Biostatistics and Informatics

BTM6000

Jochen G. Raimann

Session 2: Descriptive Statistics

Slide credit to
Prof. Levi Waldron
CUNY SPH

Material to cover for session 2

1. Review session #1
2. Frequency tables, bar plots, histograms, box plots, dot plots
3. Measuring center: Mean, Median, Mode
4. Measuring spread: Standard Deviation, Variance, Interquartile Range, Range

Vocabulary review

- **Population**: the entire group of interest
- **Sample**: a subset of the population
- **Record or observation:** data of an individual
- **Descriptive statistics**: summarization, visualization
- **Inferential statistics**: learning about the population from the sample

Vocabulary review

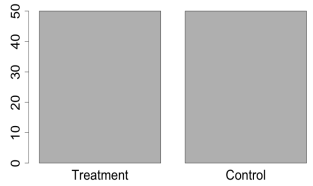
- **Variable**: any characteristic that is recorded for the subjects in a study
  - **Nominal** (**binary**, **categorical**)
    - treatment group, marital status, race
  - **Ordinal**
    - birth order among siblings, ranking
    - Sometimes nominal + ordinal lumped as categorical
  - **Quantitative** (**discrete**, **continuous**)
    - number of children
    - height, weight, age

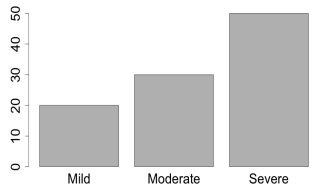
Nominal / ordinal data summary

- Frequency tables / contingency tables
- Barplot

Treatment	Control
50	50

Mild	Moderate	Severe
20	30	50





(one variable)

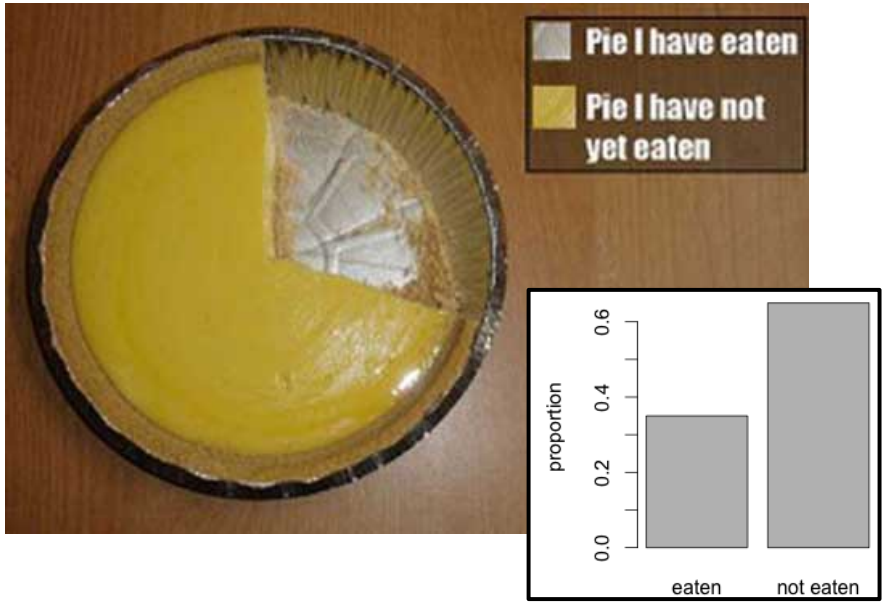
Nominal / ordinal data summary

- Pie charts are another alternative:

Generally disliked by statisticians.

Why? Can you tell what proportion of the pie I have eaten? I can't.

Anything a pie chart can show, a bar chart can show better.



Nominal / ordinal data summary: Frequency, Proportion, & Percentage

- If 4 students received an “A” out of 40 students, then:
  - the **frequency** is 4
  - the **proportion** or **relative frequency** is 0.10 =4/40
  - the **percentage** is 10% (.10*100=10%)
- Any of these can be used for a frequency table or barplot.

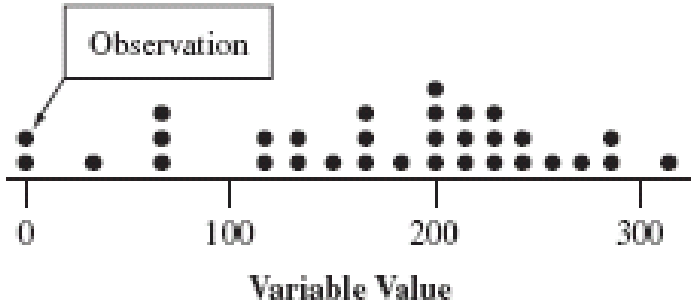
Quantitative variables: graphical summary

- **Dot plot:** shows a dot or dash for each observation
- **Histogram:** categorizes the data then shows it as a barplot
- **Box plot:** qualitative and quantitative measures together

