CS 599 A1: Assignment 2

Due: Thursday, February 19, 2026

Total: 100 pts

Instructor: Ankush Das

- This assignment is due midnight on the above date and must be submitted electronically on Gradescope.
- You are provided a `tex` file, named `asgn2.tex`. It contains environments called `solution` to enter your solutions. And please include your name and BU ID in the author section (above).
- Although not recommended, you can submit handwritten answers scanned as a PDF and clearly legible.
- Each problem specifies whether LLMs are allowed for it or not. Please adhere to it.
- Each problem is marked whether it is mandatory or optional. You must solve some of the optional problems to get to 100 points.

Definition. $\neg A$ is defined as $A \supset \bot$.

Problem 1 (Mandatory; No LLM; 10 pts) *Recall the natural deduction problems from Assignment 1. For the following propositions, provide the corresponding proof term* [2.5 pts each].

(i) $(A \supset (B \supset C)) \supset (A \land B) \supset C$

(ii) $A \supset \neg\neg A$

(iii) $(\neg A \lor \neg B) \supset \neg(A \land B)$

(iv) $(A \land (B \lor C)) \supset (A \land B) \lor (A \land C)$

Problem 2 (Mandatory; No LLM; 15 pts) *Recall the $A \land B$ operator from constructive logic. Suppose we keep the introduction rule unchanged, but update the elimination rule as follows:*

$$\dfrac{A \uparrow \qquad B \uparrow}{A \land B \uparrow} \land I \qquad\qquad \dfrac{A \land B \downarrow \qquad \begin{array}{c} \overline{A \downarrow}^{\,x} \quad \overline{B \downarrow}^{\,y} \\ \vdots \\ C \uparrow \end{array}}{C \uparrow} \land E^{x,y}$$

- *Are the rules locally sound? If yes, show a reduction. Otherwise, give an explanation of unsoundness.*
- *Are the rules locally complete? If yes, show an expansion. Otherwise, give an explanation of incompleteness.*
- *Provide an appropriate proof term for the elimination rule.*

Problem 3 (Mandatory; No LLM; 15 pts) *Recall the $A \supset B$ operator from constructive logic. Suppose we keep the introduction rule unchanged, but update the elimination rule as follows:*

$$\dfrac{\begin{array}{c} \overline{A \downarrow}^{\,x} \\ \vdots \\ B \uparrow \end{array}}{A \supset B \uparrow} \supset I^{x} \qquad\qquad \dfrac{A \supset B \downarrow \qquad A \uparrow \qquad \begin{array}{c} \overline{B \downarrow}^{\,x} \\ \vdots \\ C \uparrow \end{array}}{C \uparrow} \supset E^{x}$$

- *Are the rules locally sound? If yes, show a reduction. Otherwise, give an explanation of unsoundness.*
- *Are the rules locally complete? If yes, show an expansion. Otherwise, give an explanation of incompleteness.*
- *Provide an appropriate proof term for the elimination rule.*

Problem 4 (Mandatory; No LLMs; 15 pts) *Suppose we define a new operator $A \star B$ with the following introduction and elimination rules:*

$$\dfrac{\begin{array}{c}\overline{A\downarrow}^{\,x}\\ \vdots\\ B\uparrow\end{array}}{A\star B\uparrow}\,\star I_1^x \qquad \dfrac{\begin{array}{c}\overline{B\downarrow}^{\,x}\\ \vdots\\ A\uparrow\end{array}}{A\star B\uparrow}\,\star I_2^x \qquad \dfrac{A\star B\downarrow \quad A\uparrow \quad \begin{array}{c}\overline{B\downarrow}^{\,x}\\ \vdots\\ C\uparrow\end{array} \quad B\uparrow \quad \begin{array}{c}\overline{A\downarrow}^{\,y}\\ \vdots\\ C\uparrow\end{array}}{C\uparrow}\,\star E^{x,y}$$

- *Are the rules locally sound? If yes, show a reduction. Otherwise, give an explanation of unsoundness.*
- *Are the rules locally complete? If yes, show an expansion. Otherwise, give an explanation of incompleteness.*
- *Can $A \star B$ be expressed using the standard operators in constructive logic?*

Problem 5 (Mandatory; No LLMs; 15 pts) *Suppose we define a new operator $A\heartsuit B\Diamond C$ with the following introduction and elimination rules:*

$$\dfrac{A\uparrow}{A\heartsuit B\Diamond C\uparrow}\,\heartsuit\Diamond I_1 \qquad \dfrac{B\uparrow \quad C\uparrow}{A\heartsuit B\Diamond C\uparrow}\,\heartsuit\Diamond I_2 \qquad \dfrac{A\heartsuit B\Diamond C\downarrow \quad \begin{array}{c}\overline{B\downarrow}^{\,x} \quad \overline{C\downarrow}^{\,y}\\ \vdots\\ D\uparrow\end{array} \quad \begin{array}{c}\overline{A\downarrow}^{\,z}\\ \vdots\\ D\uparrow\end{array}}{D\uparrow}\,\heartsuit\Diamond E^{x,y,z}$$

- *Are the rules locally sound? If yes, show a reduction. Otherwise, give an explanation of unsoundness.*
- *Are the rules locally complete? If yes, show an expansion. Otherwise, give an explanation of incompleteness.*
- *Provide a derivation of $A\heartsuit B\Diamond C \supset A\heartsuit C\Diamond B$.*

Problem 6 (Optional; No LLMs; 15 pts) *Determine if the following propositions are derivable using rules of verifications ($A\uparrow$) and uses ($A\downarrow$). If yes, provide a derivation. Otherwise, briefly explain why.*

(i) $((A \vee B) \wedge (A \supset C) \wedge (B \supset C)) \supset C\uparrow$

(ii) $(A \supset B) \supset ((A \vee C) \supset (B \vee C))\uparrow$

(iii) $(A \vee B) \supset ((A \supset C) \supset ((B \supset D) \supset (C \vee D)))\uparrow$

Problem 7 (Optional; LLMs ok; 30 pts) *Given a derivation $\mathcal{D}$ of A true, can you construct a derivation $\mathcal{E}$ of $A\uparrow$.*
Hint: Try this problem with only the $A \wedge B$ and $A \supset B$ operators first.

Problem 8 (Optional; LLMs ok; 15 pts) *Design introduction and elimination rules of a new connective $A \bullet B$ such that it is locally sound but not locally complete.*

Problem 9 (Optional; LLMs ok; 15 pts) *Design introduction and elimination rules of a new connective $A \circ B$ such that it is locally complete but not locally sound.*