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Minor 1

Q1. Consider the sequence of events $e_1, e_2 \ldots \in H$ be an execution of program

Let $VC(e_i)[1..n]$ denote the vector time stamp for event e_i

Now define function convert

```
func convert ~~...~~ (VC(e_i)):
    ~~...~~ t = 0
    for k in 1,2,...n:
        t ← VC(e_i)[k] + t   // increment t by VC(e_i)[k]
    end for
return t
```

Clearly ~~...~~ $\text{logical}(e_i) = \text{Convert}(VC(e_i)) = t = \sum_k VC(e_i)[k]$

Now,

Claim: t defines weakly consistent timestamp

Proof: consider $e_i, e_j \in H \Rightarrow$

$e_i \to e_j$ where $\to$ is HB relation

Now $VC(e_i) < VC(e_j)$ - strong consistency for VC

$\Rightarrow \forall k \; VC(e_i)[k] \leq VC(e_j)[k]$ &

$\exists k' \; VC(e_i)[k'] < VC(e_j)[k']$

$\Rightarrow \sum_k VC(e_i)[k] < \sum_k VC(e_j)[k]$

$\therefore t(e_i) < t(e_j)$ (convert(VC(e_i)) < Convert(VC(e_j)))

$\therefore$ Weak consistency holds

P1. Proof:

consider $L_i = r_i^{i+2}$ & $F_j = s_j^{j+1}$

Now,

claim 1: & $\forall$ ~~(i,j)~~ $j \neq i+1$ $L_i > F_j$

Pf: as $L_i = r_i^{i+2} > r_i^{j} > s_j^{i} > s_j^{j+1} = F_j$

since $j \neq i+1$, ~~r~~ r_i^j exists

(else $j = i+1$, r_i^{i+1} doesn't Exist !)

Claim 2: $1+i = j$ $L_i \parallel F_{i+1}$

Pf: since P_{i+1} doesn't directly send message to P_i & only communication is the initial s_i^{i+1}

→ Now ~~from~~ from claim 1 & 2

– since $VC(L_i)[i] > VC(F_{i+1})[i]$

(P_i has its ~~on~~ most updated info)

& $L_i \parallel F_{i+1}$ ⇒

$\exists\ i' \ni$ ~~$VC(L_i)$~~

$VC(L_i)[i'] < VC(F_{i+1})[i']$

→ Now, we can show that for this i, this i' is unique & different $\forall i$.

Now since ~~b~~ there are N such i's, where ~~a~~ N is num. of processes ⇒ N unique i's.

So for this case, to show strong consistency ~~a~~ there should be atleast N different bits, for each pair $(i, i+1)$ which are N in number

Now we prove above by contradiction

let $\exists$ i & j, $i \neq j$ $\ni$

$VC(L_i)[i'] < VC(F_{i+1})[i']$

& $VC(L_j)[i'] < VC(F_{j+1})[i']$

i.e. $i' = j'$ (i' defined as before)

Using Claim 1 we can arrive at contradicⁿ

~~But $i \neq j \Rightarrow VC(L_i)[i'] < VC(F_{i+1})$~~

~~Now~~

→ Thus $\forall i$, $\exists$ unique i'

so for $i, i+1$ pair there is unique i'

that should exist in vector

→ Thus vector needs $\geq n$ bits