- These plots help identify central tendency, spread, skew, and outliers

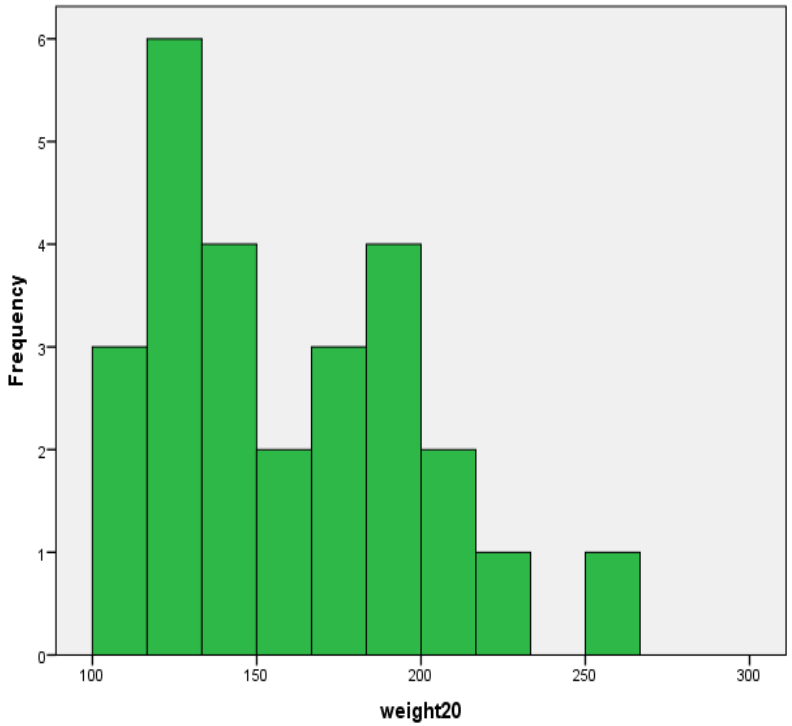
Dot Plots

Dot plots show every individual observation:



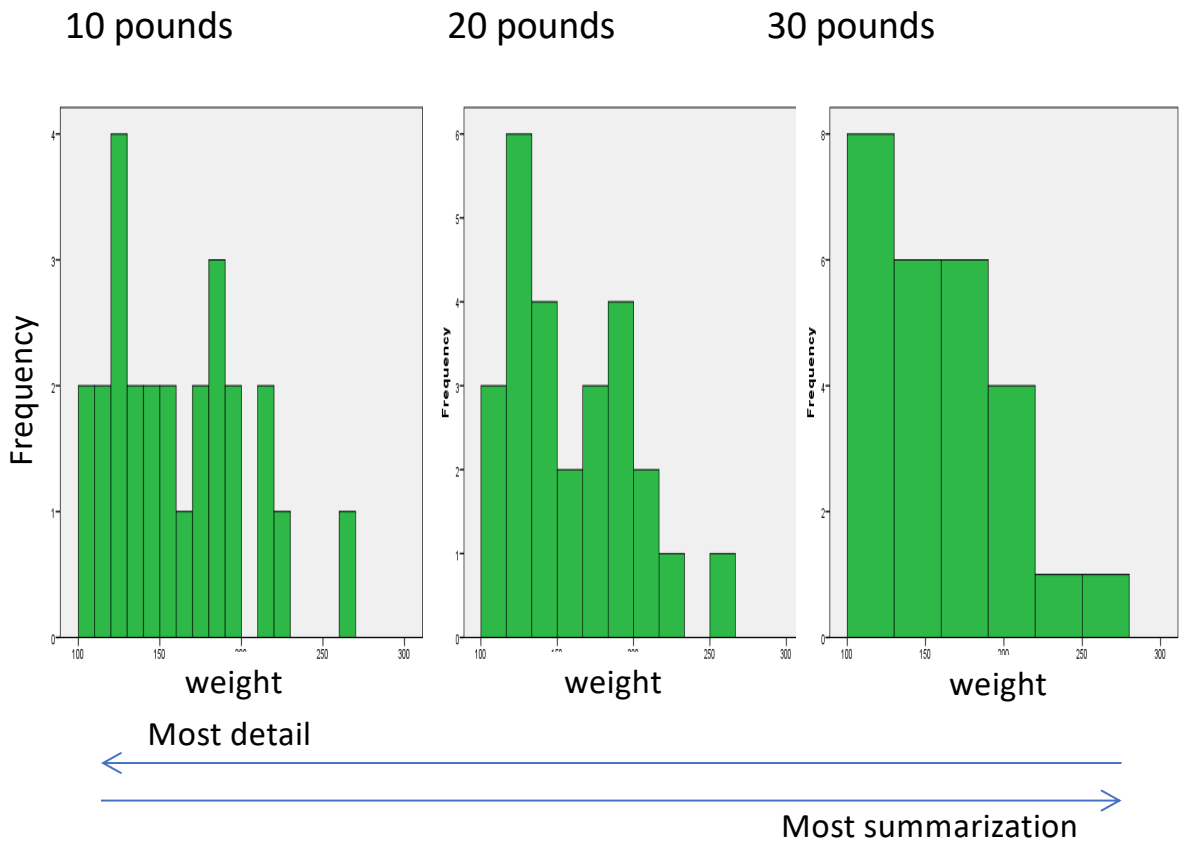
Above, some “binning” has been done, making this like a histogram

Histograms



- A **Histogram** is a graph that uses bars to portray the frequencies or the relative frequencies of the possible outcomes for a quantitative variable

Histogram: effect of the bin size



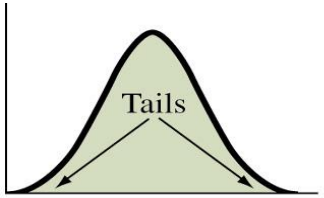
Interpreting Histograms

- Look for **center, spread, and shape:**
  - Assess where a distribution is **centered** by finding the median (half of the data points are above the median, half are below)
  - Assess the **spread** of a distribution
  - Assess the **shape** of a distribution:
    - roughly symmetric?
    - skewed to the right?
    - skewed to the left?

