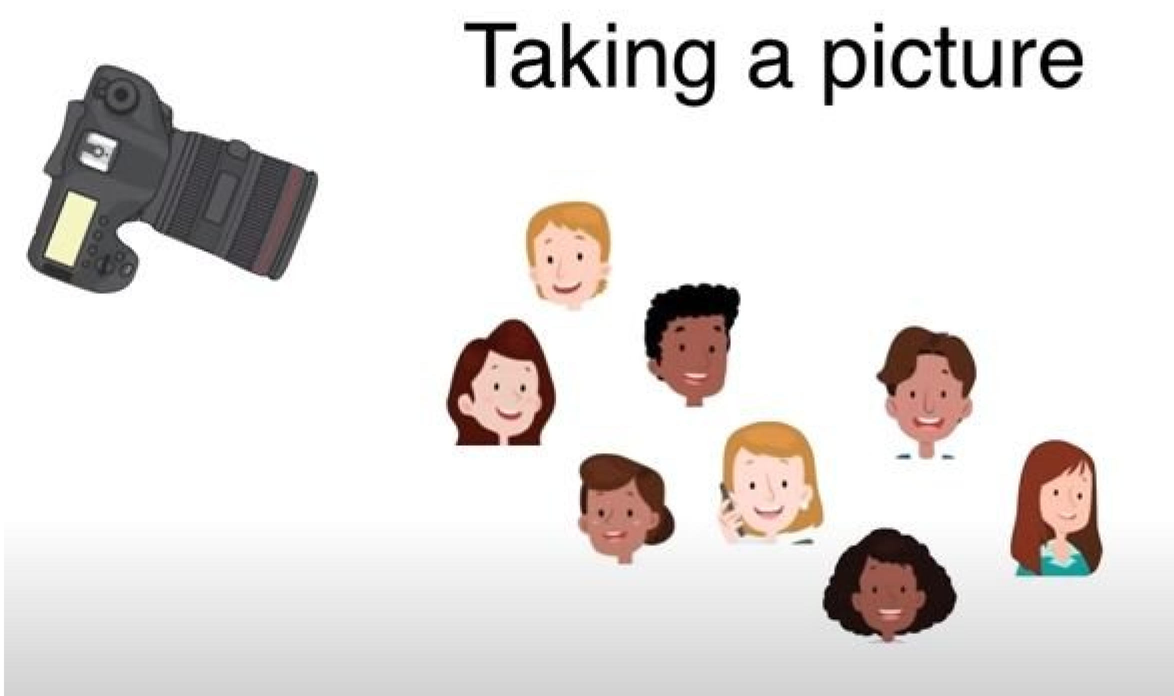
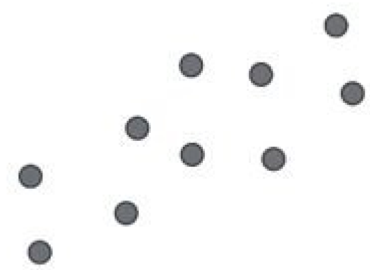
Principle Component Analysis

