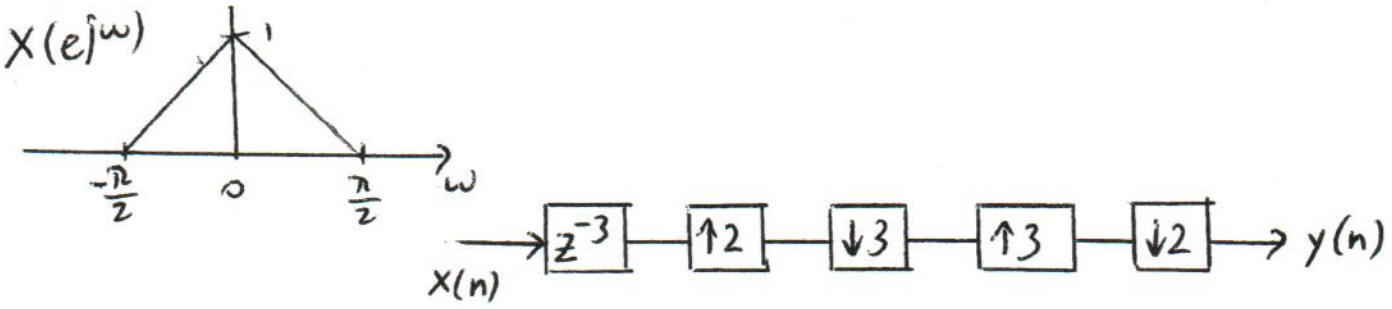
Midterm Exam — ECE 251C Fall 2011, Nguyen

Problem 1. (25pt) Consider the multirate system below. Find the input-output relation, i.e., what is $Y(z)$ in terms of $X(z)$? Sketch $|Y(e^{j\omega})|$.



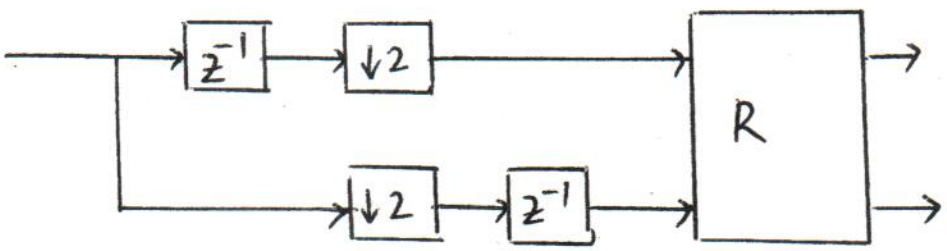
Problem 2. (20pt) Let $H(z) = \dfrac{3 + 2z^{-2} - z^{-4}}{1 - 4z^{-1} + 2z^{-2}}$. Find the two polyphase components $E_0(z)$ and $E_1(z)$ such that $H(z) = E_0(z^2) + z^{-1}E_1(z^2)$.

Problem 3. (30pt) Consider a two-channel filter bank where $H_0(z)$ is a FIR linear-phase filter with even-length N, and $H_1(z) = H_0(-z)$.

a. Find the PR condition in terms of $H_0(z)$

b. Find all possible PR solutions $(H_0(z), H_1(z), F_0(z), F_1(z))$

Problem 4. (25pt) Consider the following analysis filter bank



$R = \begin{pmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{pmatrix}$

a. Find $H_0(z)$ and $H_1(z)$.

b. Find the synthesis filters $F_0(z)$ and $F_1(z)$ for the PR system.