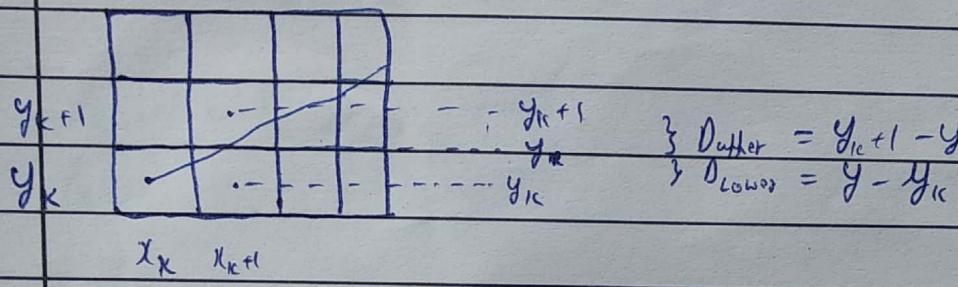
COMPUTER GRAPHICS ASSIGNMENT

* Bresenham's Line Algorithm :

⇒ Ist Octant

Here $(0 < m < 1)$



$$D_{lower} = m(x_k + 1) + b - y_k$$

$$D_{upper} = y_{k+1} - [m(x_k + 1) + b]$$

$$D_{lower} - D_{upper} = 2m(x_k + 1) - 2y_k + 2b - 1$$

$$= 2\frac{\Delta y}{\Delta x}(x_k + 1) - 2y_k + 2b - 1$$

$$\Delta x(D_{lo} - D_{up}) = 2\Delta y(x_k + 1) - 2y_k \Delta x + (2b - 1)\Delta x$$

$$P_k = 2\Delta y(x_k + 1) - 2y_k \Delta x + (2b - 1)\Delta x \quad \text{(i)}$$

$$P_{k+1} = 2\Delta y(x_{k+1} + 1) - 2y_{k+1}\Delta x + (2b - 1)\Delta x \quad \text{(ii)}$$

(ii) − (i)

$$P_{k+1} - P_k = 2\Delta y(x_{k+1} - x_k) - 2\Delta x(y_{k+1} - y_k)$$

$$P_{k+1} = P_k + 2\Delta y(x_{k+1} - x_k) + 2\Delta x(y_k - y_{k+1})$$

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→ Now if $P_k < 0$ we will choose (x_{k+1}, y_k)

$$P_{k+1} = P_k + 2\Delta y (x_{k+1} - x_k) + 2\Delta x (y_k - y_k)$$
$$P_{k+1} = P_k + 2\Delta y$$

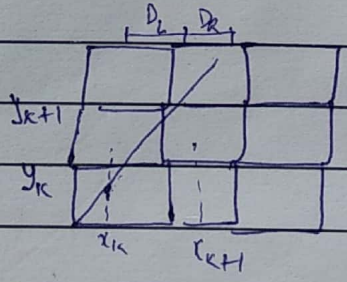
→ Now if $P_k > 0$ we will choose $(x_k + 1, y_k + 1)$

$$P_{k+1} = P_k + 2\Delta y (x_{k+1} - x_k) + 2\Delta x (y_k - y_{k-1})$$
$$P_{k+1} = P_k + 2\Delta y - 2\Delta x$$

So, when $k = 0$

$$P_0 = 2\Delta y x_0 - 2\Delta x y_0 + 2\Delta y + \Delta x (2b - 1)$$
$$\boxed{P_0 = 2\Delta y - \Delta x}$$

II nd octant



Here $(1 < m < \infty)$

$$D_{Left} = \frac{y_k + 1 - b}{m} - x_k$$

$$D_{right} = x_{k+1} - \left[\frac{y_k + 1 - b}{m}\right]$$

$$D_L - D_R = \frac{y_k + 1 - b}{m} - x_k - \left[-\left[\frac{y_k + 1 - b}{m}\right] + x_k + 1\right]$$

$$= 2\left[\frac{y_k + 1 - b}{m}\right] - 2x_k - 1$$

$$\boxed{m = \frac{\Delta y}{\Delta x}}$$

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$\Delta y[D_2 - D_1] = 2\Delta x[y_k + 1 - b] - 2\Delta y x_k - \Delta y$