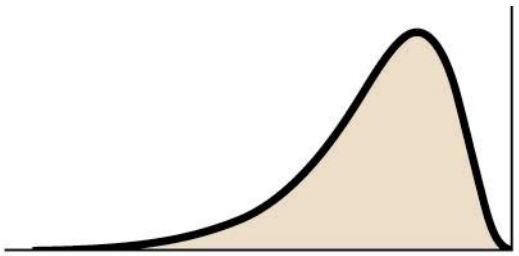
Shape: skew

- *Symmetric Distributions*:

left and right sides of the histogram are mirror images

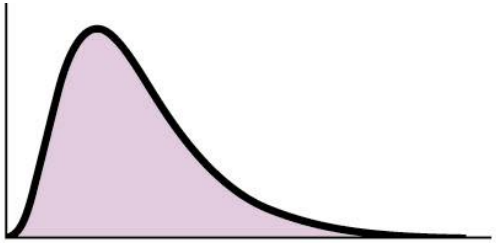


Symmetric



Skewed to the left

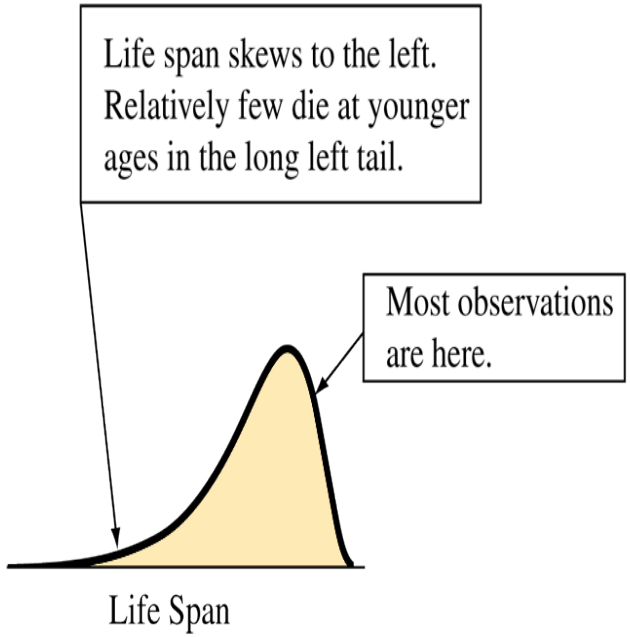
A distribution is skewed to the left if the left tail is longer than the right tail

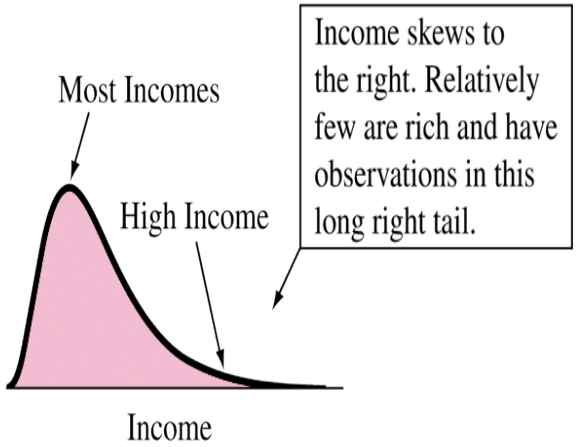


Skewed to the right

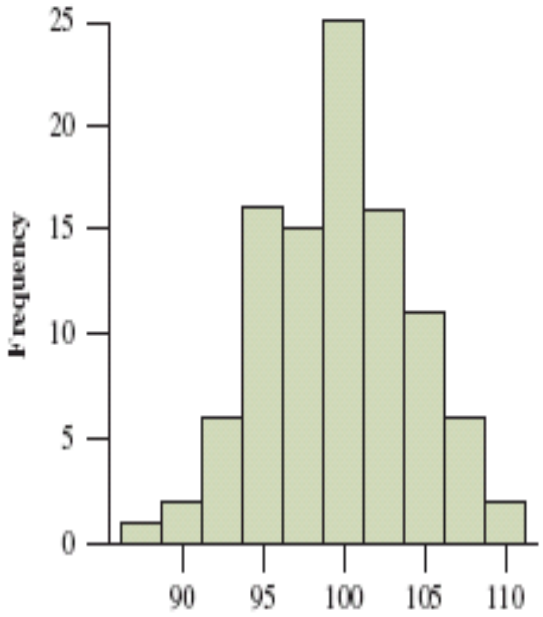
A distribution is skewed to the right if the right tail is longer than the left