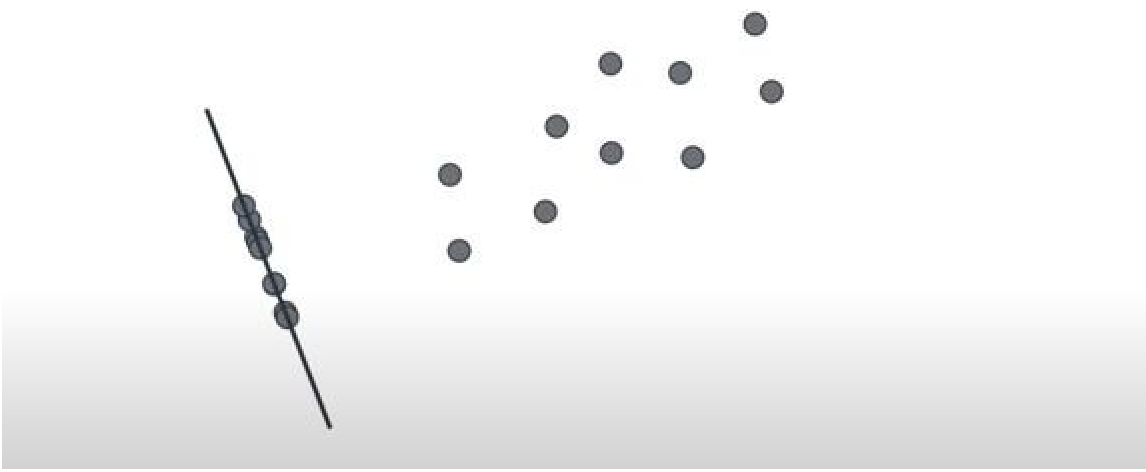

Taking a picture

Taking a picture

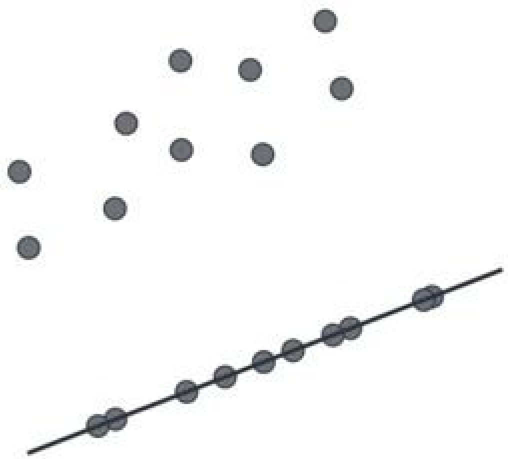
Dimensionality Reduction



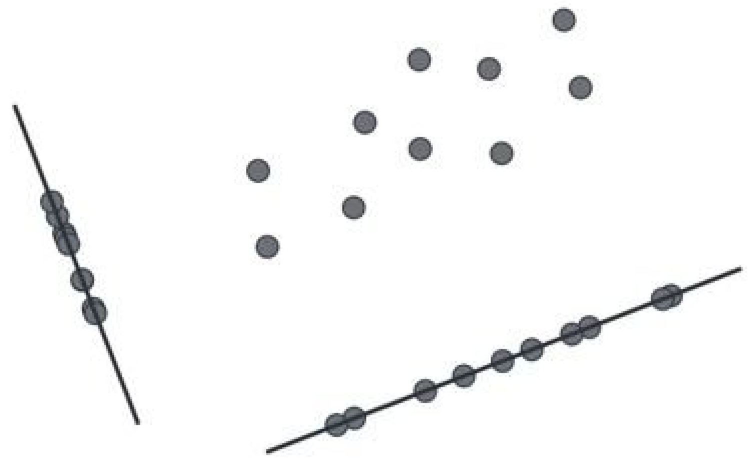
Dimensionality Reduction



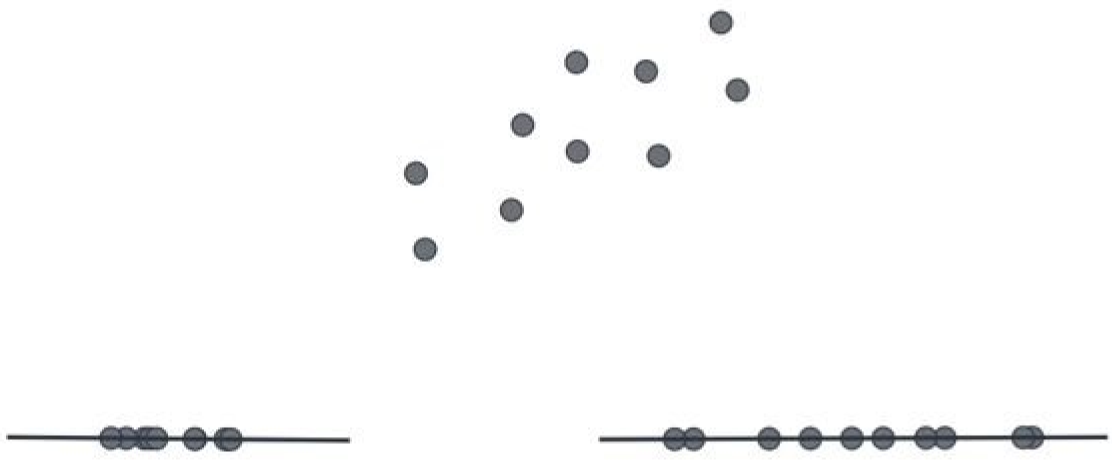
Dimensionality Reduction



Dimensionality Reduction



Dimensionality Reduction



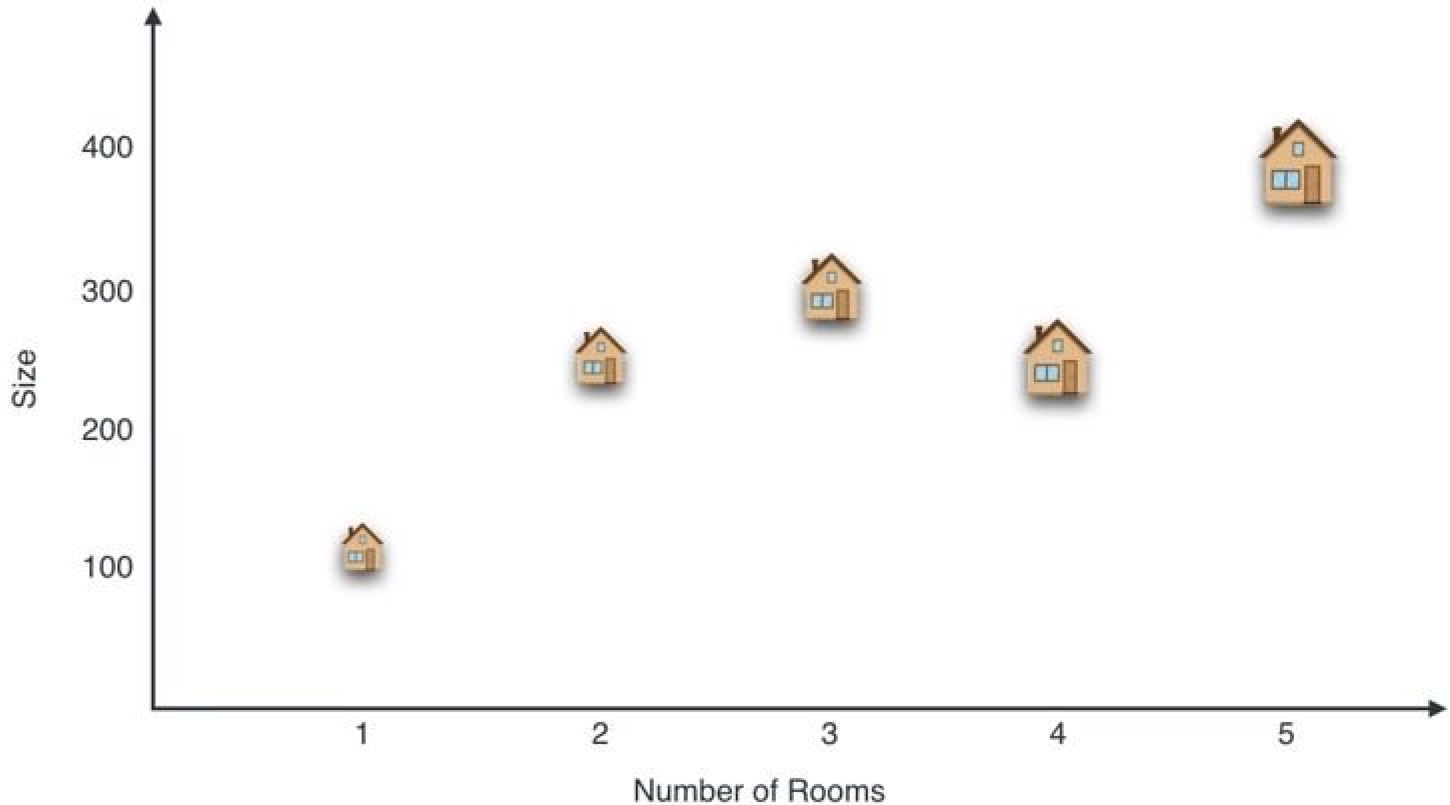
Housing Data

Size
Number of rooms
Number of bathrooms
Schools around
Crime rate

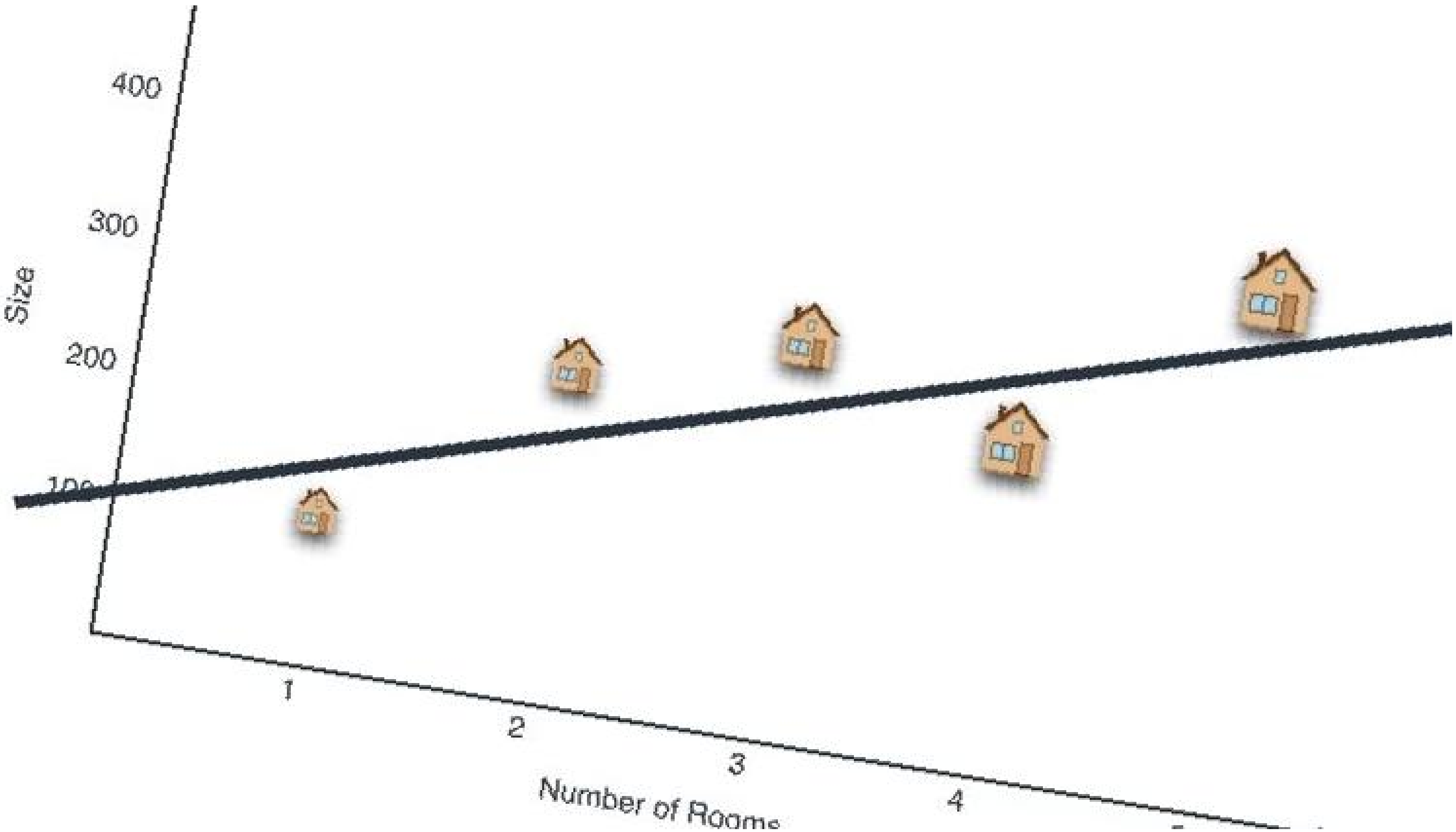
Housing Data

Size
Number of rooms → Size feature
Number of bathrooms

Schools around → Location feature
Crime rate



Size
400
300
200
100
1
2
3
4
5
Number of Rooms



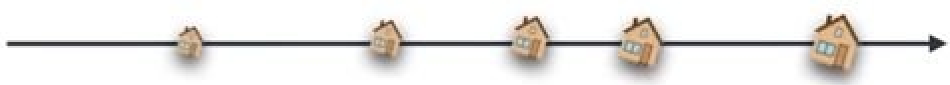
400
300
200
Size
1
2
3
4
Number of Rooms


Size feature

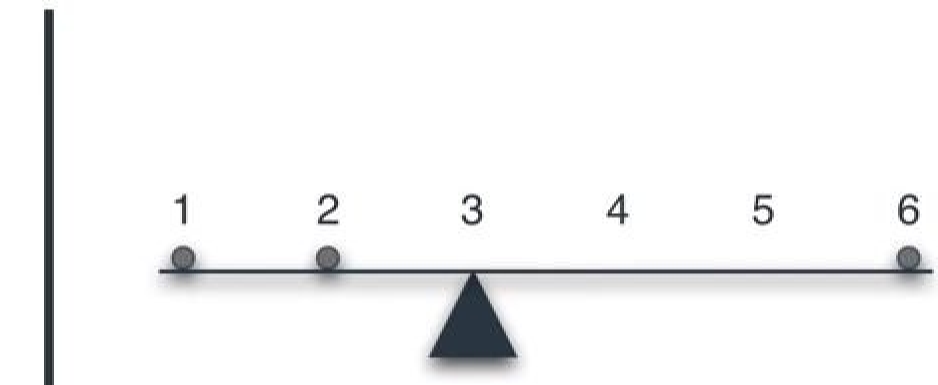
2 dimensions

size
number of rooms

1 dimension

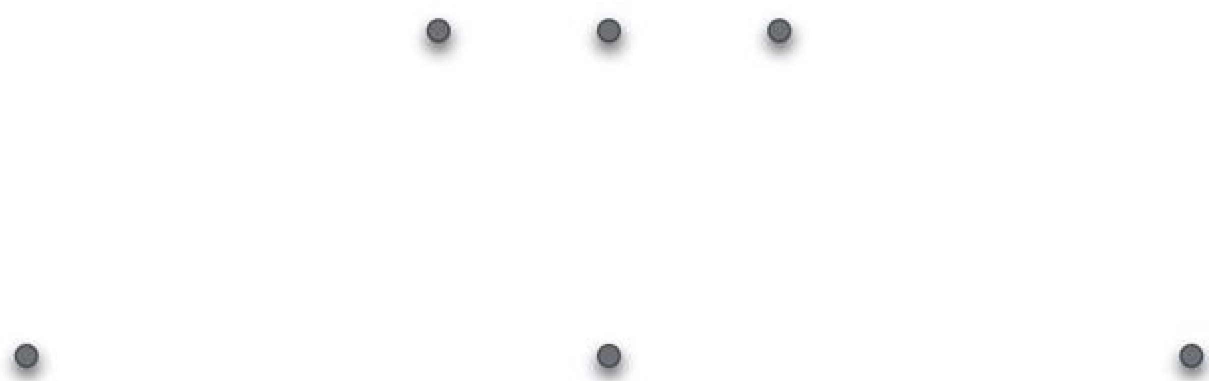


Mean

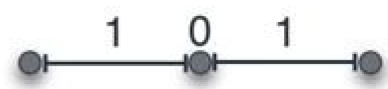


$$\text{Mean} = \frac{1+2+6}{3} = 3$$

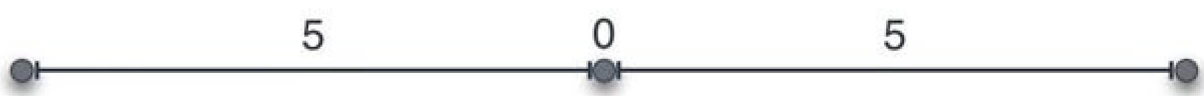
Variance



Variance

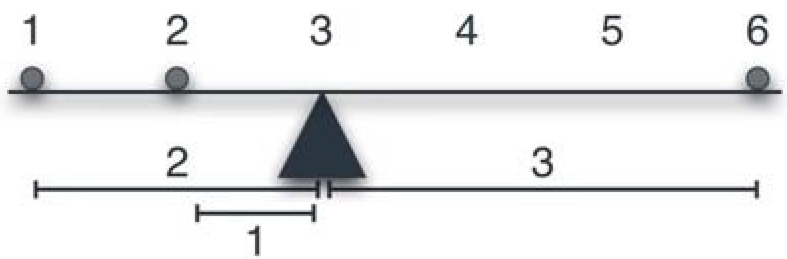


$$\text{Variance} = \frac{1^2+0^2+1^2}{3} = 2/3$$



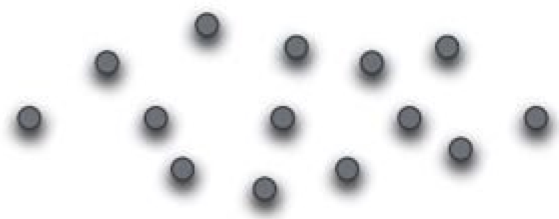
$$\text{Variance} = \frac{5^2+0^2+5^2}{3} = 50/3$$

Mean

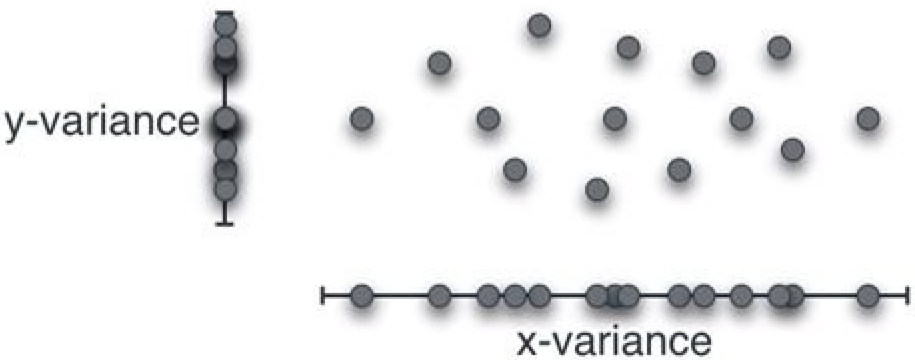


$$\text{Variance} = \frac{2^2+1^2+3^2}{3} = 14/3$$

Variance?



Variance?

