# Advanced Microeconometrics

## Exercise Set 3: Random Effects

Andreas P. A. Pedersen, Week 39 2024

UNIVERSITY OF COPENHAGEN

## Today's Plan

- Random Effects
- Hausman Test
- Your time to shine!

# Random Effects Model

- Linear Panel Data Model
$$y_{it} = c_i + \mathbf{x}_{it}\beta + u_{it} \tag{1}$$
- Let's assume no confounding time-invariant heterogeneity ($E(c_i x_{it}) = 0$) and strict strict exogeneity ($E(u_{it}x_{is}) = 0$) $\forall s, t \in T$) holds. Furthermore, our error terms are homoskedastic (FE.3: $E(u_i u_i') = \sigma_u^2 I_T \ \forall i \in N$) and individual specific effects have constant variance across time $E(c_i c_i') = \sigma_c^2 j_T j_T' \ \forall i \in N$, where $j_T = [1...1]_{T\times 1}$

# RE Covariance Matrix

- The composite error covariance matrix has non-zero off-diagonals due to the time-invariant heterogeneity, $c_i$:

$$
\begin{aligned}
E(v_i v_i') =& \sigma_c^2 \mathbf{j}_T \mathbf{j}_T' + \sigma_u^2 \mathbf{I}_T \\
=& \begin{bmatrix} \sigma_c^2 + \sigma_u^2 & \sigma_c^2 & \dots & \sigma_c^2 \\ \sigma_c^2 & \sigma_c^2 + \sigma_u^2 & \dots & \sigma_c^2 \\ \vdots & \vdots & \ddots & \vdots \\ \sigma_c^2 & \sigma_c^2 & \dots & \sigma_c^2 + \sigma_u^2 \end{bmatrix}_{T \times T}
\end{aligned}
$$

- POLS is inefficient as the composite error term is not homoskedastic
- RE accounts for the feature of the panel data and efficiently weights the data to minimize the variance

# RE Quasi-Demeaning I

- We "quasi-demean" our data and perform POLS:

$$\check{y}_{it} = \check{\mathbf{x}}_{it}\beta + \check{v}_{it} \tag{2}$$

were $\check{y}_{it} = y_{it} - \lambda\overline{y}_i$, $\mathbf{x}_{it} = \mathbf{x}_{it} - \lambda\overline{\mathbf{x}}_i$ and $\check{v}_{it} = v_{it} - \lambda\overline{v}_i$

- Quasi-demeaning subtracts the weighted mean:

$$\lambda = 1 - \sqrt{\frac{\sigma^2}{(\sigma_u^2 + T\sigma_c^2)}} \tag{3}$$

- To estimate $\lambda$ we need estimates of $\sigma_u^2$ and $\sigma_c^2$.
    - "Between estimator" returns $\sigma_v^2 = \sigma_c^2 + \frac{1}{T}\sigma_u^2$
    - Fixed Effects yields $\sigma_u^2$
    - Combine these to obtain $\sigma_c^2 = \sigma_v^2 - \frac{1}{T}\sigma_u^2$

# Re Quasi-Demeaning II

- Use perm function to quasi-demean the data. Apply the matrix $\mathbf{C}_T = \mathbf{I}_T - \hat{\lambda}\mathbf{P}_T$ where $\mathbf{P}_T = [1/T \dots 1/T]$
- Rearranging (4):

$$\hat{\lambda} = 1 - \sqrt{\frac{1}{(1 + T\widehat{\sigma}_c^2/\widehat{\sigma}_u^2)}}$$

Notice that:

$$\begin{aligned}
\hat{\lambda} &\to 1 for & T\widehat{\sigma}_c^2/\widehat{\sigma}_u^2 \to \infty \Rightarrow \hat{\beta}_{RE} \to \hat{\beta}_{FE} \\
\hat{\lambda} &\to 0 for & T\widehat{\sigma}_c^2/\widehat{\sigma}_u^2 \to 0 \Rightarrow \hat{\beta}_{RE} \to \hat{\beta}_{POLS}
\end{aligned}$$

- RE lies <u>between</u> POLS and FE estimators and puts more weight on observations for which there is more variation

# RE Assumptions

- Random Effects Assumptions

$$
\begin{aligned}
RE.1a :& E(u_{it}|x_i, c_i) = 0 \quad t \in \{1...T\} \\
RE.1b :& E(c_i|\mathbf{x}_{it}) = E(c_i|\mathbf{x}_{it}) = 0 \\
RE.2 :& rank(E(\mathbf{X}_i'\Omega^{-1}\mathbf{X}_i)) = K \\
RE.3a :& E(u_i u_i'|x_i, c_i) = \sigma_u^2 \mathbf{I}_T \qquad E(c_i^2|\mathbf{x}_i) = \sigma_c^2
\end{aligned}
$$

- We need both no confounding time-invariant heterogeneity (RE.1b) and strict exogenity (RE.1b)
- 1 and 3 implies that $E(v_i v_i') = E(v_i v_i'|x_i) = \sigma_u^2 I_T + \sigma_c^2 \mathbf{j}_T \mathbf{j}_T' \forall i \in N$

## Hausman Test: RE vs FE

- Under assumptions RE.1a(FE.1), RE.2 (and FE.2) and RE.3, we can test whether there are time invariant exogeneity i.e. whether $E(\mathbf{x}_{it}c_i = 0)$
- Hausman Test Statistic

$$H = (\hat{\beta}_{RE} - \hat{\beta}_{FE})'(\widehat{Avar}(\hat{\beta}_{RE}) - \widehat{Avar}(\hat{\beta}_{FE}))(\hat{\beta}_{RE} - \hat{\beta}_{FE}) \sim \chi^2_M$$

  Where $M \leq K$ is the number of coefficients
- Only include estimated coefficent $\beta_{RE}$ on time varying regressors (since FE doesn't have the ones on time-variant regressors)

# Your time to shine!

- Look at the new toolbox: LinearModelsWeek3.py
- Exercises this week should be less extensive than previous weeks - good time to catch up if you haven't completed previous exercise sets, and work on assignment.