} = w_n + \beta \frac{x^*(n)}{\in + \|x(n)\|^2} e(n) \tag{13}$$

In the new algorithm developed, $h'(n)$ was tried to be reconstructed using LMS algorithm. Here the desired signal was down sampled and then up sampled before being given to the filter. But it encountered the problem of sensitivity of step size as mentioned above.

The obtained $h'(n)$ varied as the step sizes varied as shown in table1. This led us to modify our algorithm by incorporating NLMS algorithm. Here the input and desired signals were given as in its original form.
The adaptive filter was fed with an input stream of impulses. Up sampling and passing the estimated output iteratively through the filter, helped in generating the scaling function for the given $h(n)$. A maximum of six iterations were required. The iterative method explained in Fig.3. was tested with the wavelet coiflet5. The obtained wavelet and scaling functions were passed into an NLMS adaptive filter. The weights of the filter obtained were used to reconstruct the respective signals. The new scaling and wavelet functions obtained were similar in properties to standard coefficients as in Fig.5

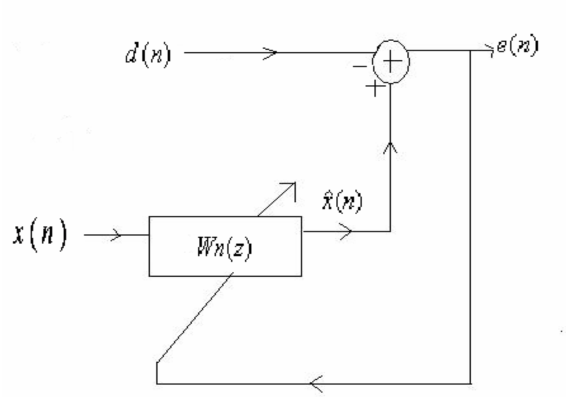

Fig. 2. The basic procedure of LMS algorithm

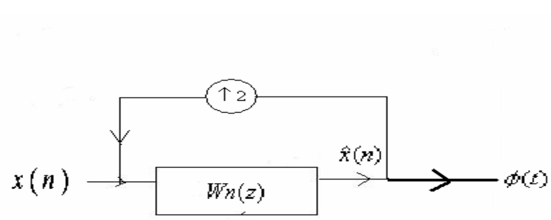

Fig. 3. The iteration to find scaling function

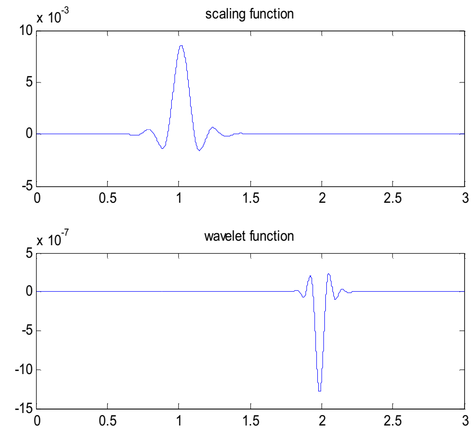

Fig.4. Scaling and wavelet functions of Coiflet5 wavelet.

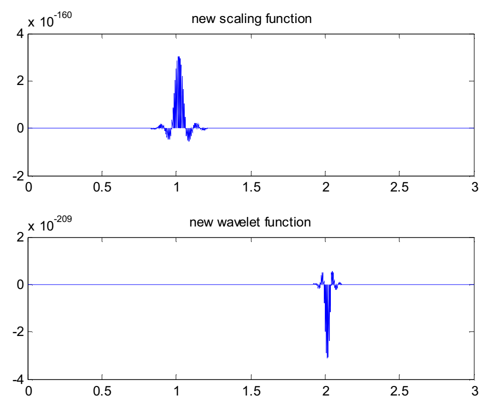

Fig.5 New scaling and wavelet functions obtained for coiflet5

4. Music signal characteristics

On analysing the signal characteristics of music notes [9], [10] played on instruments like violin and flute, there were wavelet like elements (similar events) repeating over fixed intervals as shown in Fig.6. The harmonics of these notes are different for these instruments. It is these harmonics that help us identify the instrument sounds.

5. Experimental results

a. Using LMS algorithm:

We used the LMS algorithm to reconstruct wavelet functions of the standard wavelets. But due to the sensitivity of the step-size, the reconstruction was unsuccessful. The results with respect to wavelet db2 are portrayed in Table1.

b. Using NLMS algorithm:

We repeated the same experiment with NLMS algorithm, where the step- size was not affected by any factor. With this algorithm the wavelet and scaling functions were reconstructed successfully. The results are shown in Table2.

