Problem 1

$$H(z) = \frac{2}{2 - z^{-1} - 2z^{-2} + z^{-3}} = \frac{2}{(2 - z^{-1}) - z^{-2}(2 - z^{-1})}$$

$$H(z) = \frac{2}{(2 - z^{-1})(1 - z^{-2})} = \frac{1}{(1 - \frac{1}{2}z^{-1})(1 - z^{-1})(1 + z^{-1})}$$

$$H(z) = \frac{1}{(1 - \frac{1}{2}z^{-1})(1 - z^{-1})(1 + z^{-1})} = \frac{A}{1 - \frac{1}{2}z^{-1}} + \frac{B}{1 - z^{-1}} + \frac{C}{1 + z^{-1}}$$

$$A = \left.\frac{1}{(1 - z^{-1})(1 + z^{-1})}\right|_{z^{-1} = 2} = \frac{1}{(1 - 2)(1 + 2)} = \frac{-1}{3}$$

$$B = \left.\frac{1}{(1 - \frac{1}{2}z^{-1})(1 + z^{-1})}\right|_{z^{-1} = 1} = \frac{1}{(1 - \frac{1}{2})(1 + 1)} = 1$$

$$C = \left.\frac{1}{(1 - \frac{1}{2}z^{-1})(1 - z^{-1})}\right|_{z^{-1} = -1} = \frac{1}{(1 + \frac{1}{2})(1 + 1)} = \frac{1}{3}$$

$$\Rightarrow H(z) = \frac{-1/3}{1 - \frac{1}{2}z^{-1}} + \frac{1}{1 - z^{-1}} + \frac{1/3}{1 + z^{-1}}$$

a. $H(z) = H_{even}(z^2) + z^{-1} H_{odd}(z^2)$

$$H_{even}(z) = -\frac{1}{3}\,\frac{1}{1 - \frac{1}{4}z^{-1}} + \frac{1}{1 - z^{-1}} + \frac{1/3}{1 - z^{-1}}$$

$$H_{even}(z) = \frac{-1/3}{1 - \frac{1}{4}z^{-1}} + \frac{4/3}{1 - z^{-1}}$$

$$H_{odd}(z) = \frac{-1/6}{1 - \frac{1}{4}z^{-1}} + \frac{1}{1 - z^{-1}} + \frac{1/3\,(-1)}{1 - z^{-1}}$$

$$H_{odd}(z) = \frac{-1/6}{1 - \frac{1}{4}z^{-1}} + \frac{2/3}{1 - z^{-1}}$$

b - For 3 polyphases:

$$\frac{1}{1-az^{-1}} = \frac{(1+az^{-1}+a^2z^{-2})}{(1-az^{-1})(1+az^{-1}+a^2z^{-2})} = \frac{1+az^{-1}+a^2z^{-2}}{1-a^3z^{-3}}$$

$$= \frac{1}{1-a^3z^{-3}} + z^{-1}\frac{a}{1-a^3z^{-3}} + z^{-2}\frac{a^2}{1-a^3z^{-3}}$$

Thus. $H(z) = \frac{-1/3}{1-\frac{1}{2}z^{-1}} + \frac{1}{1-z^{-1}} + \frac{1/3}{1+z^{-1}}$

$$= E_0(z^3) + z^{-1}E_1(z^3) + z^{-2}E_2(z^3)$$

- $E_0(z) = \frac{-1/3}{1-\frac{1}{8}z^{-1}} + \frac{1}{1-z^{-1}} + \frac{1/3}{1+z^{-1}}$
- $E_1(z) = \frac{-1/6}{1-\frac{1}{8}z^{-1}} + \frac{1}{1-z^{-1}} - \frac{1/3}{1+z^{-1}}$
- $E_2(z) = \frac{-1/12}{1-\frac{1}{8}z^{-1}} + \frac{1}{1-z^{-1}} + \frac{1/3}{1+z^{-1}}$

Problem 2

a-

$x(n) \to \boxed{\uparrow 2} \to \boxed{H(z^6)} \to \boxed{\downarrow 6} \to \boxed{\uparrow 3} \to y(n)$

$\to \boxed{\uparrow 2} \to \boxed{\downarrow 6} \to \boxed{\uparrow 3} \to \boxed{H(z^3)} \to$

$\to \boxed{\uparrow 2} \to \boxed{\downarrow 2} \to \boxed{\downarrow 3} \to \boxed{\uparrow 3} \to \boxed{H(z^3)} \to$

$x(n) \to \boxed{\downarrow 3} \to \boxed{\uparrow 3} \to \boxed{H(z^3)} \to y(n)$