Examples of Skewness

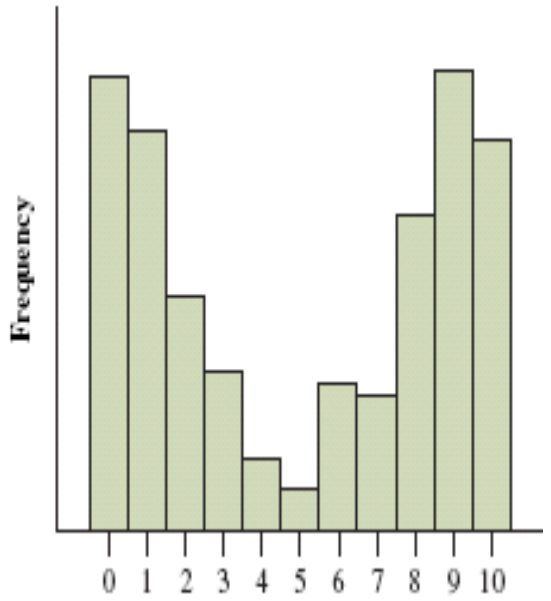




Shape: Modality



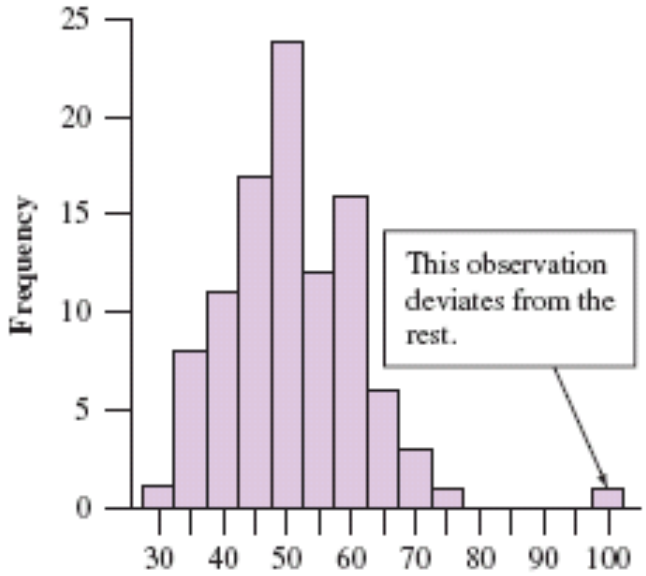
Unimodal



Bimodal

Outliers

- An **Outlier** falls far from the rest of the data:



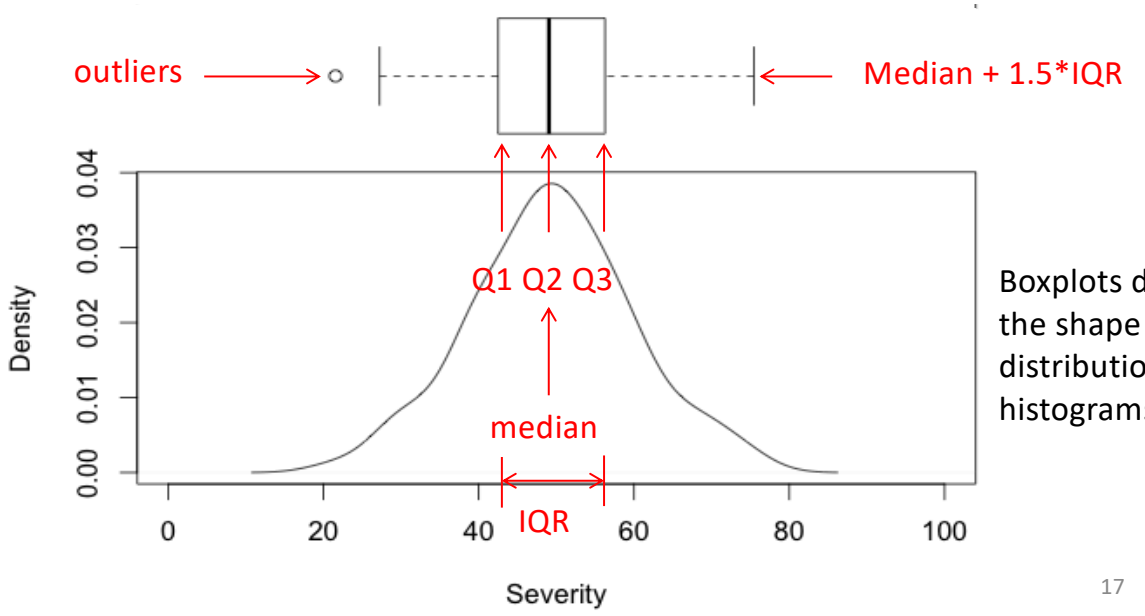
Outliers deserve special attention

It may or may not be justifiable to remove outliers

In general we are hesitant to remove any data, unless there is a very good reason

Boxplot for a single distribution

Percentile = % of data below a given value
Quartile = 25, 50, 75th percentile
Median = 50th percentile
IQR = Interquartile Range = 25th – 75th percentile