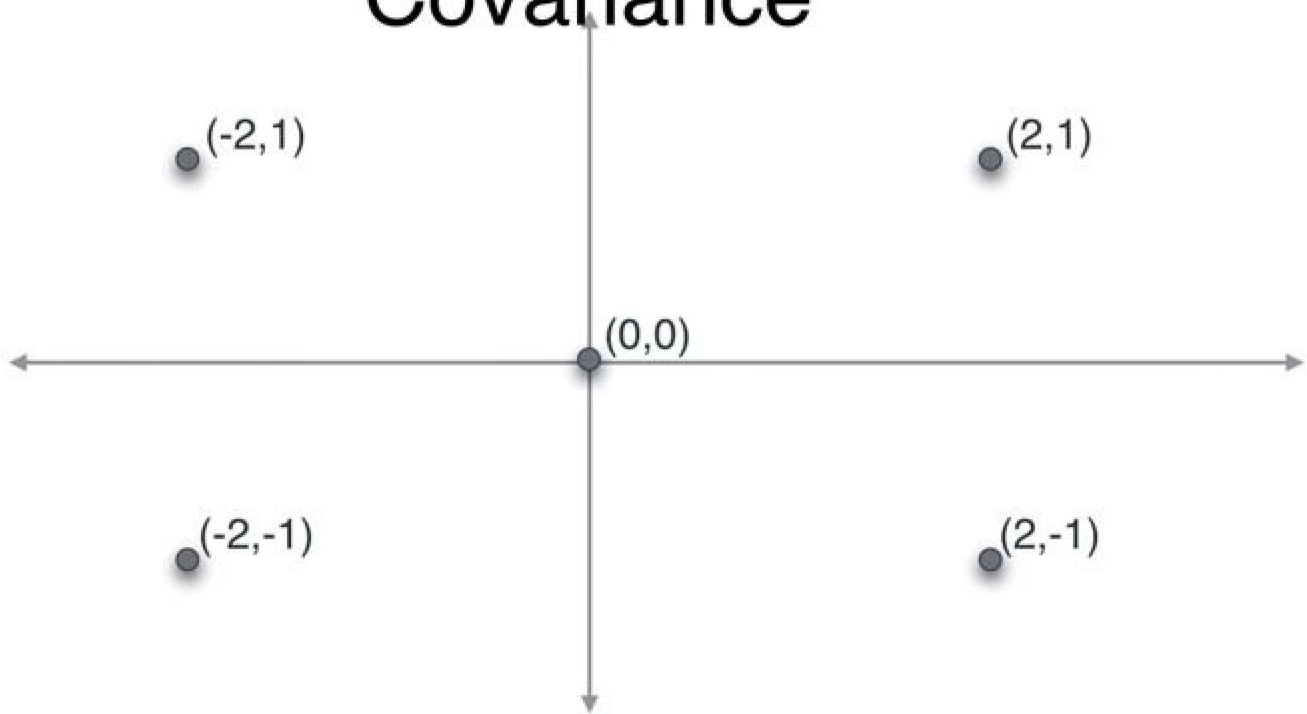


Variance?

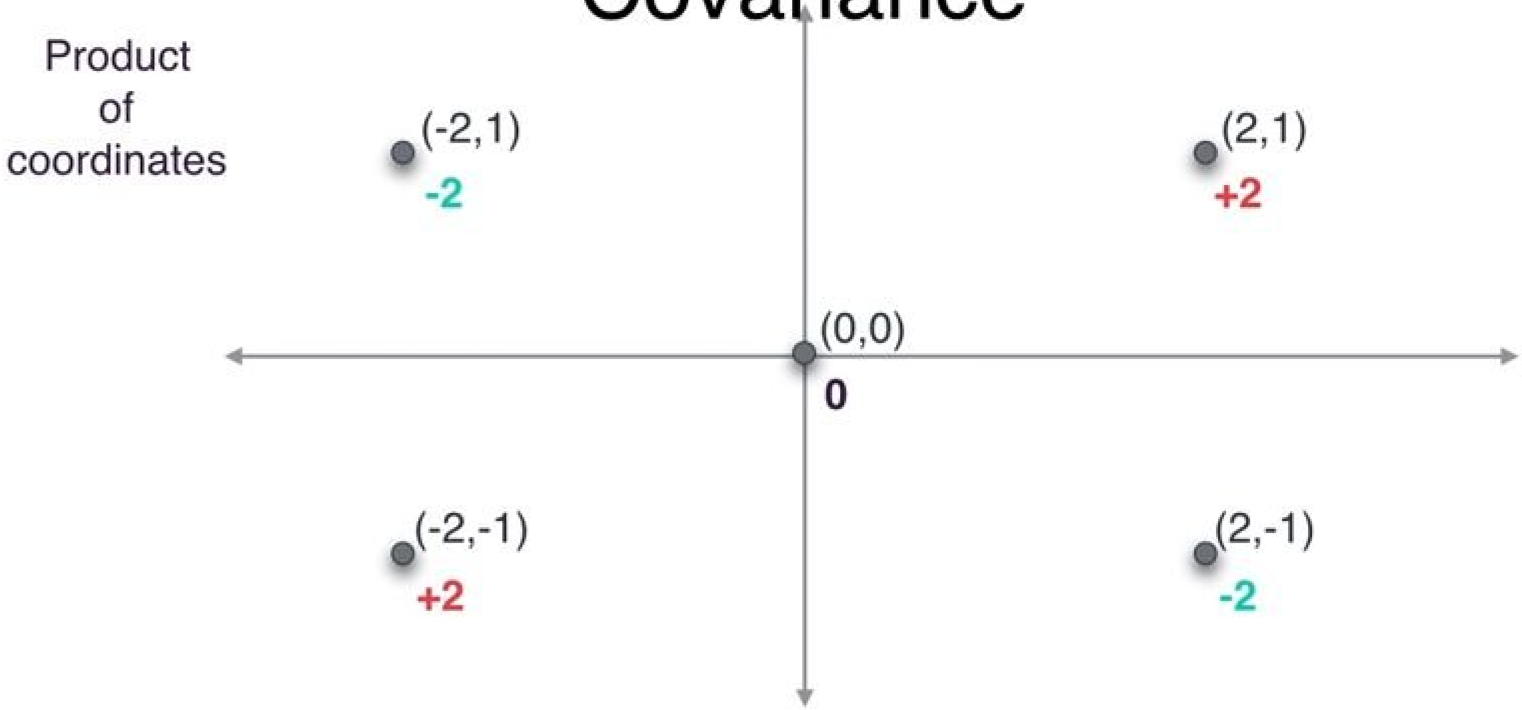
$$\text{x-variance} = \frac{2^2+0^2+2^2}{3} = 8/3$$

$$\text{y-variance} = \frac{1^2+0^2+1^2}{3} = 2/3$$

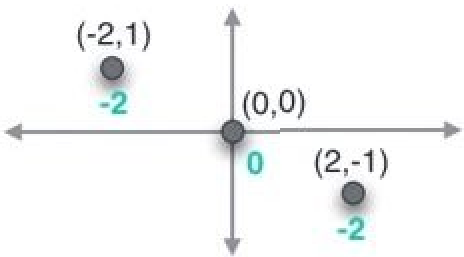
Covariance

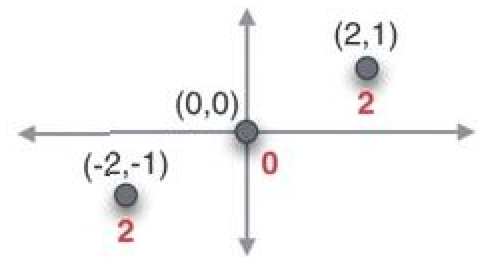


Covariance



Covariance

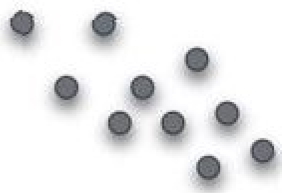




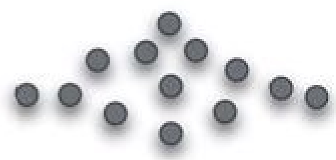
$$\text{covariance} = \frac{(-2) + 0 + (-2)}{3} = -4/3$$