Let $\Delta y(D_2 - D_1) = P_k$

$P_k = 2\Delta x[y_k + 1 - b] - \Delta y[2x_k + 1]$

$P_{k+1} = 2\Delta x[y_{k+1} + 1 - b] - \Delta y[2x_{k+1} + 1]$

$P_{k+1} - P_k = 2\Delta x[y_{k+1} + 1 - b] - \Delta y[2x_{k+1} + 1] - [2\Delta x[y_k + 1 - b] - \Delta y[2x_k + 1]]$

$= 2\Delta x[y_{k+1} - y_k] + 2\Delta y[x_k - x_{k+1}]$

$P_{k+1} = P_k + 2\Delta x[y_{k+1} - y_k] + 2\Delta y[x_k - x_k + 1]$

for $P_k < 0$ $(x_k, y_k + 1)$

for $P_k > 0$ $(x_k + 1, y_k + 1)$

$P_{k+1} = P_k + 2\Delta x + 2\Delta y \times 0$

$= P_k + 2\Delta x$

$P_{k+1} = P_k + 2\Delta x - 2\Delta y$

$P_k = 2\Delta x[y_k + 1 - b] - \Delta y[2x_k + 1]$

Put $k = 0$ for P_0

$P_0 = 2\Delta x[y_0 + 1 - b] - \Delta y[2x_0 + 1]$

& 'b' for $y_0, x_0 \Rightarrow b = y_0 - \frac{\Delta y}{\Delta x} x_0$

$P_0 = 2\Delta x\left[y_0 + 1 - y_0 + \frac{\Delta y}{\Delta x} \cdot x_0\right] - \Delta y[2x_0 + 1]$

$\boxed{P_0 = 2\Delta x - \Delta y}$

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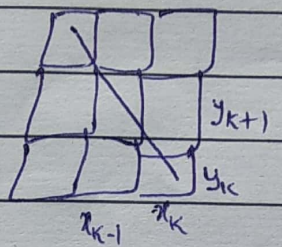
III. Octant



Here $(-\infty > m > -1)$

$$D_L = x_{k-1} - x \quad , \quad D_R = x - x_k$$

$$D_L = x_k - 1 - \left[y_k + 1 - \frac{l}{m} \right] \quad , \quad D_R = \left(y_k + 1 - \frac{l}{m} \right) - x_k$$

$$D_R - D_L = 2\left(y_{k+1} - \frac{l}{m} \right) - 2x_k + 1$$

$$\Delta y \, (D_R - D_L) = 2\Delta x \, (y_k + 1 - l) - 2\Delta y \, x_k + \Delta y$$

Let, P_k

$$P_k = 2\Delta x \, y_k - 2\Delta y \, x_k + (2\Delta x (1 - l) + \Delta y)$$

$$P_{k+1} = 2\Delta x \, y_{k+1} - 2\Delta y \, x_{k+1} + (2\Delta x (1 - l) + \Delta y)$$

$$P_{k+1} - P_k = 2\Delta x \, [y_{k+1} - y_k] + 2\Delta y \, (x_k - x_{k+1})$$

$$P_{k+1} = P_k + 2\Delta x \, [y_{k+1} - y_k] + 2\Delta y \, (x_k - x_{k+1})$$

For $P_k < 0$

(x_k, y_{k+1})

$$P_{k+1} = P_k + 2\Delta x + (2\Delta y \times 0)$$
$$= P_k + 2\Delta x$$

For $P_k \geq 0$

$(x_k - 1, y_k + 1)$

$$P_{k+1} = P_k + 2\Delta x + 2\Delta y$$
$$P_{k+1} = P_k + 2\Delta x + 2\Delta y$$

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$$P_k = 2\Delta x\, y_k - 2\Delta y\, x_k + (2\Delta x\,(1-b) + \Delta y)$$

$$P_0 = 2\Delta x\, y_0 - 2\Delta y\, x_0 + (2\Delta x\,(1-b) + \Delta y)$$

$$y_0 = mx_0 + b$$

$$\therefore = \frac{\Delta y}{\Delta x} x_0 + b$$

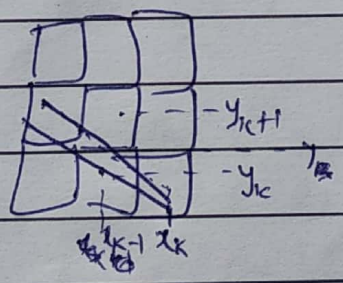
$$P_0 = 2\Delta x\, y_0 - 2\Delta y\, x_0 + \left[2\Delta x\left(1 + \frac{\Delta y}{\Delta x} x_0 - y_0\right) + \Delta y\right]$$