Boxplots do not display the shape of the distribution as clearly as histograms

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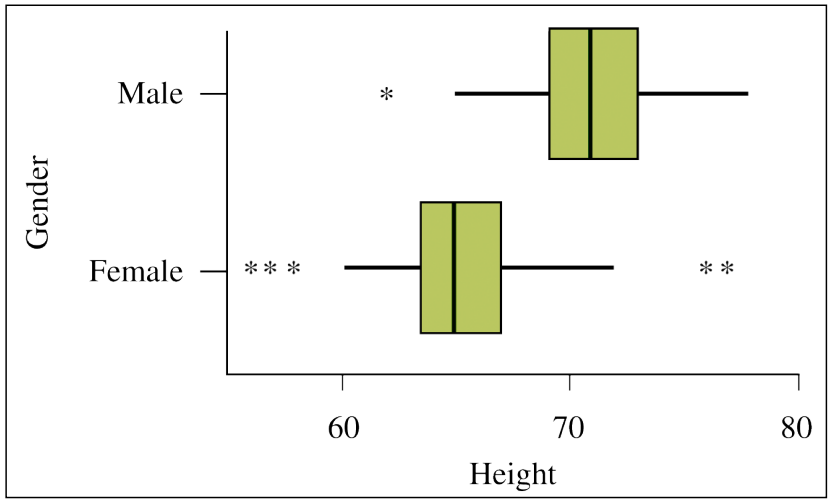
Boxplot for Comparing Distributions

However boxplots are useful for making graphical comparisons of two or more distributions.

They are useful for identifying potential outliers



Numerical summaries of quantitative variables

- Mean, median, mode
- Quartiles, percentiles
- Range
- Standard deviation

Mean

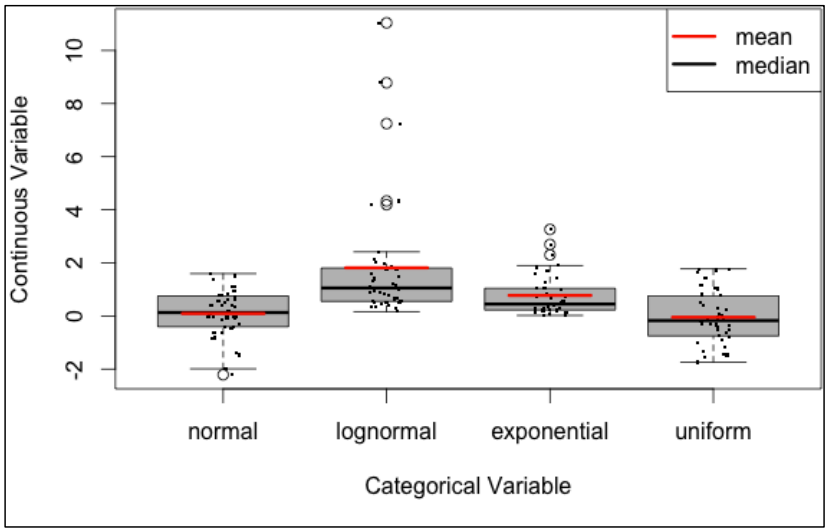
- The mean is the sum of the observations divided by the number of observations
- E.g., add up how much each person paid for lunch, divide by the number of people

$$\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$

(capital sigma notation
or summation notation)

Mean II

- The mean is not ***resisitant***.
- *It is strongly influenced by outlying values*



Median

- The **median** is the midpoint of the observations when they are ordered
- If the number of observations is:
  - Odd, then the median is the middle observation
  - Even, then the median is the average of the two middle observations
- Median is the **50^{th} percentile**

Median

- The median is the midpoint of the observations when they are ordered from the smallest to the largest (or from the largest to smallest)
- 50^{th} percentile, 2^{nd} quartile
- Examples:
  - 1, 2, **3**, 4, 5
  - 1, 2, **3**, 4, 1000000000
  - 1, 2, 3, 4: median = **2.5**

Median height of the 7 Dwarfs

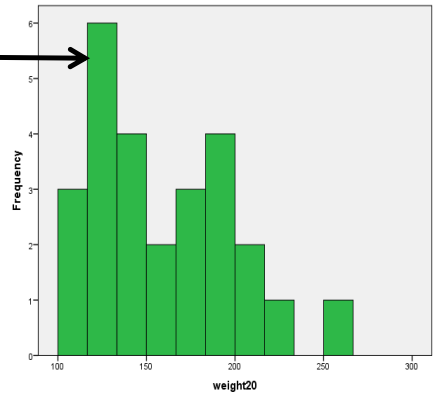
when the 7 dwarfs are arranged on order of height,
the median dwarf is in the middle (#4)

Median *is* resistant to outliers

Mode

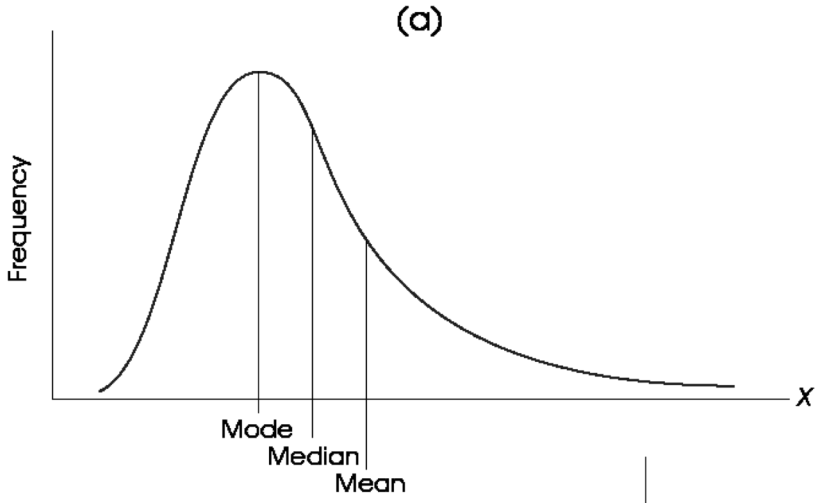
- Mode is the value that occurs most often
  - The mode is most often used with categorical data
  - The mode is the highest bar in the histogram or barchart

