2 points

Consider the piecewise function defined by

$$f(t)=\begin{cases} t, & \text{if } 0\le t<2 \\ t^2, & \text{if } 2\le t<6 \\ t^3, & \text{if } 6\le t \end{cases}$$

The function $f(t)$ is expressed in the form of unit step function $U(t)$ as

$$t+(t^2-t)U(t-2)+(t^3-t^2)U(t-6)$$

..

$$t+(t^2-t)U(t-2)+t^3U(t-6)$$

..

$$t+t^2U(t-2)+t^3U(t-6)$$

..

$$t+(t-2)^2U(t-2)+(t-6)^3U(t-6)$$

..

Clear selection

2 points

Let us consider a periodic function $f(t)$, which is defined as

$$f(t) = \begin{cases} 1, & if\ 0 \le t < a \\ -1, & if\ a \le t < 2a \end{cases}$$

If the Laplace transform of $f(t)$ is $\dfrac{\tanh(\frac{3s}{8})}{s}$ then the period of the function is

- ( ) 3/4
- ( ) 3/2
- (•) 1/2
- ( ) 3/8

Clear selection

2 points

The Fourier coefficients $\{a_n\}$ and $\{b_n\}$ of the Fourier series corresponding to the function $f(x)=e^{-x}, 0<x<2\pi$ are

$$a_n = \frac{1-e^{-2\pi}}{\pi}.\frac{1}{n^2-1}, \quad n \geq 0$$

- [ ] . .

$$a_n = \frac{1-e^{-2\pi}}{\pi}.\frac{1}{n^2+1}, \quad n \geq 0$$

- [x] . .

$$b_n = \frac{1-e^{-2\pi}}{\pi}.\frac{n}{n^2-1}, \quad n \geq 1$$

- [ ] . .

$$b_n = \frac{1-e^{-2\pi}}{\pi}.\frac{n}{n^2+1}, \quad n \geq 1$$

- [x] . .

2 points

If $L^{-1}\left\{\frac{4(3s^3-4s)}{(s^2+4)^4}\right\} = L^{-1}\{F(s).G(s)\}$, where $F(s) = L\{f(t)\}$ and $G(s) = L\{\cos 2t\}$, then $f(t)$ is given by

- ( ) $t \sin 2t$
- ( ) $t^3 \sin 2t$
- (x) $t^2 \sin 2t$
- ( ) None of these.

Clear selection

2 points

If $L\{f(t)\} = \dfrac{s+1}{(s^2+2s+2)(s^2+1)}$, then $f(t)$ is given by

$\frac{1}{5}[e^{-t}(\sin t + 2\cos t) + (\sin t - 2\cos t)]$

◯ . .

$\frac{1}{5}[e^{-t}(\cos t - 2\sin t) + (3\sin t - \cos t)]$

◉ . .

$\frac{1}{5}[-e^{-t}(\sin t + 2\cos t) + (\sin t - 2\cos t)]$

◯ . .

None of these.

◯ . .

Clear selection

2 points

If the differential equation is given by

$$\frac{d^2 y(t)}{dt^2} + ty(t) = 0, \quad y(0) = y'(0) = 0$$

Then the Laplace transform of $y(t)$ is

$ce^{\frac{s^2}{2}}$, c is an arbitrary constant

..

$c_1 e^{s} + c_2 e^{-s}$, c_1, c_2 are arbitrary constants

..

$(c_1 + c_2 s)e^{s}$, c_1, c_2 are arbitrary constants

..

$ce^{\frac{s^3}{3}}$, c is an arbitrary constant

..

Clear selection

2 points

If $P_i (i = 0, 1, 2)$ denote the Legendre polynomial of order i and $f(x)$ is defined as

$$f(x) = \begin{cases} 4P_2 + 3P_1 + 11P_0, & |x| \le 1 \\ 0, & |x| > 1 \end{cases}$$

then the value of

$$\int_{-\frac{5}{2}}^{\frac{5}{2}} (2x+1) f(x)\,dx \text{ is}$$

- 0
- 2
- 4
- 8

Clear selection

2 points

Let Fourier transform of $f(x)$ and its inverse transform be defined below as respectively

$$F\{f(x)\} \equiv \tilde{f}(p) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{ipx} f(x)dx ,$$

$$f(x) = \frac{1}{\sqrt{2\pi}} \int_{-\infty}^{\infty} e^{-ipx} \tilde{f}(p)dp$$

If $f(x) = 1, |x| < 2$, then the value of the integral $\int_{-\infty}^{\infty} \frac{\sin p \cos \frac{px}{2}}{p} dp$ is given by

$\pi, \ \forall x \in (-2,0] \cup (0,2)$

- [ ] . .

$\frac{\pi}{2}, \ \forall x \in [0,2] \cup (-2,0]$

- [ ] . .

$\pi, \ \forall x \in (-2,0)$

- [ ] . .

$\frac{\pi}{2}, \ \forall x \in [0,2)$

- [x] . .

2 points

Which of the following statement(s) is/are correct?

For the function $f(x) = |x|, -L < x \le L$, the integral $\int_{-L}^{L} |f(x)|\,dx$ does not exist.

☑ . .

The conditions for the convergence of Fourier series (Dirichlet Conditions) provide the set of sufficient but not necessary conditions.

☑ . .

$\ln x$ cannot be expanded into a Fourier series in the interval [0, 1].

☐ . .

$\cos(\frac{2\pi nx}{L})$ and $\sin(\frac{2\pi mx}{L})$ defined in $-L \le x \le L$, represents two non-orthogonal functions.

☑ . .

2 points

If the function $f(x)=\begin{cases}-\pi, & -\pi<x<0\\ x\ , & 0<x<\pi\end{cases}$ represented by the trigonometrical series then consider the following statements:

A. $f(x)$ is periodic with period 2π.

B. $V=1-\frac{1}{3}+\frac{1}{5}-$..........

C. At $x=0$, the trigonometrical series converges to $\frac{1}{2}[\lim_{x\to 0+} f(x)+\lim_{x\to 0-} f(x)]$.

- [ ] A is true and $V=\frac{3\pi}{4}$ ..
- [x] C is true and $V=\frac{\pi}{4}$ ..
- [ ] A is true and C is false. ..
- [x] $V=\frac{\pi}{4}$. ..

A copy of your responses will be emailed to ankushbhagat_co20b2_68@dtu.ac.in.