Taxation and government spending

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Econtwitter

Econtwitter flagships

- Beatrice Cherrier (History of economic thought)
- Khoa Vu (Memes)
- Ben Moll (Theory/policy – German)
- Jon Steinsson (Theory/policy – US)
- Jeppe Druedahl (Policy – Denmark)
- Claudia Sahm (Policy – US)

- Econtwitter is very active (also on BlueSky)
- Very current on policy debates and economic research

Robert Solow on simple models

I am trying to express an attitude towards the building of very simple models. I don't think that models like this lead directly to prescription for policy or even to detailed diagnosis. [...] They are more like reconnaissance exercises. If you want to know what it's like out there, it's all right to send two or three fellows in sneakers to find out the lay of the land and whether it will support human life. If it turns out to be worth settling, then that requires an altogether bigger operation. The job of building usable larger-scale econometric models on the basis of whatever analytical insights come from simple models is much more difficult and less glamorous (Solow 1970, 105).

- Simple models do not yield policy prescriptions, but insights

Agenda

Recap of Ramsey dynamics

- Transition after shock

Taxation and Ricardian Equivalence

- Simple model, partial equilibrium
- Full Ramsey model with taxation
- Distortionary capital taxation

Ramsey dynamics

Ramsey economy in equations

$$k_{t+1} = (1-\delta)k_t + f(k_t) - c_t$$
$$u'(c_t) = \beta(1 + f'(k_{t+1}) - \delta)u'(c_{t+1})$$
$$k_0, \text{ Transversality condition }, \text{No-Ponzi condition}$$

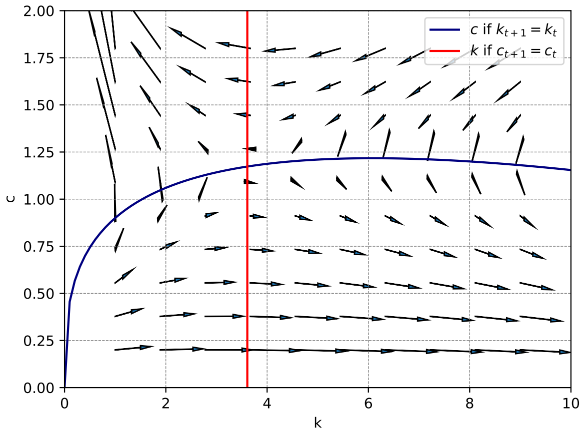
- Difference equations → dynamic
- Transversality & No-Ponzi game conditions rule out explosions
- The **only** unknown parameter is c_0

Solving the model

- Find the correct c_0 (guess and verify, bisection method)
- Plug in and solve forwards (very easy on a computer)

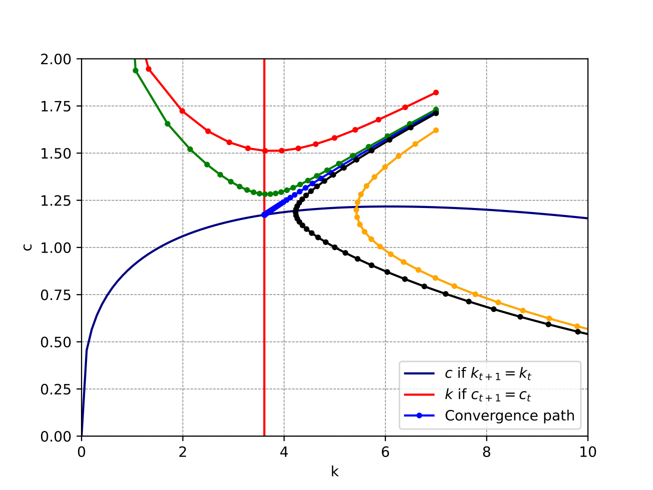
$$k_1 = (1-\delta)k_0 + f(k_0) - c_0$$
$$u'(c_0) = \beta(1 + f'(k_1) - \delta)u'(c_1)$$

Phase diagram



- Arrows: $k_{t+1} - k_t$ and $c_{t+1} - c_t$ implied by equations, given $\{k_t, c_t\}$

Phase diagram



- Given the dynamic equations and k_0, only one c_0 ends in steady state

Shocks to parameter values

Deterministic models

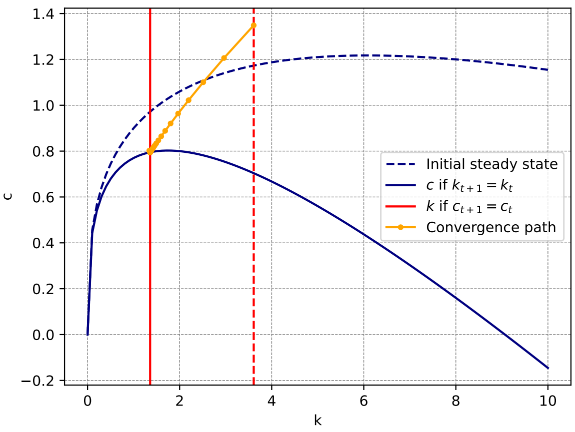
- The Ramsey model (and many others) are entirely deterministic
- Once initial conditions are known, there are no surprises
- Agents in the model perfectly predict the future

How can there be shocks? Any “shock” has probability $= 0$.