$$\text{covariance} = \frac{2 + 0 + 2}{3} = 4/3$$

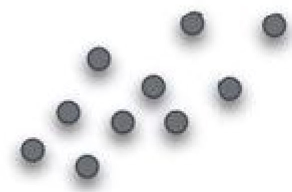
Covariance



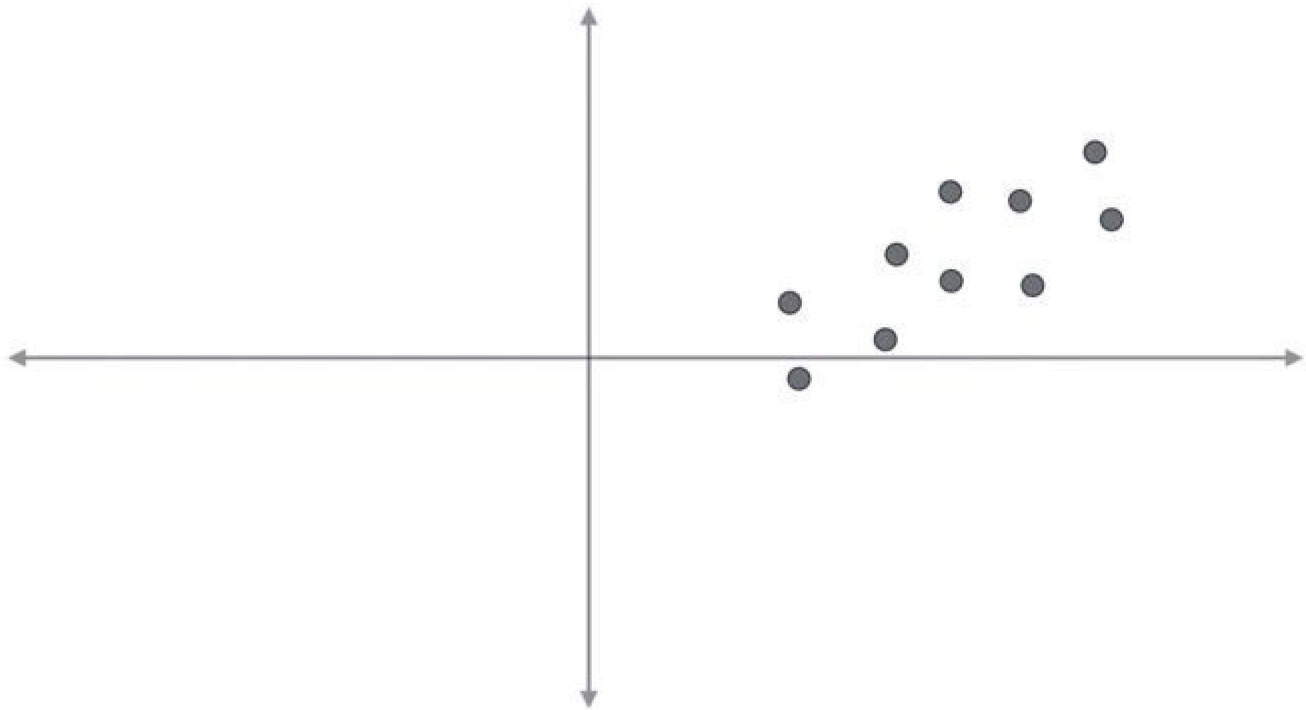
negative
covariance

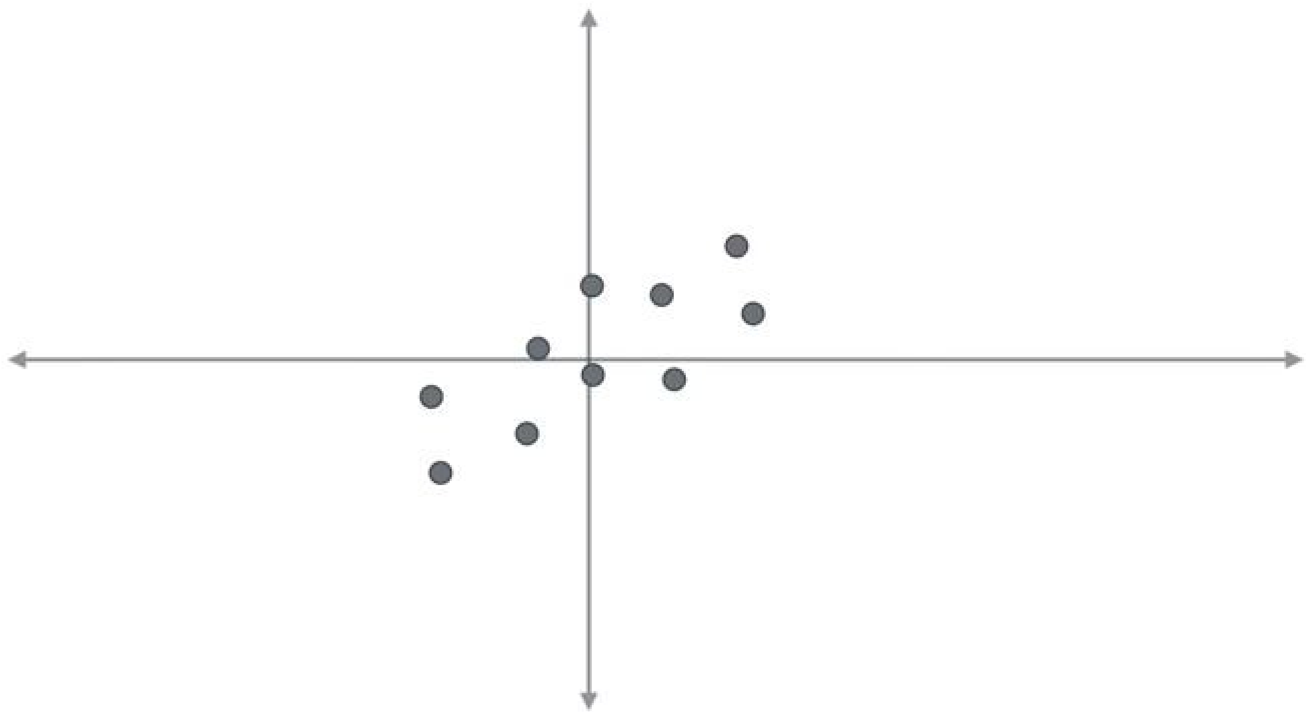


covariance zero
(or very small)



positive
covariance



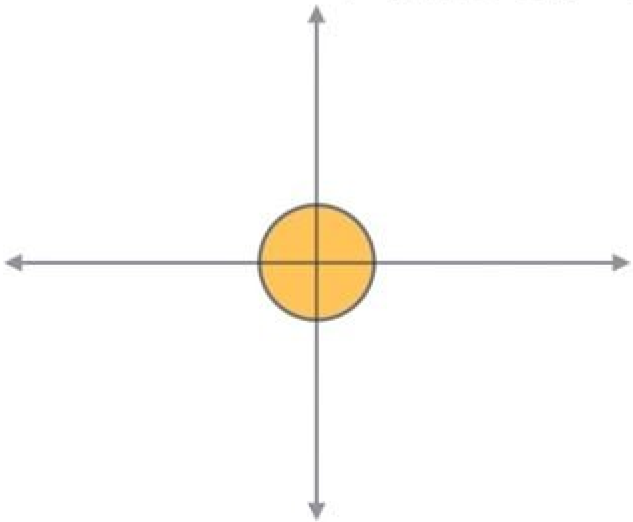


Covariance matrix

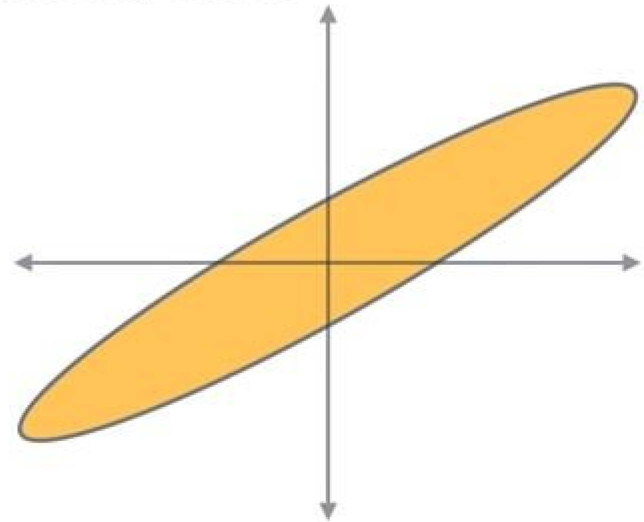
$$\Sigma = \begin{pmatrix} \mathrm{Var}(X) & \mathrm{Cov}(X,Y) \\ \mathrm{Cov}(X,Y) & \mathrm{Var}(Y) \end{pmatrix}$$

$$\begin{pmatrix} 9 & 4 \\ 4 & 3 \end{pmatrix}$$

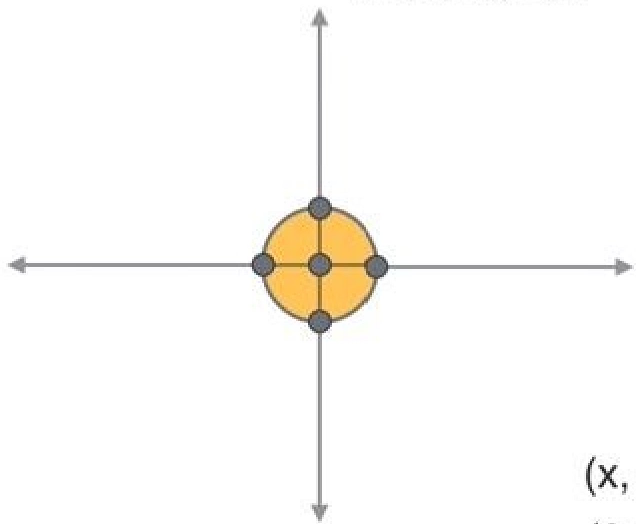
Linear Transformations



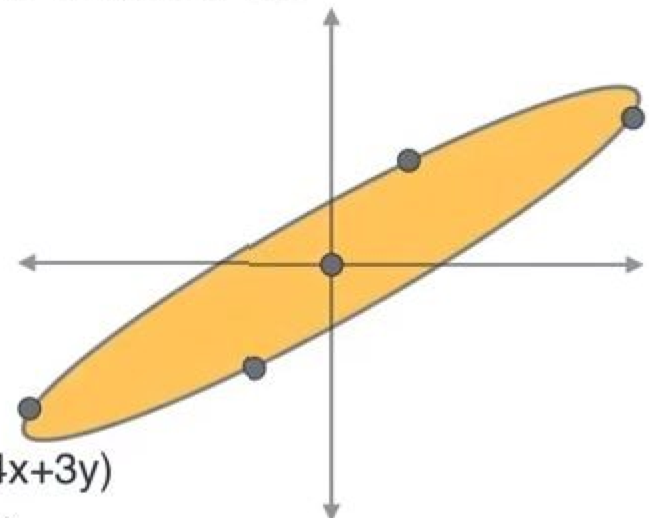
$$\begin{pmatrix} 9 & 4 \\ 4 & 3 \end{pmatrix} \longrightarrow$$



Linear Transformations



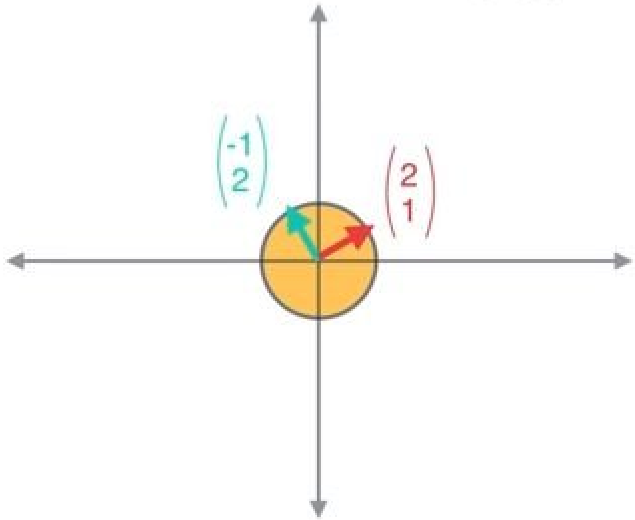
$$\begin{pmatrix} 9 & 4 \\ 4 & 3 \end{pmatrix} \longrightarrow$$



$(x, y) \longrightarrow (9x+4y,\ 4x+3y)$

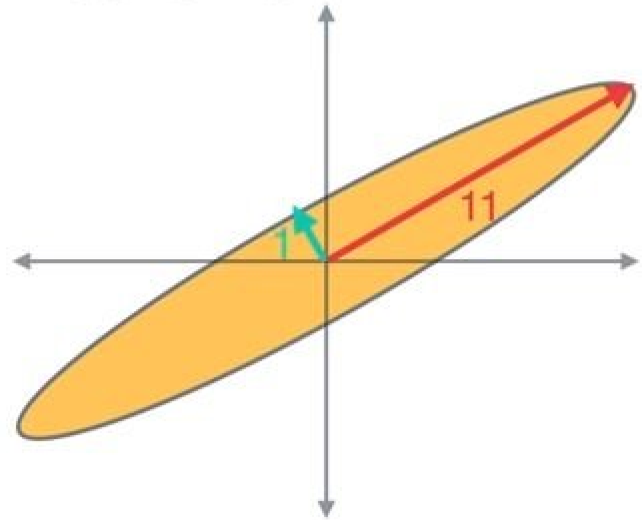
(0,0)	(0,0)
(1,0)	(9,4)
(0,1)	(4,3)
(-1,0)	(-9,-4)
(0,-1)	(-4,-3)