$$= 2\Delta x\, y_0 - 2\Delta y\, x_0 - 2\Delta x\, y_0 + 2\Delta y\, x_0 + 2\Delta x + \Delta y$$

$$= 2\Delta x + \Delta y$$

$$\boxed{P_0 = 2\Delta x + \Delta y}$$

⇒ **IV Octant**



$$D_{up} = y_{k+1} - y$$

$$D_l = y - y_k$$

$$D_{up} = y_{k+1} - [m(x_k - 1) + b]$$

$$D_l = m(x_k - 1) + b - y_k$$

$$D_l - D_u = m(x_k - 1) + b - y_k - [y_k + 1 - mx_k + m - b]$$

$$= mx_k - m + b - y_k - y_k - 1 + mx_k - m + b$$

$$= 2mx_k - 2y_k - 2m + 2b - 1$$

$$m = \frac{\Delta y}{\Delta x}$$

$$D_l - D_u = \frac{2\Delta y}{\Delta x} x_k - 2y_k - \frac{2\Delta y}{\Delta x} + 2b - 1$$

$$\Delta x(D_l - D_u) = 2\Delta y\, x_k - 2\Delta x\, y_k - 2\Delta y + (2b - 1)\Delta x$$

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$$P_k = 2\Delta y\, x_k - 2\Delta x\, y_k - 2\Delta y + \Delta x(2b-1)$$
$$P_{k+1} = 2\Delta y\, x_{k+1} - 2\Delta x\, y_{k+1} - 2\Delta y + \Delta x(2b-1)$$

$$P_{k+1} - P_k = 2\Delta y(x_{k+1} - x_k) + 2\Delta x(y_k - y_{k+1})$$

$$P_{k+1} = P_k + 2\Delta y(x_{k+1} - x_k) + 2\Delta x(y_k - y_{k+1})$$

for $P < 0$
use $(x_k - 1, y_k)$
$P_{k+1} = P_k - 2\Delta y$

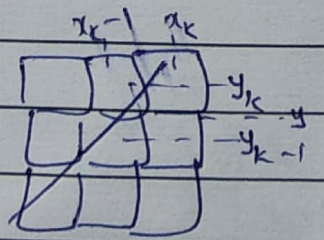
for $P > 0$
use $(x_k - 1, y_k + 1)$
$P_{k+1} = P_k - 2\Delta y + 2\Delta x$

$$P_0 = 2\Delta y\, x_0 - 2\Delta x\, y_0 - 2\Delta y + \Delta x(2b-1)$$

$$b = y_0 - \frac{\Delta y}{\Delta x} x_0$$

$$P_0 = 2\Delta y x_0 - 2\Delta x y_0 - 2\Delta y + \Delta x\left(2y_0 - \frac{2\Delta y}{\Delta x} \cdot x_0 - 1\right)$$
$$= 2\Delta y x_0 - 2\Delta x y_0 - 2\Delta y + 2\Delta x y_0 - 2\Delta y x_0 - \Delta x$$
$$= -2\Delta y_0 - \Delta x = -(\Delta x + 2\Delta y)$$

✓ -octant



$D_u = y - y_k$
$D_l = y_k - 1 - y$

$$D_u = m(x_k - 1) + b - y_k$$
$$D_l = y_k - 1 - [m(x_k - 1) + b]$$

$$D_u - D_l = m(x_k - 1) + b - y_k - [y_k - 1 - mx_k + m - b]$$
$$= mx_k - m + b - y_k - y_k + 1 + mx_k - m + b$$

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$D_u - D_l = 2mx_k - 2m + 2b - 2y_k + 1$

$= 2mx_k - 2y_k + 2b - 2m + 1$

$\Delta x(D_u - D_l) = 2\Delta y x_k - 2\Delta x y_k + 2b\Delta x - 2\Delta y + \Delta x$

$p_k = 2\Delta y x_k - 2\Delta x y_k + 2b\Delta x - 2\Delta y + \Delta x$

$p_{k+1} = 2\Delta y x_{k+1} - 2\Delta x y_{k+1} + 2b\Delta x - 2\Delta y + \Delta x$

