Prob 1

a. $2y(n) = 2x(n) - 2x(n-1) - 5y(n-1) - 2y(n-2)$

$$2y(n) + 5y(n-1) + 2y(n-2) = 2x(n) - 2x(n-1)$$

$$H(z) = \frac{2 - 2z^{-1}}{2 + 5z^{-1} + 2z^{-2}} = \frac{1 - z^{-1}}{1 + \frac{5}{2}z^{-1} + z^{-2}}$$

$$= \frac{1 - z^{-1}}{(1 + 2z^{-1})(1 + \frac{1}{2}z^{-1})} = \frac{2}{1 + 2z^{-1}} + \frac{-1}{1 + \frac{1}{2}z^{-1}}$$

Note that $$\frac{1}{1 + az^{-1}} = \frac{1}{(1 + az^{-1})} \frac{(1 - az^{-1})}{(1 - az^{-1})} = \frac{1}{1 - a^2z^{-2}} + z^{-1}\frac{(-a)}{1 - a^2z^{-2}}$$

Thus,

$$H(z) = \frac{2(1 - 2z^{-1})}{(1 + 2z^{-1})(1 - 2z^{-1})} - \frac{(1 - \frac{1}{2}z^{-1})}{(1 + \frac{1}{2}z^{-1})(1 - \frac{1}{2}z^{-1})}$$

$$= \left[\frac{2}{1 - 4z^{-2}} - \frac{1}{1 - \frac{1}{4}z^{-2}}\right] + z^{-1}\left[\frac{-4}{1 - 4z^{-2}} + \frac{\frac{1}{2}}{1 - \frac{1}{4}z^{-2}}\right]$$

$$\Rightarrow H_{even}(z) = \frac{2}{1 - 4z^{-1}} - \frac{1}{1 - \frac{1}{4}z^{-1}}$$

$$H_{odd}(z) = \frac{-4}{1 - 4z^{-1}} + \frac{\frac{1}{2}}{1 - \frac{1}{4}z^{-1}}$$

b- <u>For 4-polyphases</u>:

$$H(z)=\left(\frac{2(1+4z^{-2})}{(1-4z^{-2})(1+4z^{-2})}-\frac{1(1+\frac{1}{4}z^{-2})}{(1-\frac{1}{4}z^{-2})(1+\frac{1}{4}z^{-2})}\right)+z^{-1}\left[\frac{-4(1+4z^{-2})}{(1-4z^{-2})(1+4z^{-2})}+\frac{1}{2}\frac{(1+\frac{1}{4}z^{-2})}{(1-\frac{1}{4}z^{-2})(1+\frac{1}{4}z^{-2})}\right]$$

$$=\left(\frac{2}{1-16z^{-4}}-\frac{1}{1-\frac{1}{16}z^{-4}}\right)+z^{-1}\left(\frac{-4}{1-16z^{-4}}+\frac{\frac{1}{2}}{1-\frac{1}{16}z^{-4}}\right)$$

$$+z^{-2}\left(\frac{8}{1-16z^{-4}}-\frac{\frac{1}{4}}{1-\frac{1}{16}z^{-4}}\right)+z^{-3}\left(\frac{-16}{1-16z^{-4}}+\frac{\frac{1}{8}}{1-\frac{1}{16}z^{-4}}\right)$$

$$\Rightarrow \quad H(z)=E_0(z^4)+z^{-1}E_1(z^4)+z^{-2}E_2(z^4)+z^{-3}E_3(z^4)$$

$$\Rightarrow \quad E_0(z)=\frac{2}{1-16z^{-1}}-\frac{1}{1-\frac{1}{16}z^{-1}}$$

$$E_1(z)=\frac{-4}{1-16z^{-1}}+\frac{\frac{1}{2}}{1-\frac{1}{16}z^{-1}}$$

$$E_2(z)=\frac{8}{1-16z^{-1}}-\frac{\frac{1}{4}}{1-\frac{1}{16}z^{-1}}$$

$$E_3(z)=\frac{-16}{1-16z^{-1}}+\frac{\frac{1}{8}}{1-\frac{1}{16}z^{-1}}$$

Prob 2

$$-\boxed{\downarrow 2}-\boxed{1-z^{-1}}-\boxed{\uparrow 2}-\boxed{1+z^{-2}}-\boxed{\downarrow 4}-$$

#2

$$-\boxed{\downarrow 2}-\boxed{\uparrow 2}-\boxed{1-z^{-2}}-\boxed{1+z^{-2}}-\boxed{\downarrow 4}-$$

$$-\boxed{\downarrow 2}-\boxed{\uparrow 2}-\boxed{1-z^{-4}}-\boxed{\downarrow 4}-$$