Linear Transformations

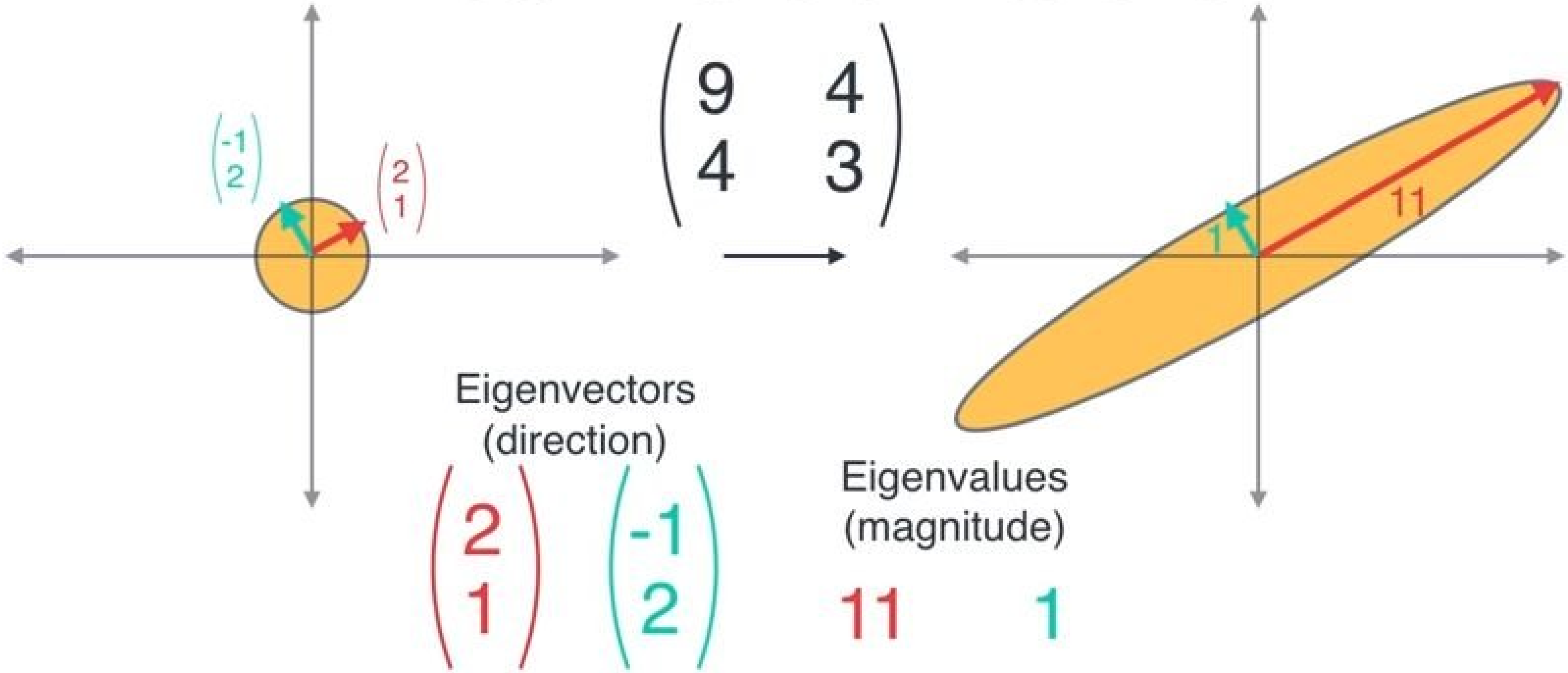


$$\begin{pmatrix} 9 & 4 \\ 4 & 3 \end{pmatrix}$$

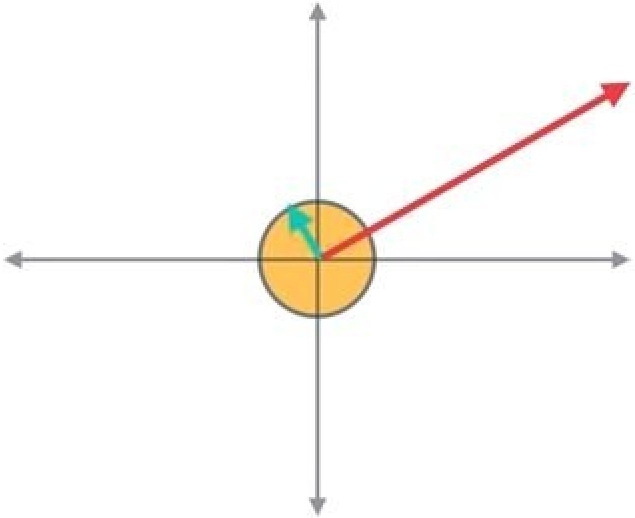
$$\longrightarrow$$



Linear Transformations



Linear Transformations



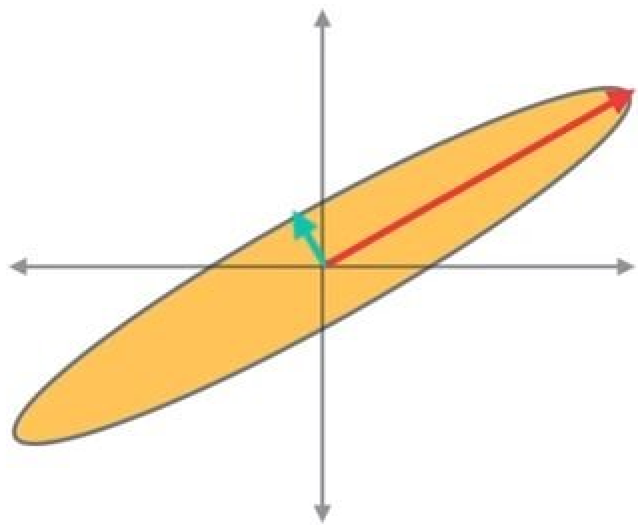
Eigenvectors

$$\begin{pmatrix} 2 \\ 1 \end{pmatrix} \quad \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

Eigenvalues

11 1

Linear Transformations



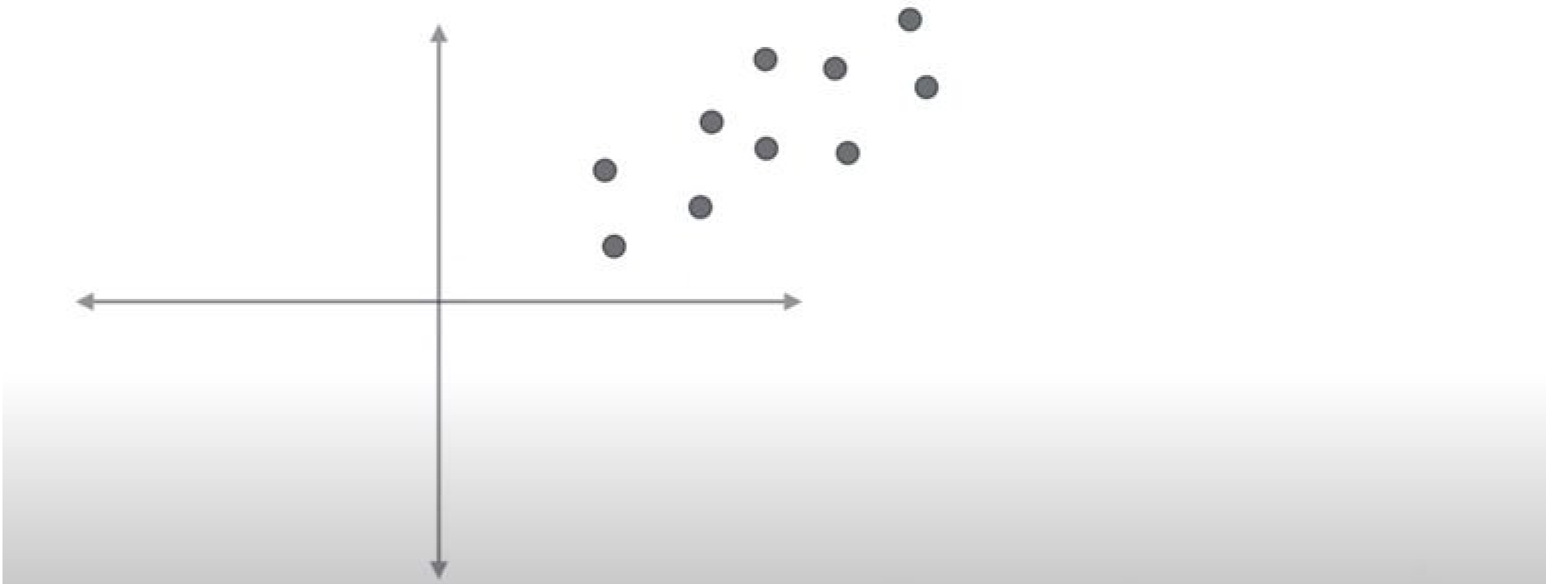
Eigenvectors
(direction)

$$\begin{pmatrix} 2 \\ 1 \end{pmatrix} \quad \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$

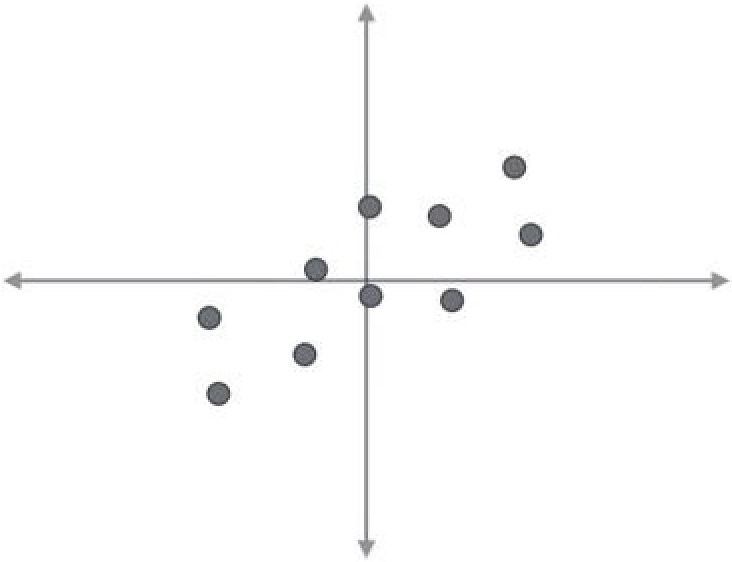
Eigenvalues
(magnitude)

11 1

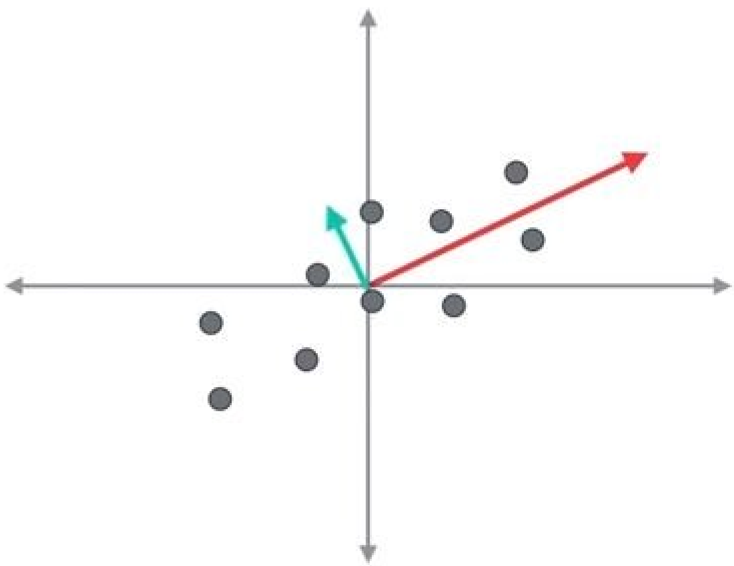
Principal Component Analysis (PCA)



Principal Component Analysis (PCA)



Principal Component Analysis (PCA)

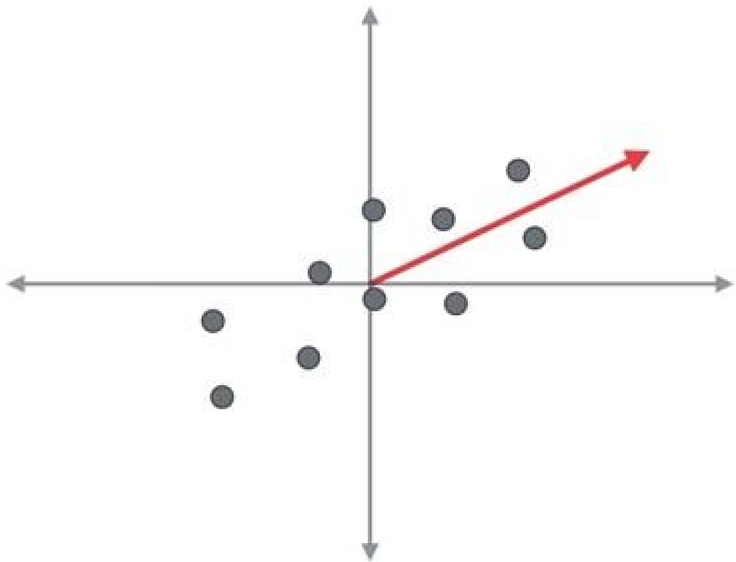


$$\Sigma = \begin{pmatrix} 9 & 4 \\ 4 & 3 \end{pmatrix}$$

$$\begin{pmatrix} 2 \\ 1 \end{pmatrix} \quad \begin{pmatrix} -1 \\ 2 \end{pmatrix}$$ Eigenvectors (direction)

$$11 \quad 1$$ Eigenvalues (magnitude)