mode

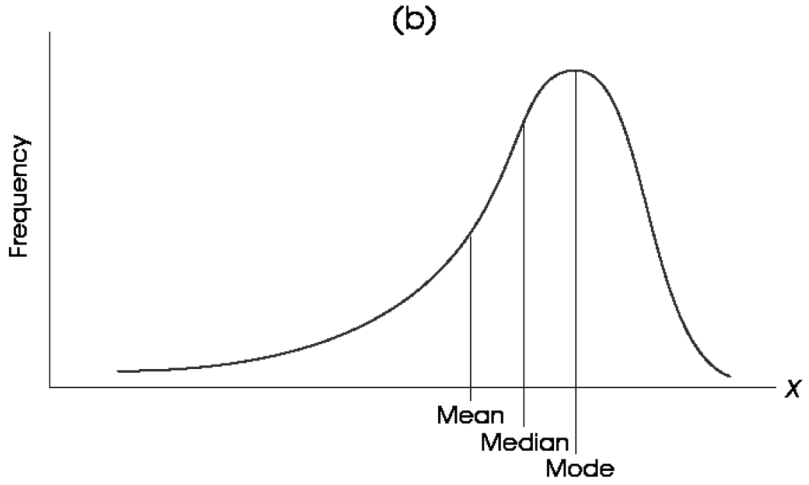


Note that for a histogram, the mode changes with the width of the bins

Central Tendency & Skewness

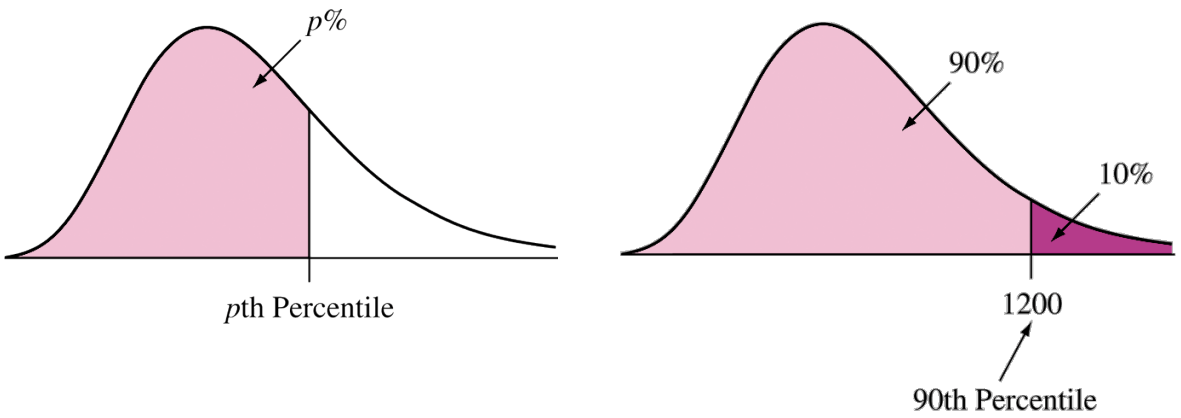


Can infer skewness from differences between mean and median



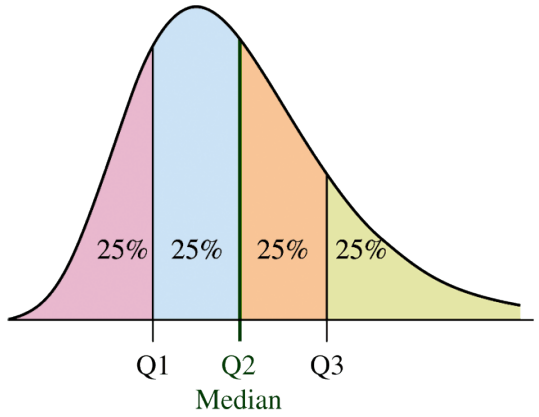
Percentile

- The **p^{th} percentile** is a value such that p percent of the observations fall <u>below or at that value</u>



Quartiles

- Quartiles split the data into four equal parts
  - The median is the second quartile, Q2. 50% of observations are below, 50% are above Q2.
  - The first quartile, Q1, is the median of the lower half of the observations. 25% of observations are below Q1.
  - The third quartile, Q3, is the median of the upper half of the observations. 75% are below.

