Sol for Midterm, ECE 251C, Fall 2011

Problem 1

$$\to \boxed{z^{-3}} - \boxed{\uparrow 2} - \boxed{\downarrow 3} - \boxed{\uparrow 3} - \boxed{\downarrow 2} -$$

By Noble identity #3,

$$- \boxed{z^{-3}} - \boxed{\downarrow 3} - \boxed{\uparrow 2} - \boxed{\downarrow 2} - \boxed{\uparrow 3} -$$

which simplifies to

$$- \boxed{z^{-3}} - \boxed{\downarrow 3} - \boxed{\uparrow 3} -$$

By Noble identity #1,

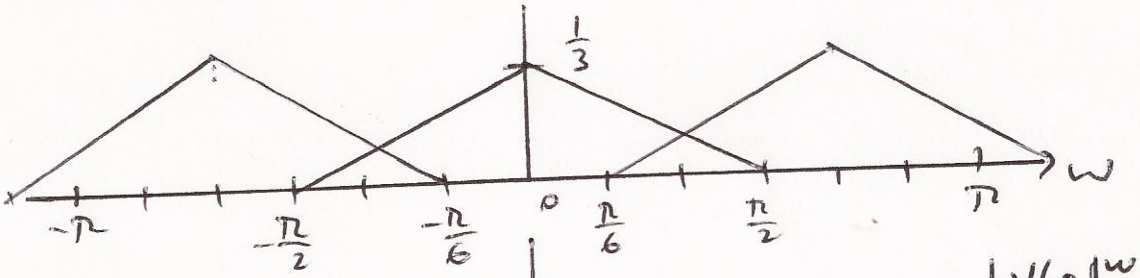
$$- \boxed{\downarrow 3} - \boxed{z^{-1}} - \boxed{\uparrow 3} -$$

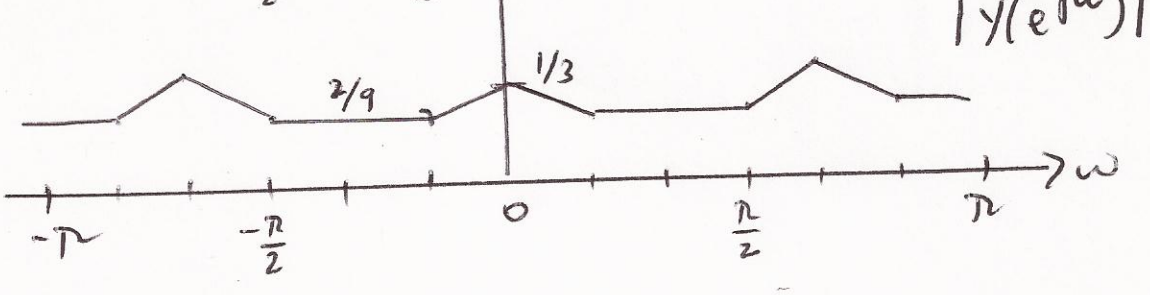
By Noble identity #2,

$$x(n) - \boxed{\downarrow 3} - \boxed{\uparrow 3} - \boxed{z^{-3}} - y(n)$$

$$Y(z) = \frac{1}{3} \sum_{k=0}^{2} X(z e^{j 2k\pi/3}) \cdot z^{-3}$$

$$|Y(e^{j\omega})| = \frac{1}{3} \left| \sum_{k=0}^{2} X(e^{j(\omega - 2k\pi/3)}) \right|$$





Problem 2

$$H(z) = \frac{3 + 2z^{-2} - z^{-4}}{1 - 4z^{-1} + 2z^{-2}} = E_0(z^2) + z^{-1} E_1(z^2)$$

$$= \frac{(3 + 2z^{-2} - z^{-4})\left[(1 + 2z^{-2}) + 4z^{-1}\right]}{\left[(1 + 2z^{-2}) - 4z^{-1}\right]\left[(1 + 2z^{-2}) + 4z^{-1}\right]}$$

$$= \frac{(3 + 2z^{-2} - z^{-4})(1 + 2z^{-2})}{(1 + 2z^{-2})^2 - 16z^{-2}} + z^{-1} \frac{4(3 + 2z^{-2} - \cancel{4} z^{-4})}{(1 + 2z^{-2})^2 - 16z^{-2}}$$

$$E_0(z) = \frac{(3 + 2z^{-1} - z^{-2})(1 + 2z^{-1})}{(1 + 2z^{-1})^2 - 16z^{-1}}$$

$$E_1(z) = \frac{4(3 + 2z^{-1} - \cancel{4} z^{-2})}{(1 + 2z^{-1})^2 - 16z^{-1}}$$