Principal Component Analysis (PCA)



$$\Sigma = \begin{pmatrix} 9 & 4 \\ 4 & 3 \end{pmatrix}$$

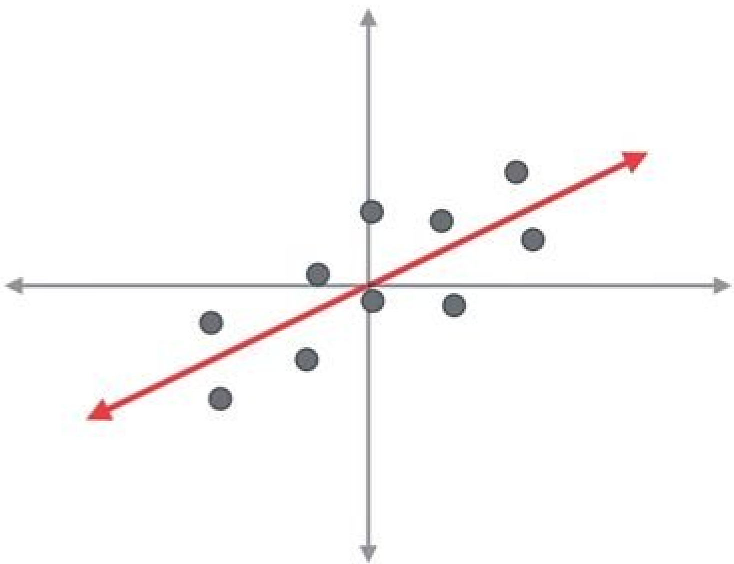
$$\begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

Eigenvectors
(direction)

11

Eigenvalues
(magnitude)

Principal Component Analysis (PCA)



$$\Sigma = \begin{pmatrix} 9 & 4 \\ 4 & 3 \end{pmatrix}$$

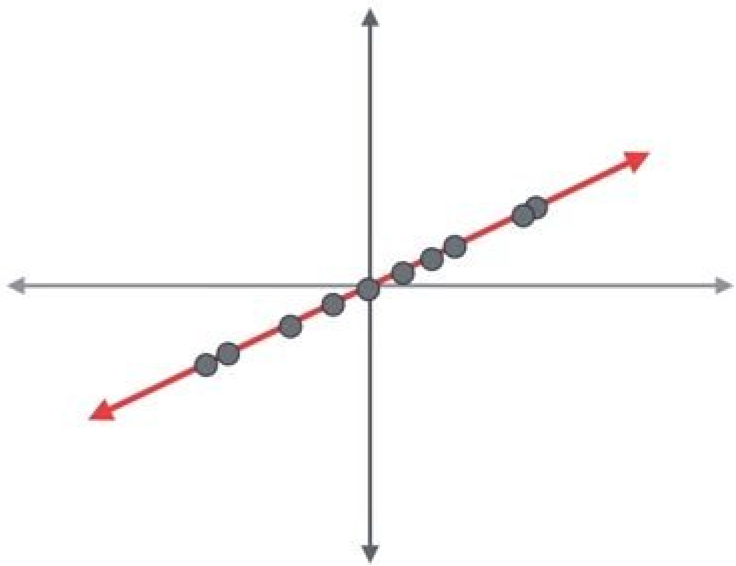
$$\begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

Eigenvectors
(direction)

11

Eigenvalues
(magnitude)

Principal Component Analysis (PCA)



$$\Sigma = \begin{pmatrix} 9 & 4 \\ 4 & 3 \end{pmatrix}$$

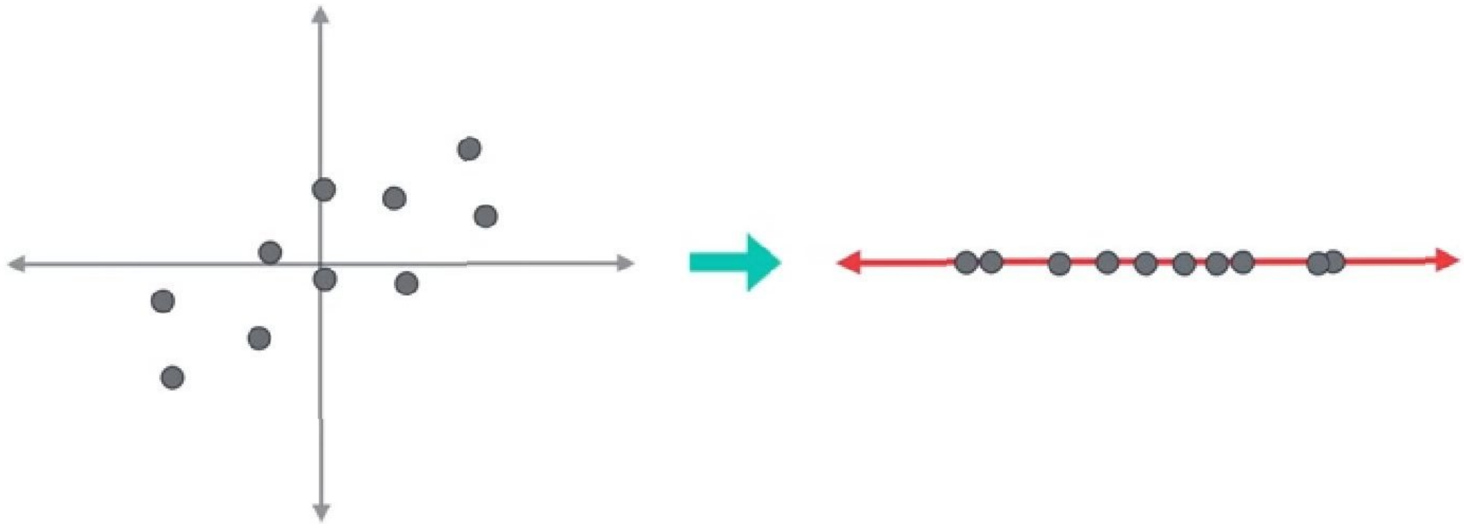
$$\begin{pmatrix} 2 \\ 1 \end{pmatrix}$$

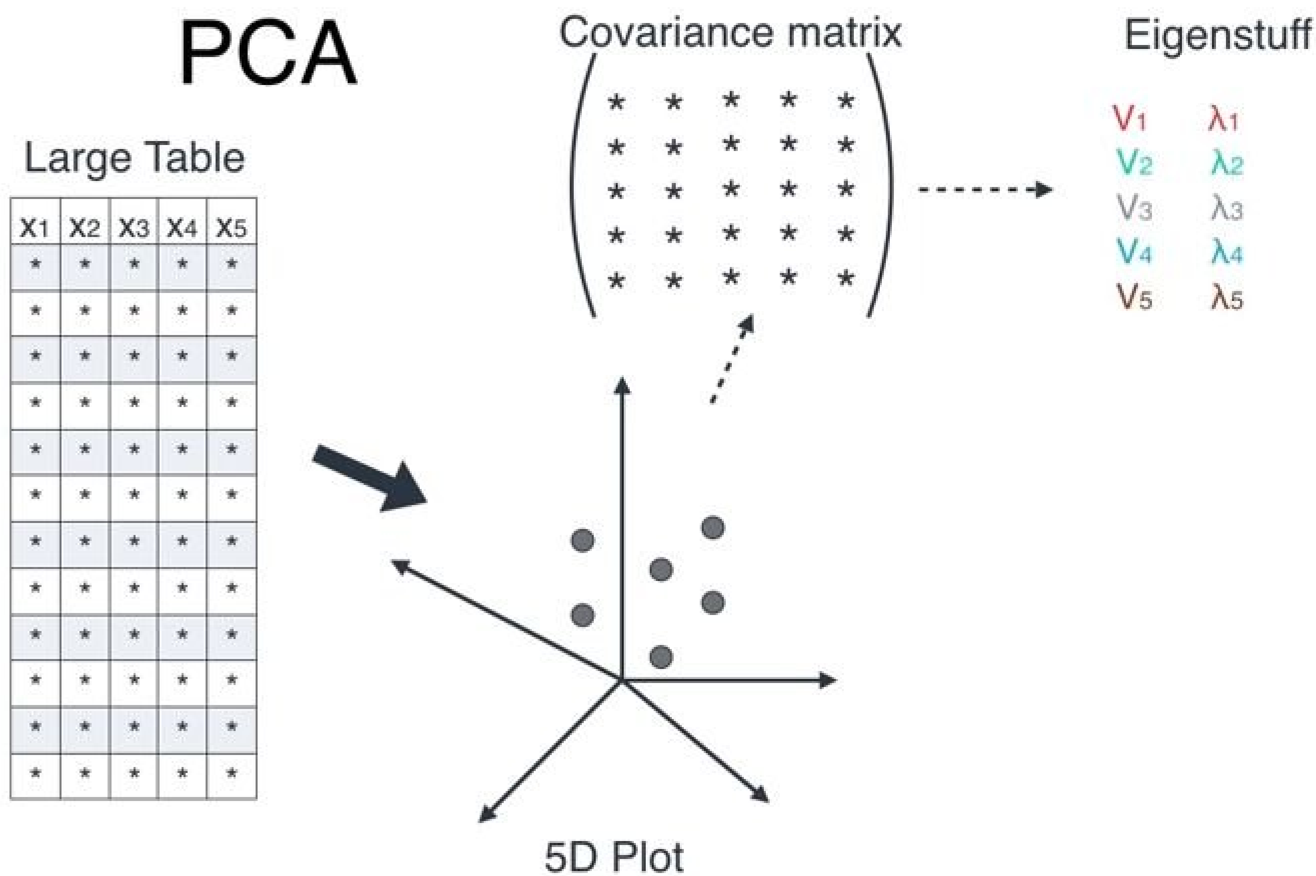
Eigenvectors
(direction)

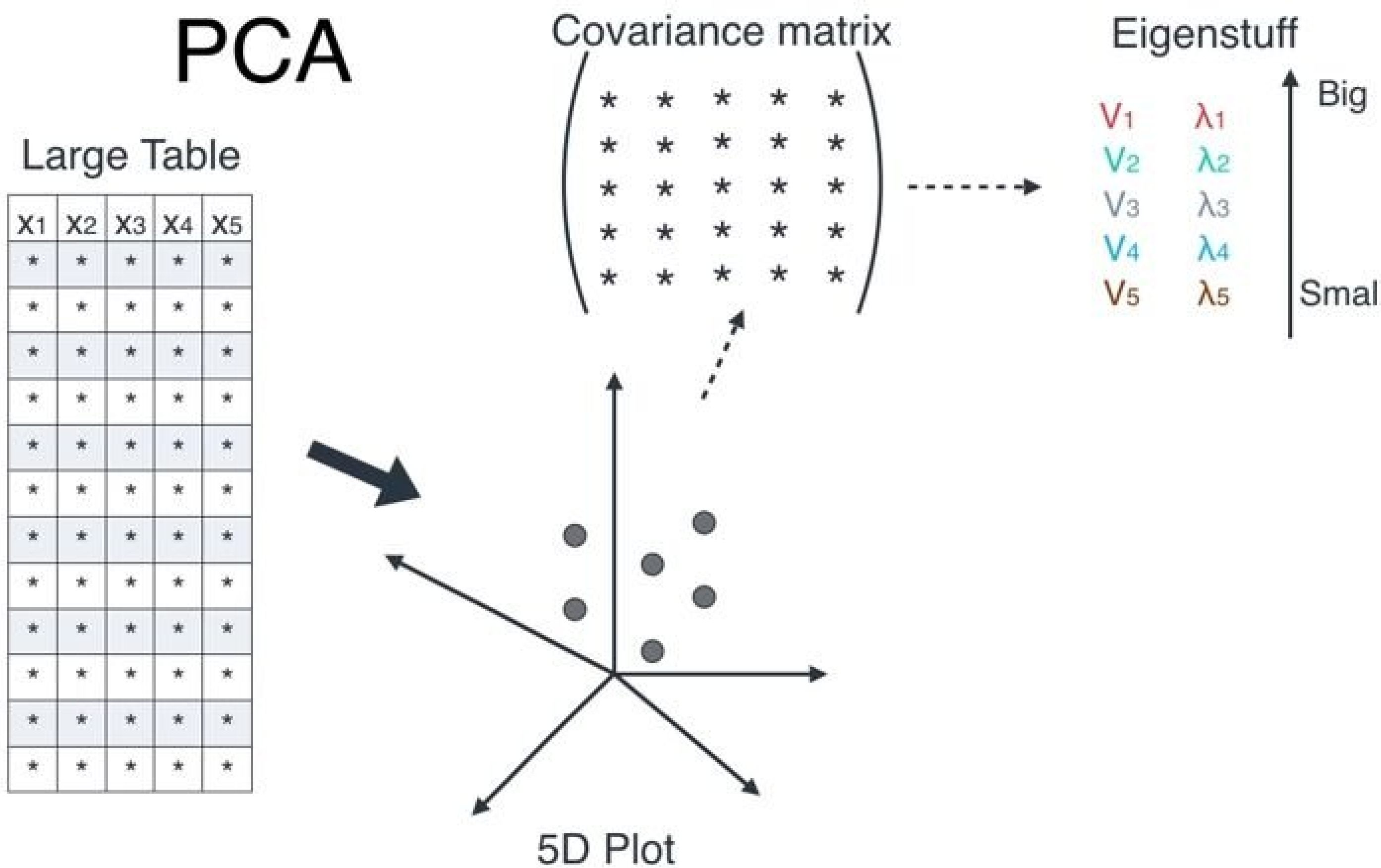
$$11$$

Eigenvalues
(magnitude)