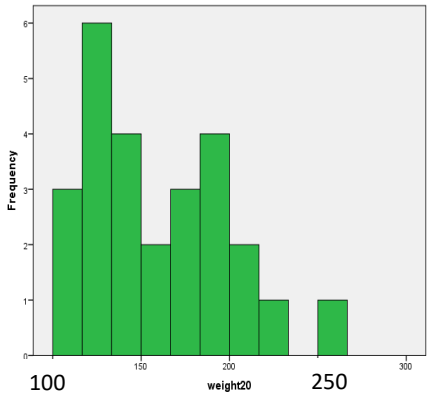


Range

- **Range** is one way to measure the spread of a quantitative variable
- The range is the difference between the largest and smallest values in the data set;

Range = max – min

Range ~ 250 – 100 = 150



Note that range is strongly affected by outliers

Variance and Standard Deviation (SD)

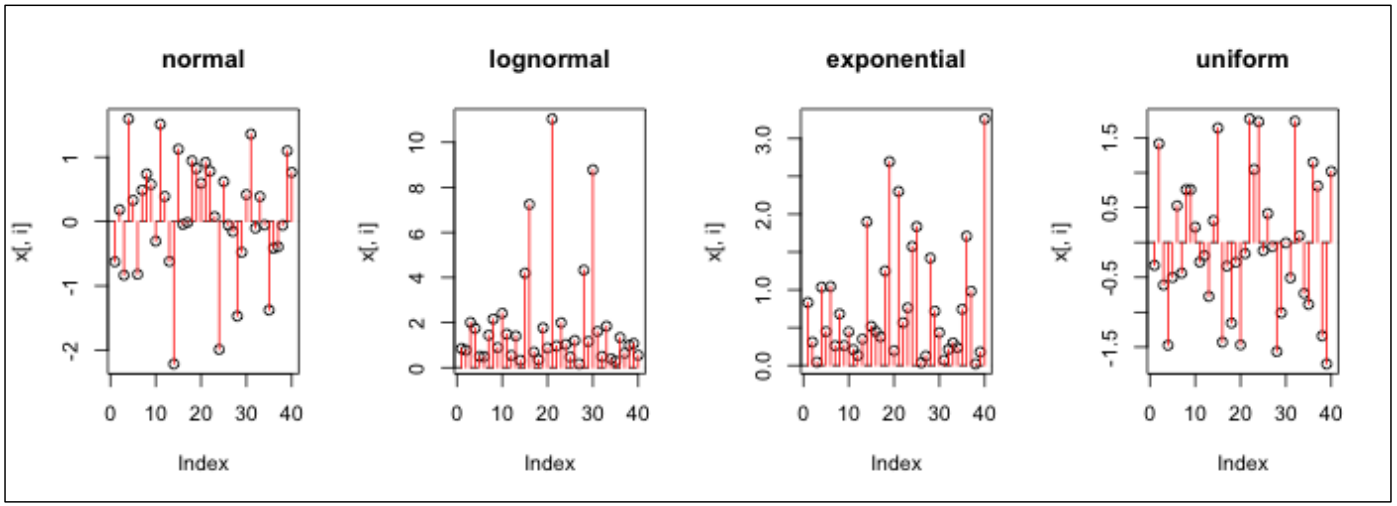
- Gives a measure of variation by:
  - Calculating squared *deviations* of each observation from the mean
  - Calculating the *adjusted mean* of squared deviations

- Variance is the square of standard deviation

n-1 for a *sample*
n for a *population*

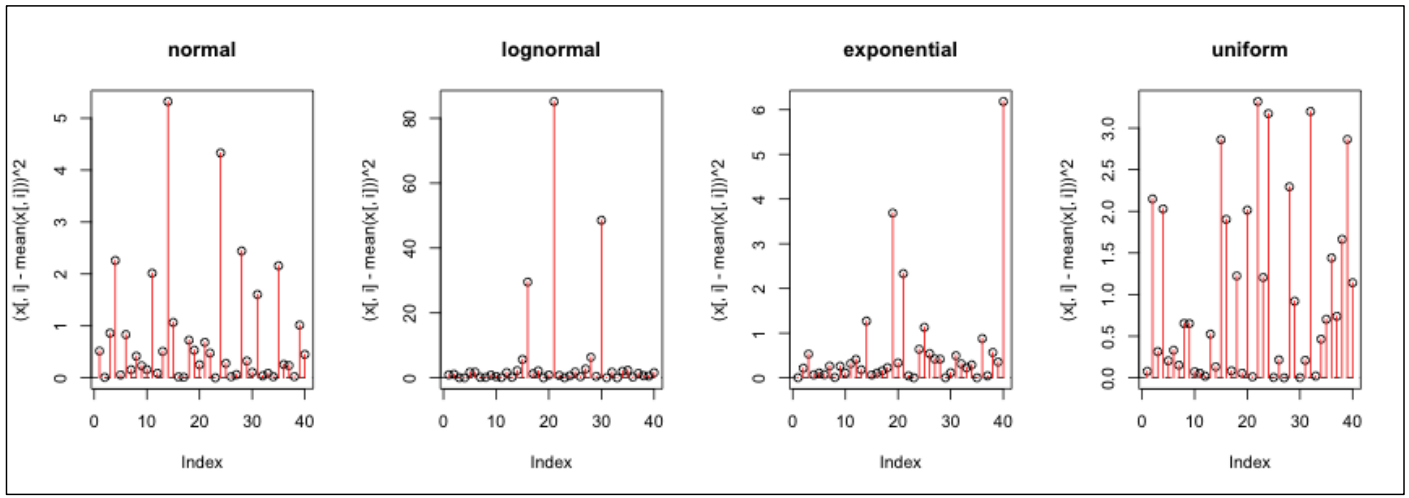
$$\text{var} = s^2 = \frac{1}{n-1}\sum_{i=1}^{n}(x_i - \overline{x})^2$$

