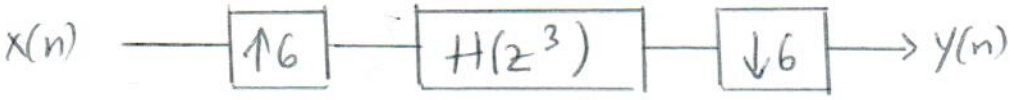
Midterm Exam — ECE 251C Fall 2019, Nguyen

Problem 1. (20pt) Consider the following LTI system $H(z)$:

$$H(z) = \frac{2z^{-1} + 5z^{-3}}{1 - \frac{3}{2}z^{-1} - z^{-2}}$$

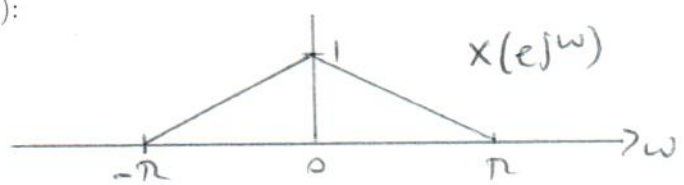
Find the two polyphases $H_{even}(z)$ and $H_{odd}(z)$, i.e., $H(z) = H_{even}(z^2) + z^{-1}H_{odd}(z^2)$

Problem 2. (40pt) Consider the multirate system below:



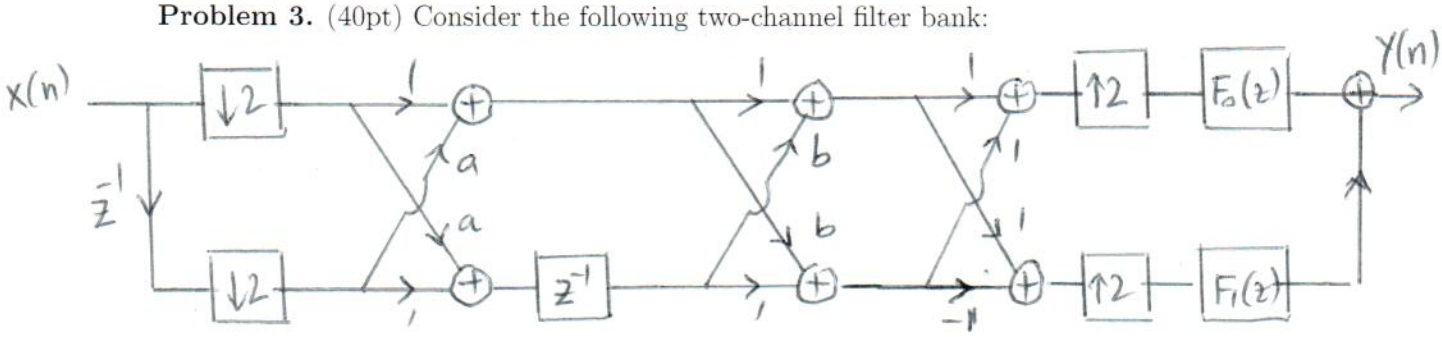
a. (10pt) Find $Y(z)$ in terms of $X(z)$ and $H(z)$.

b. (15pt) Sketch $|Y(e^{j\omega})|$ for $H(e^{j\omega})$ being an ideal lowpass filter with cutoff frequency at $\frac{\pi}{2}$ and $X(e^{j\omega})$:



c. (15pt) Sketch $|Y(e^{j\omega})|$ for $H(z) = -1 + 9z^{-2} + 16z^{-3} + 9z^{-4} - z^{-6}$ and $x(n) = (-1)^n$.

Problem 3. (40pt) Consider the following two-channel filter bank:



a = 2, b = 1/2

a.(7pt) Find the polyphase matrix $\mathbf{H}_p(z)$

b.(7pt) Find the analysis filters $H_0(z)$ and $H_1(z)$.

c.(7pt) Find all zeros and poles of $H_0(z)$ and $H_1(z)$ and sketch their pole-zero plots.

d.(7pt) Find the PR synthesis filters $F_0(z)$ and $F_1(z)$ by inverting $\mathbf{H}_p(z)$.

e.(7pt) Verify that the system is PR by the aliasing condition and halfband condition.

f.(5pt) Find the delay L, i.e., $y(n) = x(n - L)$.