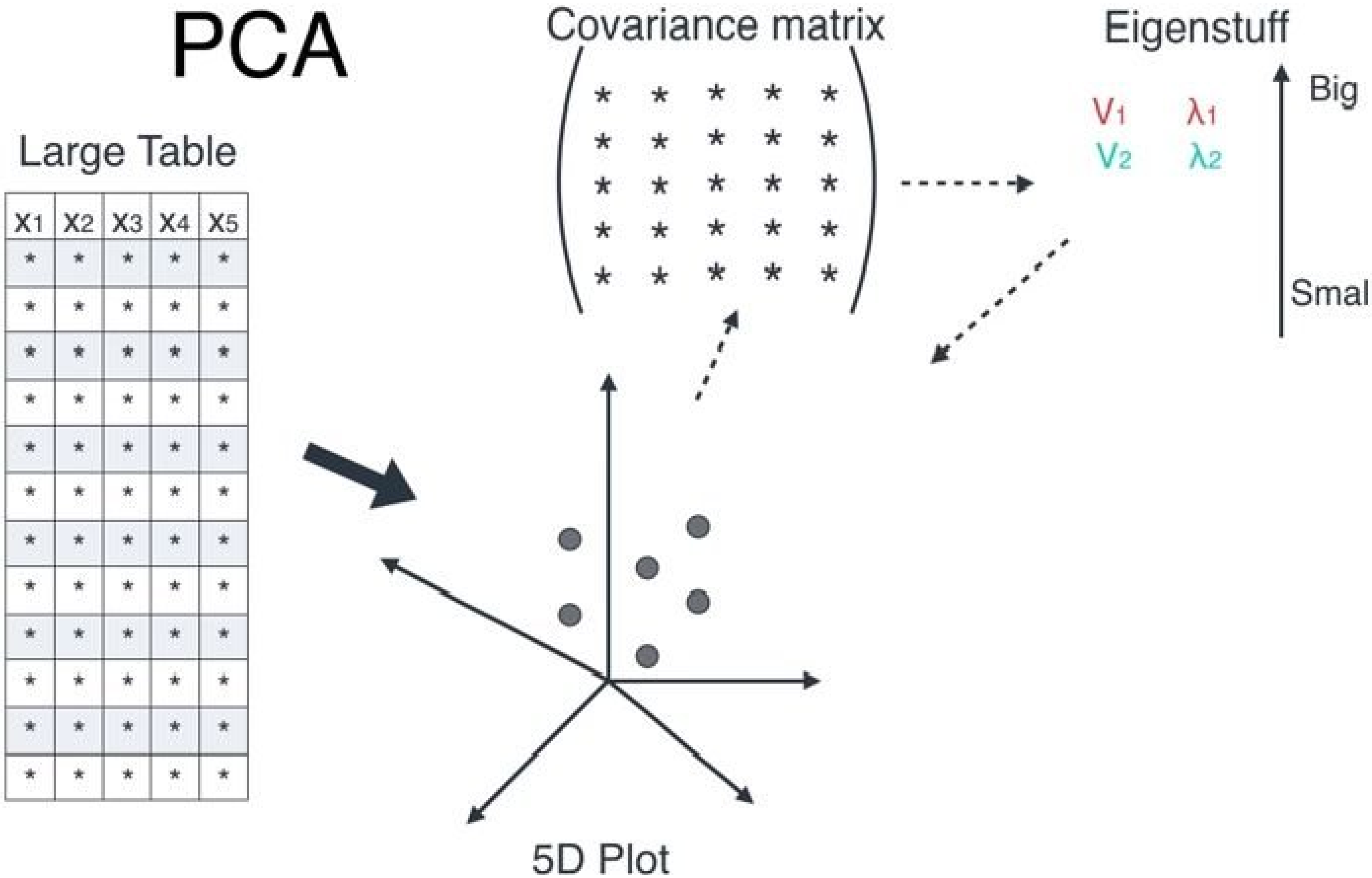
Principal Component Analysis (PCA)

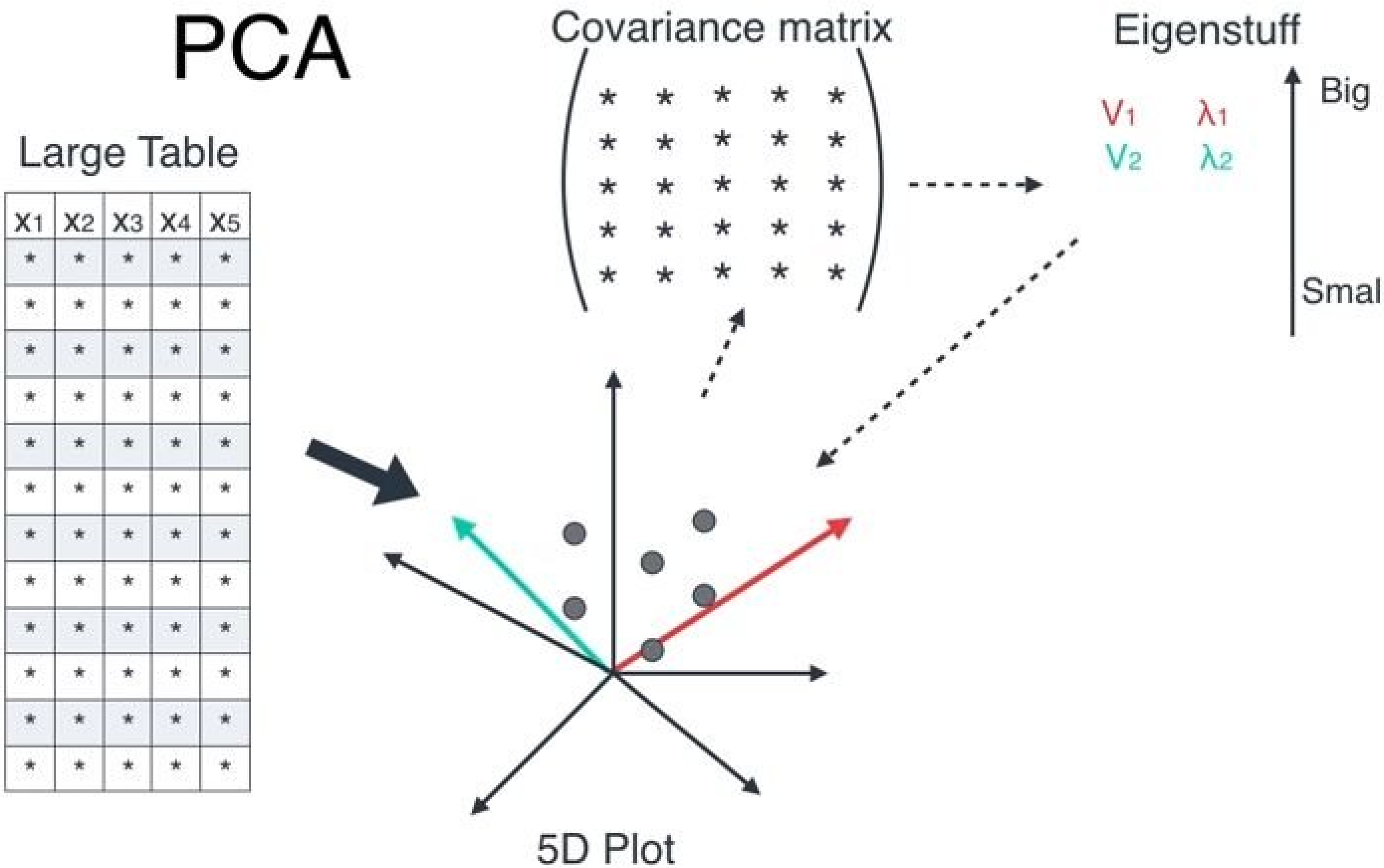



PCA
Large Table
x1 x2 x3 x4 x5
Covariance matrix
Eigenstuff
v1 λ1
v2 λ2
v3 λ3
v4 λ4
v5 λ5
5D Plot

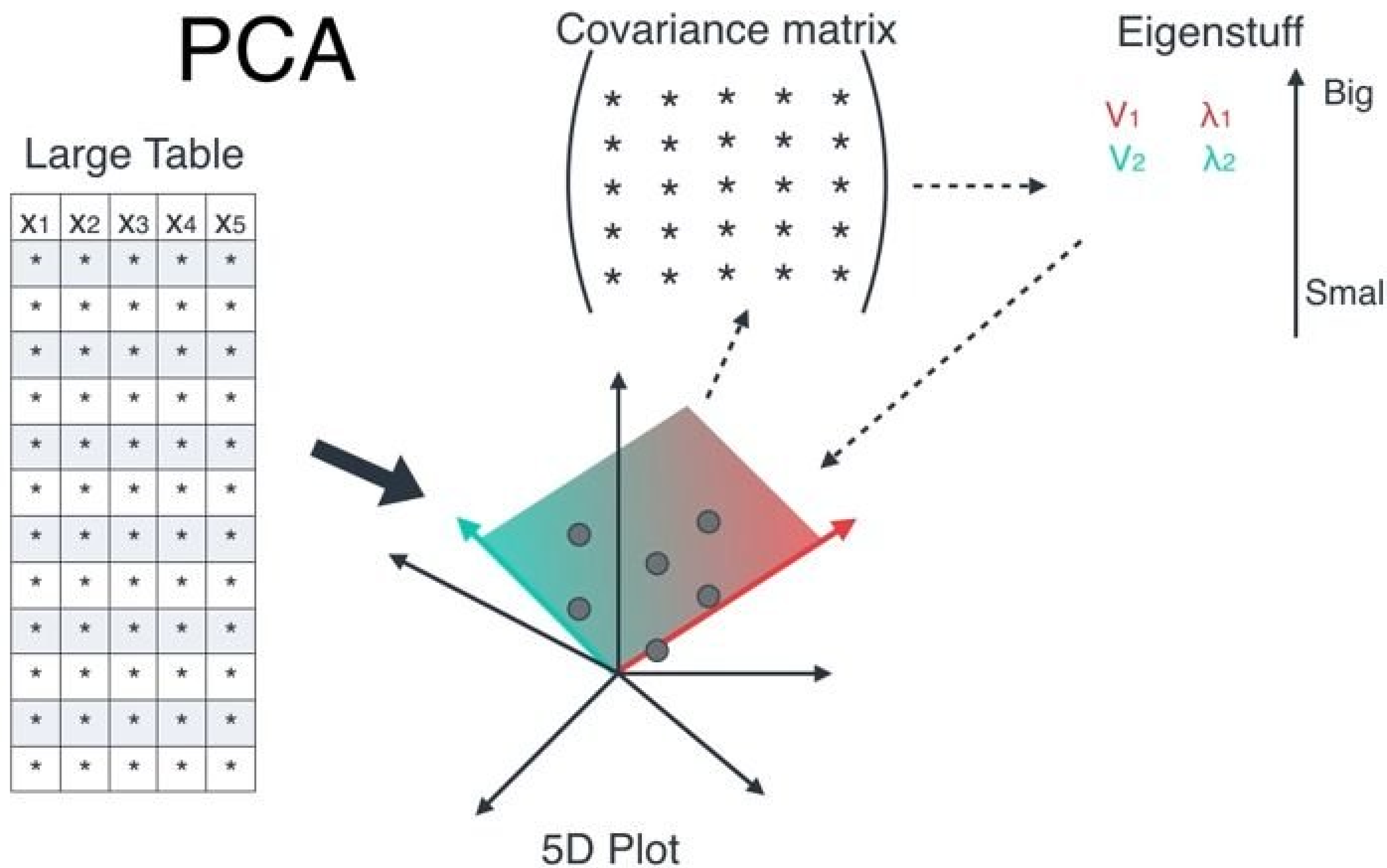

PCA
Large Table
X1 X2 X3 X4 X5
Covariance matrix
Eigenstuff
V1 λ1
V2 λ2
V3 λ3
V4 λ4
V5 λ5
Big
Small
5D Plot