Deviations from mean: $x - \bar{x}$



$$s = \sqrt{\frac{\sum (x - \bar{x})^2}{n - 1}}$$

Squared deviations: $\left(x - \bar{x}\right)^2$

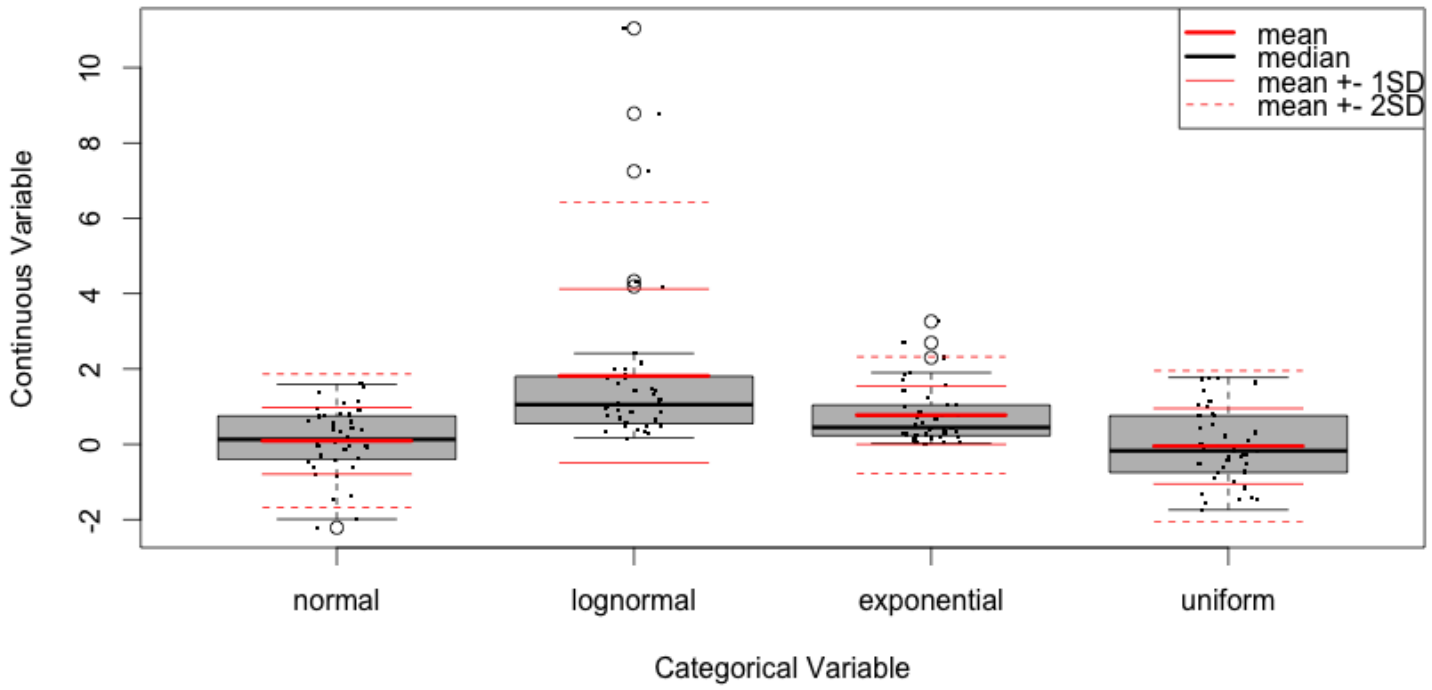


- SD not robust to outliers/skew, but convenient for describing normal distributions

Standard Deviation

- Has the same units as the observed variable
- s measures the spread of the data
- s = 0 only when all observations have the same value, otherwise s > 0. As the spread of the data increases, s gets larger.
- s is not *resistant*. Strong skewness or a few outliers can greatly increase s.

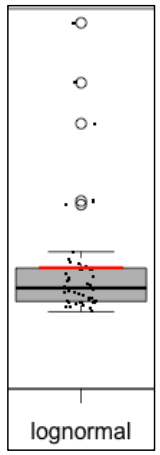
Standard Deviation



Alternative measures of spread

* Interquartile Range (IQR):
  - Q3 – Q1 (75^{th} percentile minus 25^{th} percentile)
* Median Absolute Deviation (MAD):
  - median(x – median(x))
- Range:
  - max(x) – min(x)

***robust to outliers and skew**



Vocabulary Summary

- Dot plot, histogram, box plot
- Mean, Median, Mode
- Percentiles, quartiles, Q1, Q2, Q3
- Standard deviation, IQR, MAD, range
- Capital-Sigma (summation) notation
- Unimodal, bimodal distributions
- Skew, left skew, right skew
- Outliers

Learning Summary

- Define and calculate Mean, Median, Mode, Range, Standard Deviation
- Create histograms, boxplots, barplots, frequency tables
- Identify skewed and symmetric distributions
- Identify outlier data points