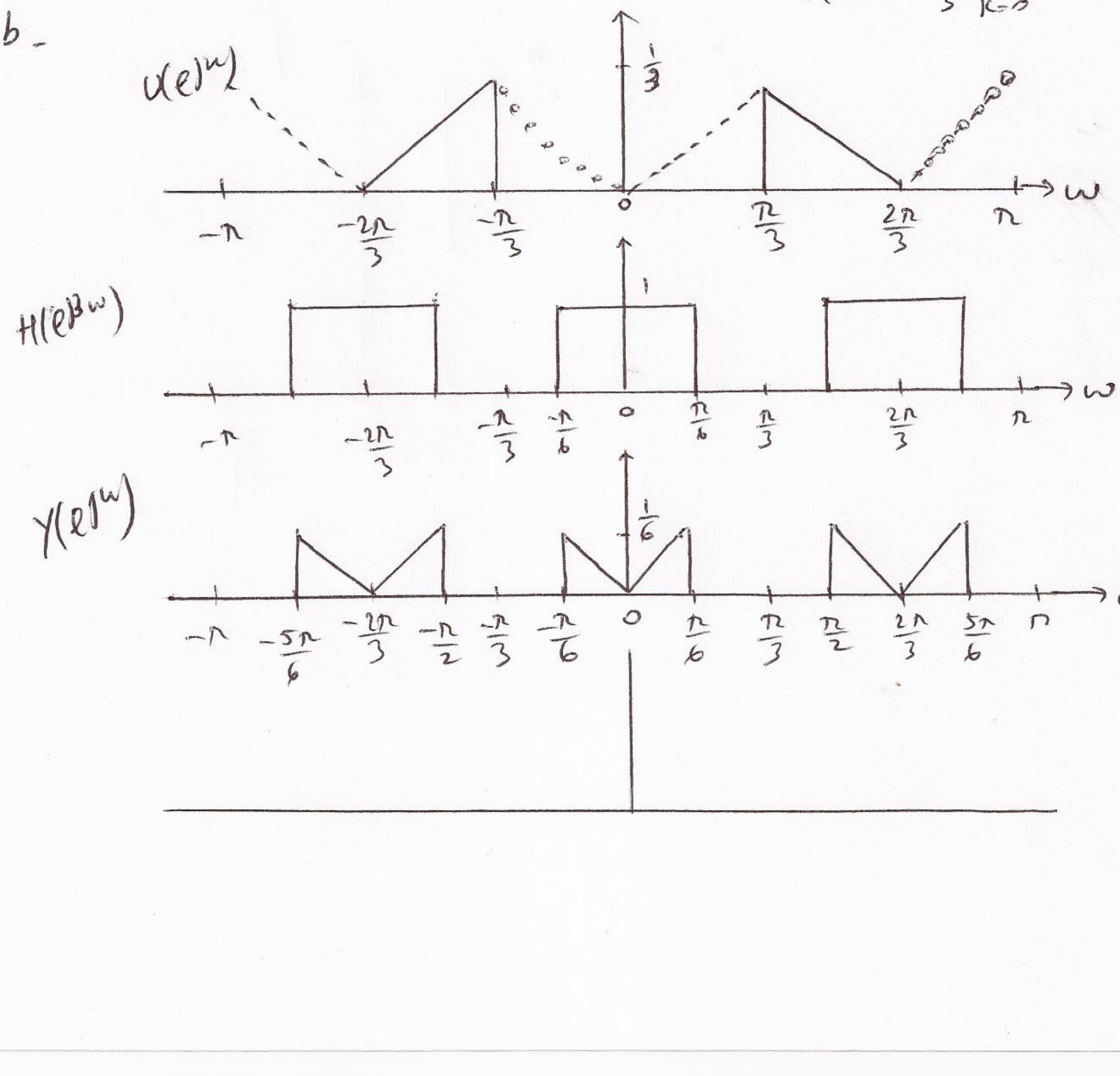
$X(z)$

$$\frac{1}{3} \sum_{k=0}^{2} X\left(z e^{j\frac{2k\pi}{3}}\right)$$

$$Y(z) = \frac{1}{3} H(z^3) \sum_{k=0}^{2} X\left(z e^{j\frac{2k\pi}{3}}\right)$$

$$U(e^{j\omega}) = \frac{1}{3} \sum_{k=0}^{2} X\left(e^{j\left(\omega - \frac{2k\pi}{3}\right)}\right)$$

b-



Prob 3

a. $F_p(z) = \begin{pmatrix} 1 & 0 \\ -2 & 1 \end{pmatrix} \begin{pmatrix} z^{-1} & 0 \\ 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} z^{-1} & 0 \\ -2z^{-1} & 1 \end{pmatrix} \begin{pmatrix} 1 & 2 \\ 0 & 1 \end{pmatrix}$ (4)

$$F_p(z) = \begin{pmatrix} z^{-1} & 2z^{-1} \\ -2z^{-1} & 1-4z^{-1} \end{pmatrix}$$