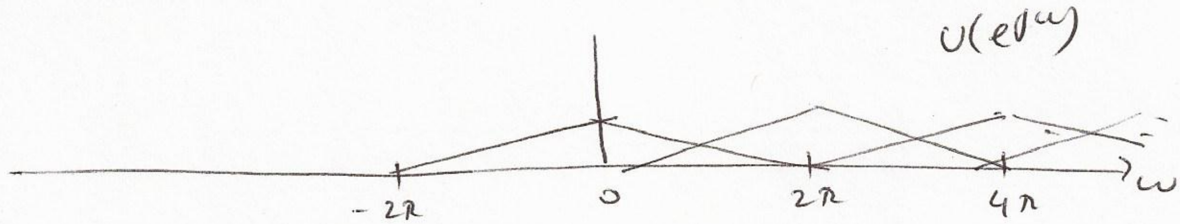
#1

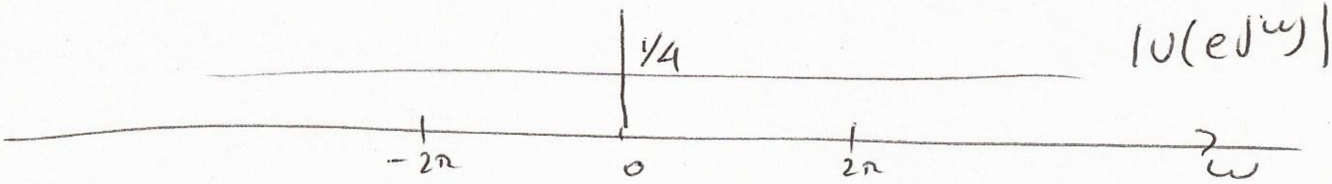
$$-\boxed{\downarrow 2}-\boxed{\uparrow 2}-\boxed{\downarrow 4}-\boxed{1-z^{-1}}-$$

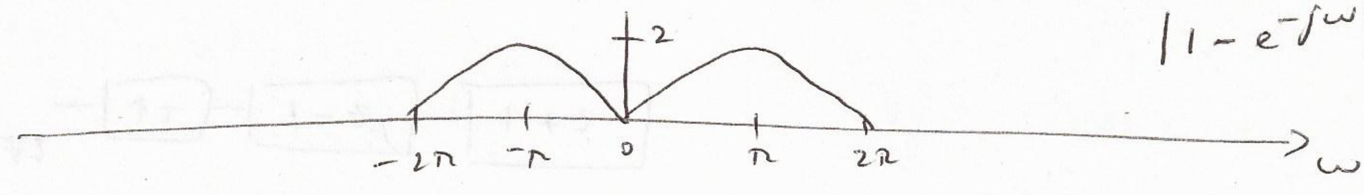
$$-\boxed{\downarrow 2}-\boxed{\downarrow 2}-\boxed{1-z^{-1}}-$$

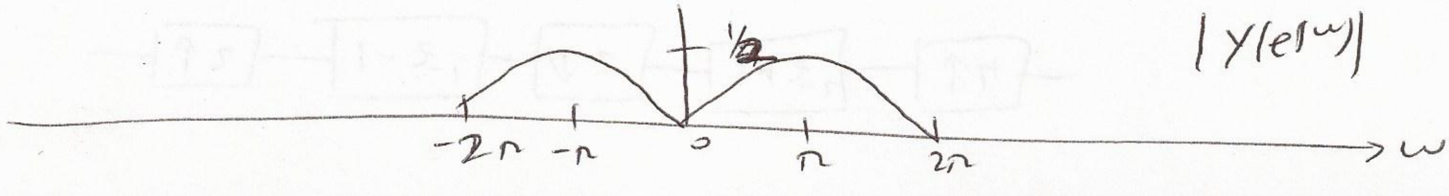
$$X(z) \quad -\boxed{\downarrow 4}-\boxed{1-z^{-1}}- \quad (1-z^{-1})\frac{1}{4}\sum_{k=0}^{3} X\left(z^{1/4} e^{j\frac{2k\pi}{4}}\right)$$

$$U(z) = \frac{1}{4}\sum_{k=0}^{3} X\left(z^{1/4} e^{j\frac{2k\pi}{4}}\right)$$









Prob 4

- orthogonal FB $\Rightarrow$ $F_0(z) = z^{-N} H_0(z^{-1})$
- Linear-phase filter: $H_0(z) = z^{-N} H_0(z^{-1})$ (only symmetric filter because $H_0(z)$ is lowpass)
- orthogonal + Linear phase:

$$F_0(z) = H_0(z)$$

- PR condition:

$$F_0(z) H_0(z) - F_0(-z) H_0(-z) = 2z^{-L}$$

$$H_0^2(z) - H_0^2(-z) = 2z^{-L} = 2z^{-(2k+1)}$$

$$\big(H_0(z) + H_0(-z)\big)\big(H_0(z) - H_0(-z)\big) = 2z^{-(2k+1)}$$

$$\Rightarrow \begin{cases} H_0(z) + H_0(-z) = \alpha_0 z^{-\beta_0} \\ H_0(z) - H_0(-z) = \alpha_1 z^{-\beta_1} \end{cases} \qquad \begin{cases} \alpha_0 \alpha_1 = 2 \\ \beta_0 + \beta_1 = 2k+1 \end{cases}$$

$$H_0(z) = \frac{1}{2}\left(\alpha_0 z^{-\beta_0} + \alpha_1 z^{-\beta_1}\right)$$

Since $H_0(z)$ is symmetric linear phase $\alpha_0 = \alpha_1 = \sqrt{2}$

$$\Rightarrow \begin{cases} H_0(z) = \frac{1}{\sqrt{2}} z^{-\beta_0} + \frac{1}{\sqrt{2}} z^{-\beta_1} \\ F_0(z) = H_0(z) = \frac{1}{\sqrt{2}} z^{-\beta_0} + \frac{1}{\sqrt{2}} z^{-\beta_1} \\ H_1(z) = F_0(-z) = H_0(-z) = \frac{1}{\sqrt{2}} z^{-\beta_0} - \frac{1}{\sqrt{2}} z^{-\beta_1} \\ F_1(z) = -H_0(-z) = -\frac{1}{\sqrt{2}} z^{-\beta_0} + \frac{1}{\sqrt{2}} z^{-\beta_1} \end{cases}$$

* Note that when $k=0$, $\beta_0 + \beta_1 = 1 \Rightarrow \beta_0 = 0, \beta_1 = 1$ we have Haar solution.