$p_{k+1} - p_k = 2\Delta y(x_{k+1} - x_k) + 2\Delta x(y_k - y_{k+1})$

$p_{k+1} = p_k + 2\Delta y(x_{k+1} - x_k) + 2\Delta x(y_k - y_{k+1})$

$p_k < 0$

$(x_k - 1, y_k)$

$p_{k+1} = p_k - 2\Delta y$

$p_k > 0$

$(x_k - 1, y_k - 1)$

$p_{k+1} = p_k - 2\Delta y + 2\Delta x$

$p_0 = 2\Delta y x_0 - 2\Delta x y_0 + 2b\Delta x - 2\Delta y + \Delta x$

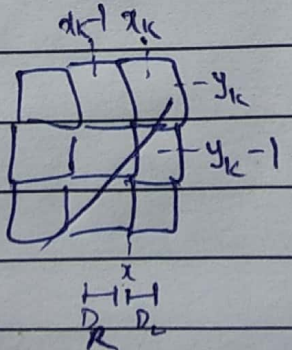
$b = y_0 - \frac{\Delta y}{\Delta x} x_0$

$p_0 = 2\Delta y x_0 - 2\Delta x y_0 + 2\left[y_0 - \frac{\Delta y}{\Delta x} \cdot x_0\right]\Delta x - 2\Delta y + \Delta x$

$= 2\Delta y x_0 - 2\Delta x y_0 + 2\Delta x y_0 - 2\Delta y x_0 - 2\Delta y + \Delta x$

$= \Delta x - 2\Delta y$

(VI) Octant



$D_L = x - (x_k - 1)$

$D_R = x_k - x$

$D_L = x_k - [x_k - 1]$

$D_R = x_k - x$

$D_L = \frac{y_k - 1 - b}{m} - [x_k - 1]$

$D_R = x_k - \left[\frac{y_k - 1 - b}{m}\right]$

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$$D_L - D_R = \frac{y_k - 1 - b}{m} - [x_k - 1] - \left[x_k - \left[\frac{y_k - 1 - b}{m}\right]\right]$$

$$= \frac{y_k - 1 - b}{m} - x_k + 1 - x_k + \frac{y_k - 1 - b}{m}$$

$$= 2\left[\frac{y_k - 1 - b}{m}\right] - 2x_k + 1$$

$$\Delta y(D_L - D_R) = 2\Delta x[y_k - 1 - b] - 2\Delta y x_k + \Delta y$$

$$p_k = 2\Delta x y_k - 2\Delta x - 2\Delta x b - 2\Delta y x_k + \Delta y$$

$$= 2\Delta x y_k - 2\Delta y x_k - 2\Delta x - 2\Delta x b + \Delta y$$

$$p_{k+1} = 2\Delta x y_{k+1} - 2\Delta y x_{k+1} - 2\Delta x - 2\Delta x b + \Delta y$$

$$p_{k+1} - p_k = 2\Delta x(y_{k+1} - y_k) + 2\Delta y(x_k - x_{k+1})$$

$$p_{k+1} = p_k + 2\Delta x(y_{k+1} - y_k) + 2\Delta y(x_k - x_{k+1})$$

If $p_k < 0$

$(x_k, y_k - 1)$

$p_{k+1} = p_k - 2\Delta x$

If $p_k > 0$

$(x_k - 1, y_k - 1)$

$p_{k+1} = p_k - 2\Delta x + 2\Delta y$

$$p_0 = 2\Delta x y_0 - 2\Delta y x_0 - 2\Delta x - 2\Delta x b + \Delta y$$

$$b = y_0 - \frac{\Delta y}{\Delta x} x_0$$

$$p_0 = 2\Delta x y_0 - 2\Delta y x_0 - 2\Delta x - 2\Delta x\left[y_0 - \frac{\Delta y}{\Delta x} \cdot x_0\right] + \Delta y$$

$$= 2\Delta x y_0 - 2\Delta y x_0 - 2\Delta x - 2\Delta x y_0 + 2\Delta y x_0 + \Delta y$$

$$= \Delta y - 2\Delta x$$

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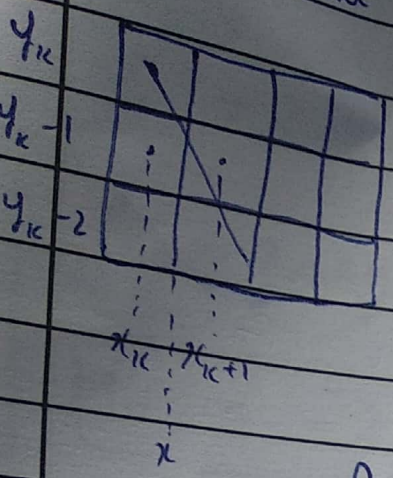
⇒ VII th Octant