Problem 3

$H_0(z)$: even-length N, linear-phase

$H_1(z) = H_0(-z)$.

a- PR cond for FIR 2 channel FB.

$$H_0(z)\,H_1(-z) - H_0(-z)\,H_1(z) = 2z^{-L} = 2z^{-(2K+1)}$$

substitute $H_1(z) = H_0(-z)$, yields

$$H_0^2(z) - H_0^2(-z) = 2z^{-(2K+1)}$$

$$\left(H_0(z) + H_0(-z)\right)\left(H_0(z) - H_0(-z)\right) = 2z^{-(2K+1)}$$

$$\left[E_0(z^2)\right]\left[z^{-1}E_1(z^2)\right] = 2z^{-(2K+1)}$$

$$E_0(z)\,E_1(z) = 2z^{-K}$$

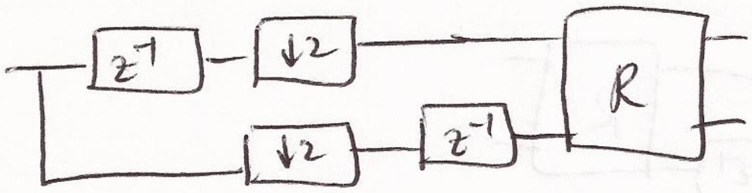
b- Since $H_0(z)$ & $H_1(z)$ are FIR filters, E_0 & E_1 are polynomials, the only solution is (taking account symmetry in $h_0(n)$)

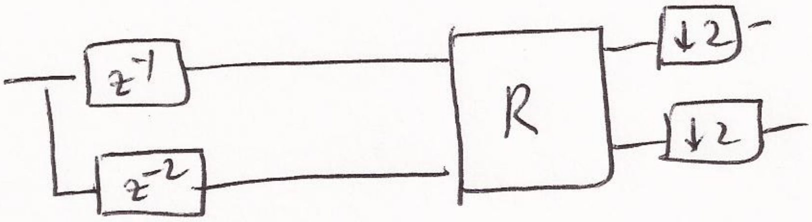
$$\begin{cases} E_0(z) = \sqrt{2}\,z^{-K_1} \\ E_1(z) = \sqrt{2}\,z^{-K_2} \end{cases}$$ where $K_1 + K_2 = K$

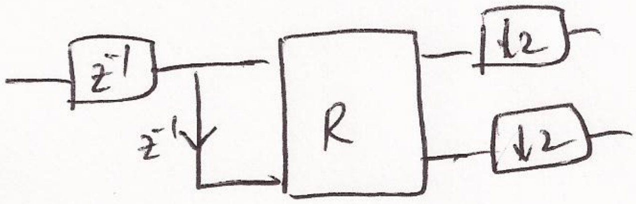
Thus, the PR filter bank is:

$$\begin{cases} H_0(z) = \sqrt{2}\left(z^{-2K_1} + z^{-(2K_2+1)}\right) \\ H_1(z) = \sqrt{2}\left(z^{-2K_1} - z^{-(2K_2+1)}\right) \\ F_0(z) = H_1(-z) = \sqrt{2}\left(z^{-2K_1} + z^{-(2K_2+1)}\right) \\ F_1(z) = -H_0(-z) = \sqrt{2}\left(-z^{-2K_1} + z^{-(2K_2+1)}\right) \end{cases}$$

Problem 4







a. $$\begin{pmatrix} H_0(z) \\ H_1(z) \end{pmatrix} = \begin{pmatrix} c & s \\ -s & c \end{pmatrix} \begin{pmatrix} 1 \\ z^{-1} \end{pmatrix} z^{-1} = \begin{pmatrix} c z^{-1} + s z^{-2} \\ -s z^{-1} + c z^{-2} \end{pmatrix}$$

b. PR:
$$F_0(z) = H_1(-z) = s z^{-1} + c z^{-2}$$
$$F_1(z) = -H_0(-z) = c z^{-1} - s z^{-2}$$

Check halfband

$$P(z) = F_0(z) H_0(z) = z^{-1}(s + c z^{-1}) z^{-1} (c + s z^{-1})$$
$$= z^{-2}(sc + z^{-1} + sc z^{-2})$$

halfband $l = 3$.