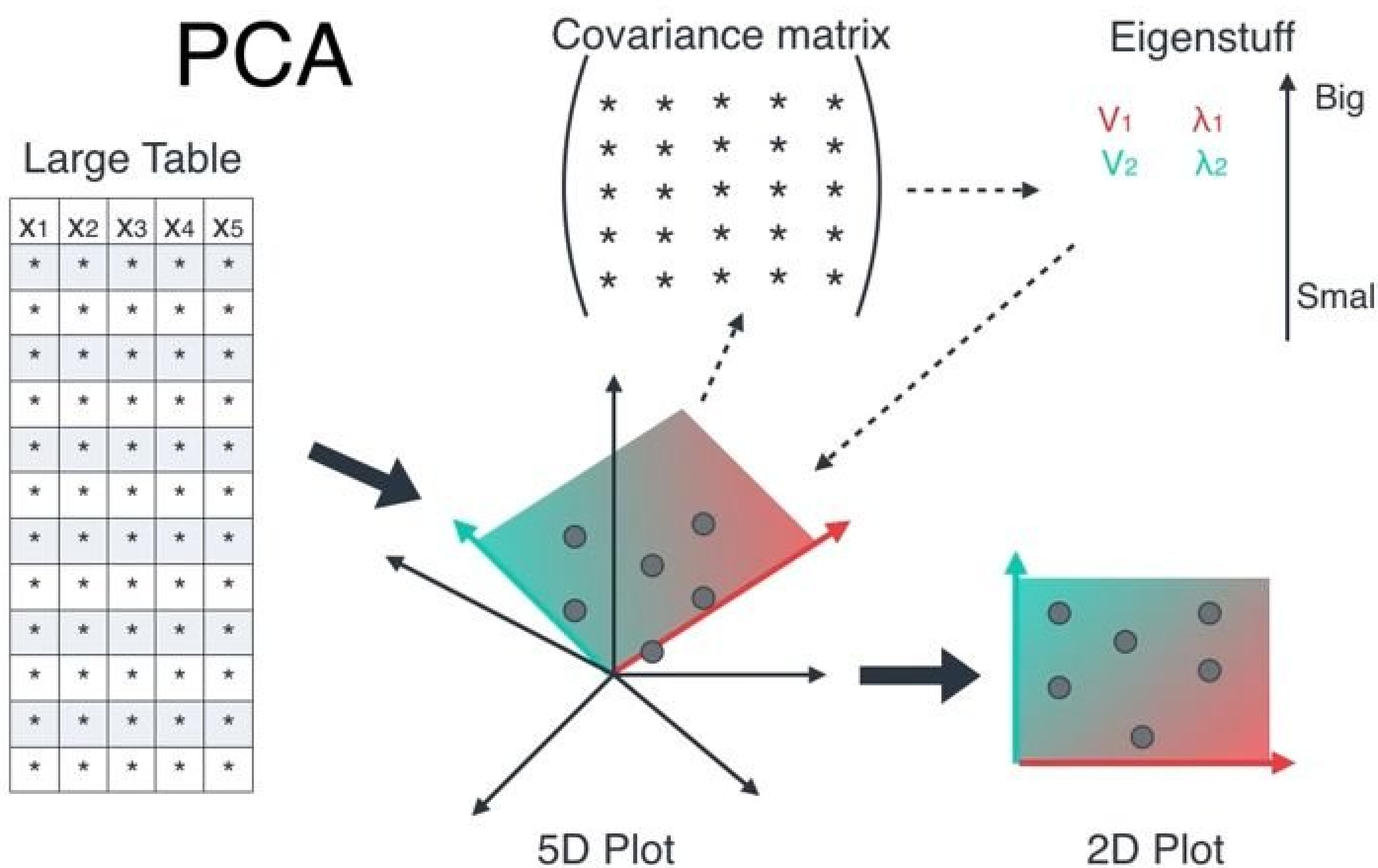
PCA

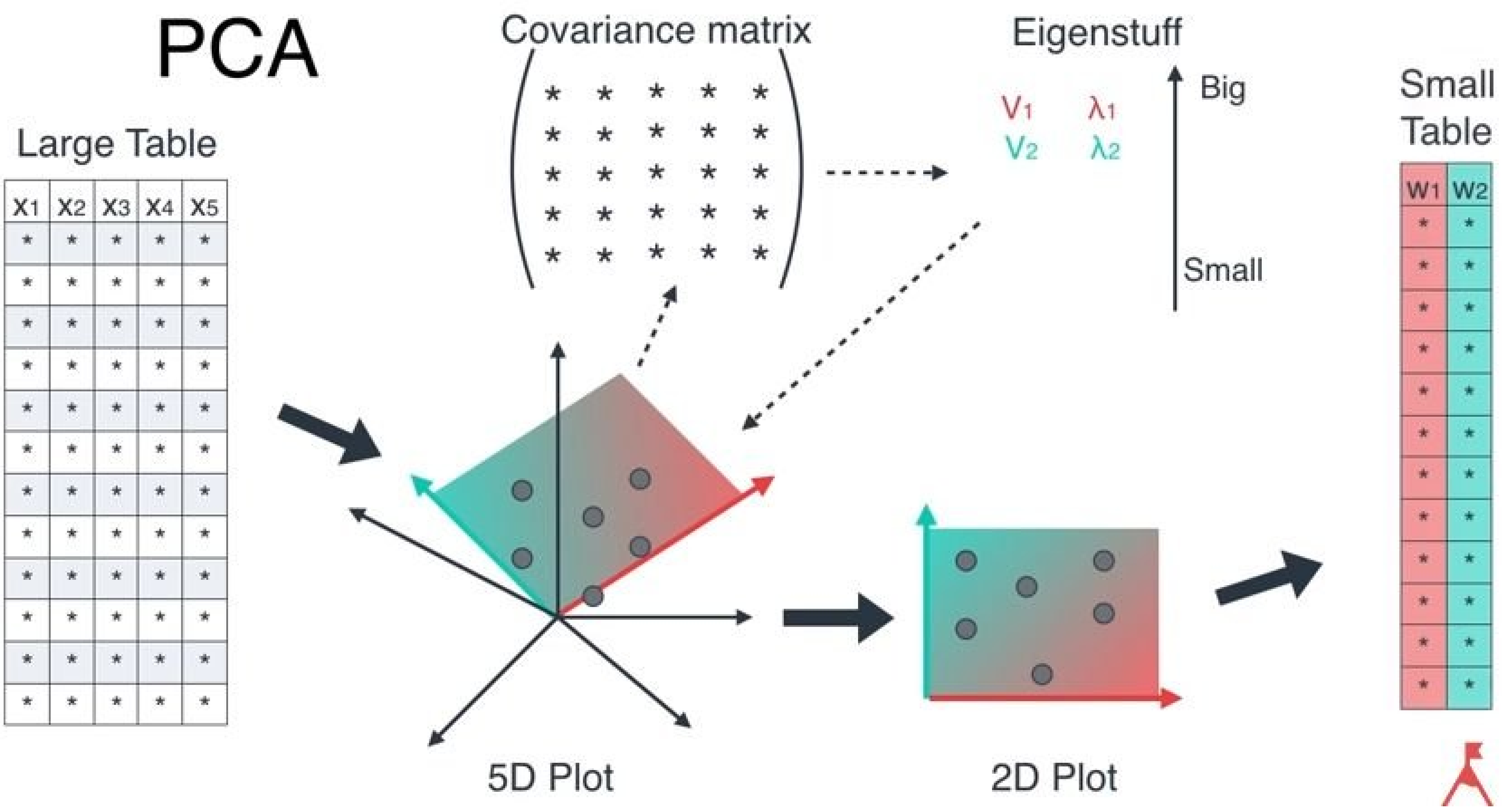

Large Table
x1 x2 x3 x4 x5
Covariance matrix
Eigenstuff
V1 λ1
V2 λ2
Big
Small
5D Plot


PCA
Large Table
x1 x2 x3 x4 x5
Covariance matrix
Eigenstuff
v1 λ1
v2 λ2
Big
Small
5D Plot

PCA


Covariance matrix
Eigenstuff
Big
v_1 λ_1
v_2 λ_2
Small
Large Table
x_1 x_2 x_3 x_4 x_5
5D Plot


PCA
Large Table
x1 x2 x3 x4 x5
Covariance matrix
Eigenstuff
Big
V1 λ1
V2 λ2
Small
5D Plot
2D Plot



PCA
Covariance matrix
Eigenstuff
Big
Small
Large Table
X1 X2 X3 X4 X5
V1 λ1
V2 λ2
Small Table
W1 W2
5D Plot
2D Plot

PCA Computation Steps

- PCA converts dataset with a large number of correlated/uncorrelated features into the dataset with less number of linearly uncorrelated features.
- **Step 1**: **Standardization of the values in the dataset:** The various analysis algorithms can work efficiently on the dataset in which all feature values are on the same scale.
- **Step 2: Computing the covariance matrix**
- In the PCA aim is to find the set of features that have high variance and these are highly uncorrelated to each other. The covariance shows a correlation between two or more features, the positive correlation means features values are increasing/decreasing with each other in the same direction. A negative correlation means features values are increasing/decreasing in opposite direction. The covariance matrix represents the variance between the features, the dataset is of ‘n’ dimension the covariance matrix is of ‘n X n’ dimension.
- **Step 3**: **Computing the eigenvalues and eigenvectors** :The eigenvectors are the principal components of the covariance matrix i.e. it is representing the direction of the maximum covariance and its magnitude is the eigenvalue. The eigenvalues and eigenvectors are calculated and a matrix of eigenvectors is formed.

- **Step 4**: Sorting of the eigenvalues and their corresponding eigenvectors: The eigenvalues are sorted with a magnitude of eigenvalues. The sorting is done as in PCA, features that have high variance are selected as principal components.
- **Step 5**: Select k, i.e. number of principal components: In this step, the dimensionality of the dataset is determined i.e. which features and how many features should be taken ahead for analysis is determined.
- **Step 6**: Transform the dataset in newly computed k-dimension: This is the last step of PCA, the ‘n’; dimension dataset is converted to the ‘k’ dimension dataset.