here $(-\infty > m > -1)$



$$y = mx + c$$

$$y_k - 1 = mx + b$$

$$x = \frac{y_k - 1 - b}{m}$$

$$D_L = x - x_k$$

$$D_L = \frac{y_k - 1 - b}{m} - x_k$$

$$D_R = x_k + 1 - x$$

$$D_R = x_k + 1 - \left(\frac{y_k - 1 - b}{m}\right)$$

$$D_L - D_R = 2\left(\frac{y_k - 1 - b}{m}\right) - 2x_k - 1$$

$$\underbrace{\Delta y (D_L - D_R)}_{\downarrow P_k} = 2\Delta x (y_k - 1 - b) - 2\Delta y x_k - \Delta y$$

$$P_k = 2\Delta x y_k - 2\Delta y x_k - \Delta y - 2\Delta x (1 + b) \quad \text{--- (i)}$$

$$P_{k+1} = 2\Delta x y_{k+1} - 2\Delta y x_{k+1} - \Delta y - 2\Delta x (1 + b) \quad \text{--- (ii)}$$

(ii) − (i)

$$P_{k+1} = P_k + 2\Delta x (y_{k+1} - y_k) - 2\Delta y (x_{k+1} - x_k)$$

⇒ If $P_k > 0$, use $(x_k, y_k - 1)$

$$P_{k+1} = P_k - 2\Delta x$$

⇒ If $P_k < 0$, use $(x_k + 1, y_k - 1)$

$$P_{k+1} = P_k - 2\Delta x - 2\Delta y$$

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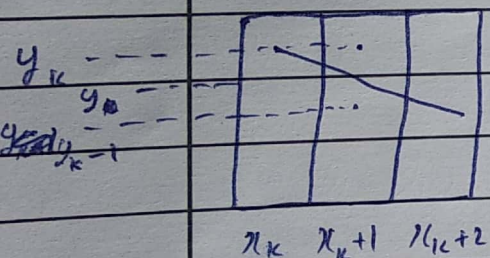
$$P_k = 2\Delta x\, y_k - 2\Delta y\, x_k - \Delta y - 2\Delta x(1+b)$$

$$P_0 = 2\Delta x\, y_0 - 2\Delta y\, x_0 - \Delta y - 2\Delta x\left[1 + y_0 - \left(\frac{\Delta y}{\Delta x}\right)x_0\right]$$

$$P_0 = 2\Delta x\, y_0 - 2\Delta y\, x_0 - \Delta y - 2\Delta x - 2\Delta x\, y_0 + 2\Delta y\, x_0$$

$$\boxed{P_0 = -2\Delta x - \Delta y}$$

$\Rightarrow$ VIII th Octant here $(0 > m > -1)$



$$y = mx + c$$
$$y = m(x_k+1) + b$$

$$D_{lo} = y - y_k + 1 \qquad D_{up} = y_k - y$$

$$D_{lo} = m(x_k+1) + b - y_k + 1 \qquad D_{up} = y_k - m(x_k+1) - b$$

$$P_0 - D_{up} = 2m(x_k+1) + 2b - 2y_k + 1$$
$$\Delta x(P_0 - D_{up}) = 2\Delta y(x_k+1) + 2\Delta x b - 2\Delta x\, y_k + \Delta x$$

$$P_k = 2\Delta y\, x_k - 2\Delta x\, y_k + 2\Delta y + 2\Delta x b + \Delta x \quad \text{— (i)}$$

$$P_{k+1} = 2\Delta y\, x_{k+1} - 2\Delta x\, y_{k+1} + 2\Delta y + 2\Delta x b + \Delta x \quad \text{— (ii)}$$

(ii) − (i)

$$P_{k+1} = P_k + 2\Delta y(x_{k+1} - x_k) - 2\Delta x(y_{k+1} - y_k)$$

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If $P_k > 0$, use (x_{k+1}, y_k)

$$P_{k+1} = P_k + 2\Delta y$$

If $P_k < 0$, use $(x_{k+1}, y_k - 1)$

$$P_{k+1} = P_k + 2\Delta y + 2\Delta x$$

Now, $P_k = 2\Delta y\, x_k - 2\Delta x\, y_k + 2\Delta y + \Delta x + 2\Delta x\, b$

$k = 0$, Putting $b = y_0 - \left(\frac{\Delta y}{\Delta x}\right) x_0$

$$P_0 = 2\Delta y\, x_0 - 2\Delta x\, y_0 + 2\Delta y + \Delta x + 2\Delta x \left[y_0 + \frac{\Delta y}{\Delta x} x_0\right]$$

$$\boxed{P_0 = 2\Delta y + \Delta x}$$

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