b. $\begin{pmatrix} F_0(z) & F_1(z) \end{pmatrix} = \begin{pmatrix} z^{-1} & 1 \end{pmatrix} F_p(z^2)$

$$= \begin{pmatrix} z^{-1} & 1 \end{pmatrix} \begin{pmatrix} z^{-2} & 2z^{-2} \\ -2z^{-2} & 1-4z^{-2} \end{pmatrix}$$

$$\begin{cases} F_0(z) = -2z^{-2} + z^{-3} \\ F_1(z) = 1 - 4z^{-2} + 2z^{-3} \end{cases}$$

c. $H_p(z) = F_p^{-1}(z) = \dfrac{1}{\Delta(z)} \begin{pmatrix} 1-4z^{-1} & -2z^{-1} \\ 2z^{-1} & z^{-1} \end{pmatrix}$

$$\Delta(z) = z^{-1}(1-4z^{-1}) + 4z^{-2} = z^{-1}$$

$\rightarrow$ Causal $H_p(z) = z^{-1} F_p^{-1}(z) = \begin{pmatrix} 1-4z^{-1} & -2z^{-1} \\ 2z^{-1} & z^{-1} \end{pmatrix}$

$$\begin{pmatrix} H_0(z) \\ H_1(z) \end{pmatrix} = H_p(z^2) \begin{pmatrix} 1 \\ z^{-1} \end{pmatrix} = \begin{pmatrix} 1-4z^{-2} & -2z^{-2} \\ 2z^{-2} & z^{-2} \end{pmatrix} \begin{pmatrix} 1 \\ z^{-1} \end{pmatrix}$$

$$\begin{cases} \begin{pmatrix} H_0(z) \\ H_1(z) \end{pmatrix} = \begin{pmatrix} 1-4z^{-2}-2z^{-3} \\ 2z^{-2}+z^{-3} \end{pmatrix} \end{cases}$$

d- Aliasing cond :

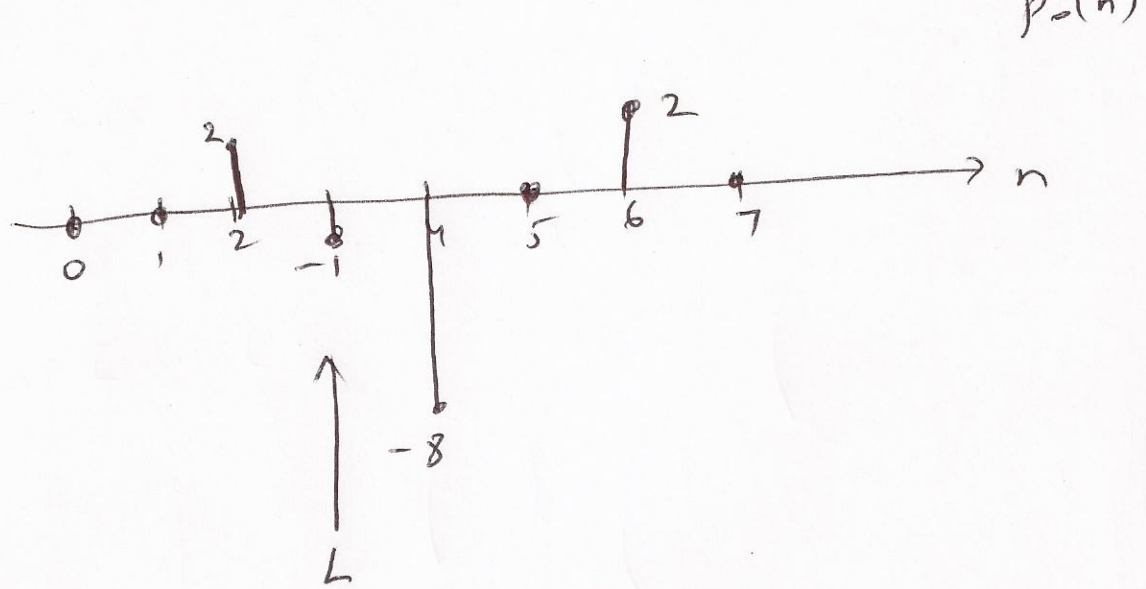
$$\begin{cases} H_0(z) = F_1(-z) = 1 - 4z^{-2} - 2z^{-3} \\ H_1(z) = -F_0(-z) = 2z^{-2} + z^{-3} \end{cases}$$

Halfband cond :

$$P_0(z) = F_0(z)H_0(z) = (-2z^{-2} + z^{-3})(1 - 4z^{-2} - 2z^{-3})$$

$$= z^{-2}(-2 + z^{-1})(1 - 4z^{-2} - 2z^{-3})$$

$$= z^{-2}(2 - z^{-1} - 8z^{-2} + 2z^{-4})$$



Halfband with $L = 3$.

e- $y(n) = x(n - L)$

$L = 3$