We had developed new scaling functions and wavelet functions for two music instruments-violin and flute. The repeating elements were extracted from the classical music signals played. Average $h(n)$ of these signals were found out with respect to each instrument. With this average $h(n)$ being fed to the algorithm defined, scaling and wavelet functions of each instrument were obtained. The weights of the filter were updated in each step. The outputs are displayed in Fig.7, Fig.8 and Fig.9.

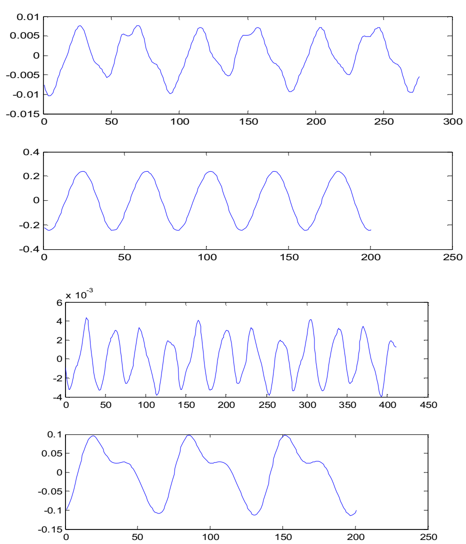

Fig.6. waveforms of notes ga and sa using flutes and violin

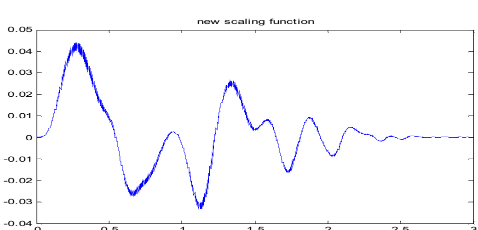

Fig.7. Scaling function of violin

Table I result of LMS algorithm on wavelet db2

Actual Filter Coefficients h(n)	Step-Size	Obtained filter coefficients
[0.683,1.183,0.317,-0.183]	0.3	[0.527,0.5101,0.46,0.4209]
	0.7	[0.5426,0.5425,0.4449,0.4005]
	1.2	[1.865,-11.3923,2.6026,11.0971]
	1.9	[1.6965,8.1151,-1.4531,-6.9507]*10^6

TABLE II Result of NLMS algorithm on wavelet db2

Actual Filter Coefficients h(n)	Step-Size	Obtained filter coefficients
[0.683,1.183,0.317,-0.183]	0.3	[0.682,1.666,0.3172,-0.1801]
	0.7	[0.683,1.183,0.317,-0.183]
	1.2	[0.6829 1.183 0.317 -0.183]
	1.9	[0.726,1.994,0.329, -0.1802]

TABLE III Average h(n) of instruments

Instrument	Average h(n)
Violin	[0.0388, 0.0640 , 0.0777,0.0839, 0.0865,0.0871 , 0.0878,0.0902, 0.0955,0.1017, 0.1088,0.1139, 0.1175,0.1191, 0.1191,0.1201, 0.1233,0.1283, 0.1344,0.1402]
Flute	[0.0833,0.1249, 0.1458,0.1562, 0.1614,0.1640, 0.1654,0.1660, 0.1663,0.1665, 0.1666,0.1666, 0.1666,0.1667,0.1667,0.1667, 0.1667,0.1667,0.1667,0.1667]

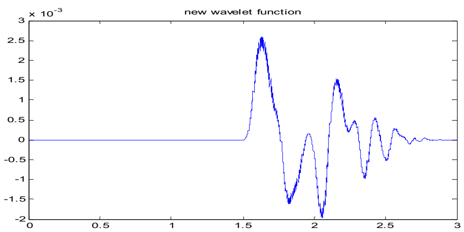

Fig.8. Wavelet function of violin

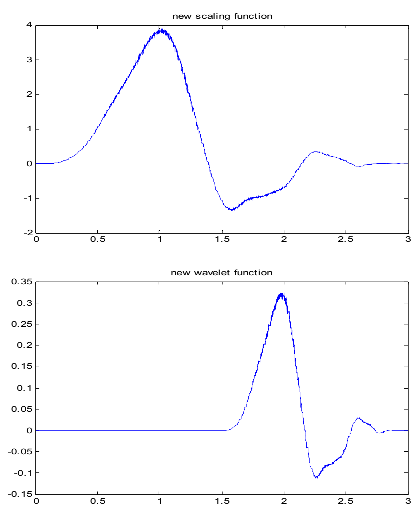

Fig.9. Scaling & wavelet function of flute

6. Conclusion

In this paper, we conclude that wavelets can be applied to many areas of signal processing, of which our concern is related to music signals. With the new algorithms developed in our experiments, it is possible to reconstruct wavelets. With the wavelets developed for the instruments, it is possible to develop music notes for those instruments. This makes it feasible for wavelets to be used for blind source separation of music signal mixtures. This can be extended to speech signals as well. Our future work is concentrated on blind source separation of music signals with the wavelets of the instruments developed.

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