“MIT shocks”

- Coined by Tom Sargeant (U of Minnesota) dismissive of MIT econ
- Unanticipated (by agents) change of parameter in the model
- Model “starts over” in new reality (according to new equations)
- Agents didn’t expect the change **and never expect another**
- Solve the model along the new deterministic equations

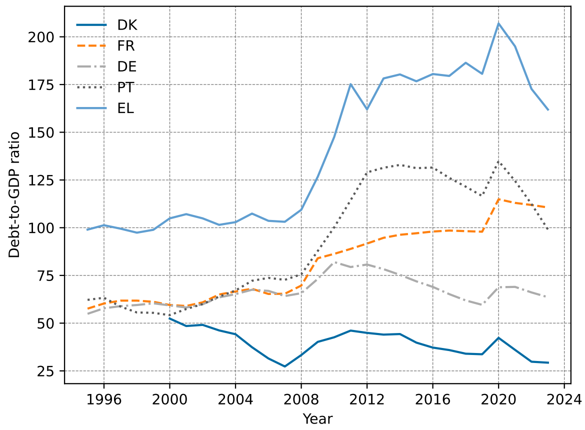
Problem set: $\delta \uparrow$



- k_0 is given, staying at old c implies $k \to \infty$
- Violates transversality condition $\to$ must choose new c_0

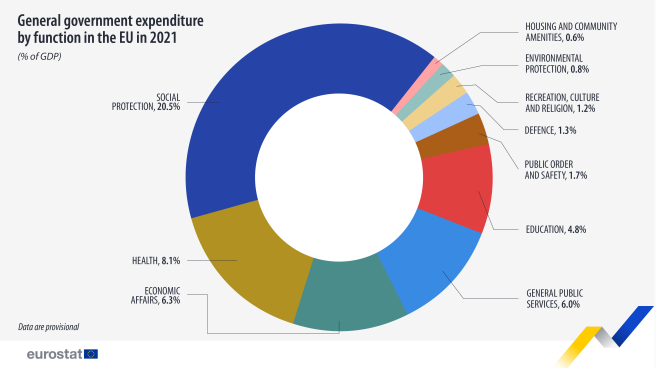
Government spending

Government spending



- Debt ratios are very different across countries Eurostat

Government spending



- Governments fund wide arrays of activities
- US: 14% national defense

Government spending

Multiple forms

- Wasteful (throw the money in the sea)
- Useful (rebates to citizens, redistribution)

Financing

- Balanced budget (no government debt)
- Debt financed
- Different forms of taxation

The government in the model

Taxation

- For now, the government taxes households "lump sum"
- Taxes don't distort relative prices (here: interest rates or wages)
- Later: distortionary taxes

Government spending

- The government wants to spend G_t in each period t
- Assume that spending doesn't affect representative agent's utility or budget
- Example: military spending

Two period model

Two period model

Agents

- As before: live 2 periods, endowment each period
- Interest rate is given

$$
\begin{aligned}
U &= u(c_0) + \beta u(c_1) \\
s.t. \quad c_0 + a_1 &= w_0 - \tau_0 \\
c_1 &= w_1 + a_1(1+r) - \tau_1
\end{aligned}
$$

Government spending

- The government spends G_0 and G_1, taxes τ_0 and τ_1
- Budget each period:

$$
\begin{aligned}
b_0 &= g_0 - \tau_0 \\
0 &= g_1 - \tau_1 + b_0(1+r)
\end{aligned}
$$

Simple analysis

Experiment

- Hold the government's spending plan constant
- Spending "disappears" out of the model (wasteful)
- Shift around **when** the representative agent gets taxed

$$g_0 + \frac{g_1}{1+r} = \tau_0 + \frac{\tau_1}{1+r}$$

Graphical analysis

- Same as in previous lectures
- Households are on their Euler equations

Mathematical intuition

Combine budget constraints

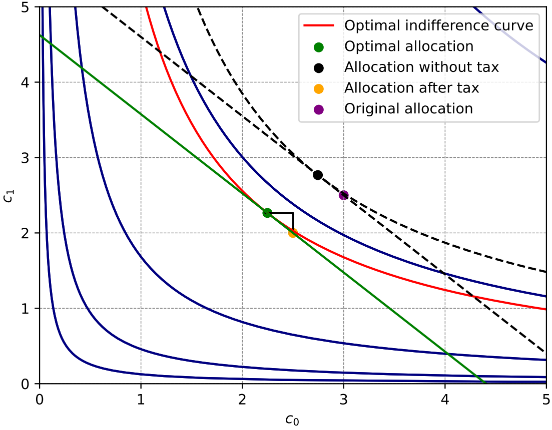
$$
\begin{aligned}
c_0 + \frac{c_1}{1+r} &= w_0 + \frac{w_1}{1+r} - \left(\tau_0 + \frac{\tau_1}{1+r}\right) \\
&= w_0 + \frac{w_1}{1+r} - \left(g_0 + \frac{g_1}{1+r}\right)
\end{aligned}
$$

- Taxes drop out
- Households only care about the PV of government spending

Intuition

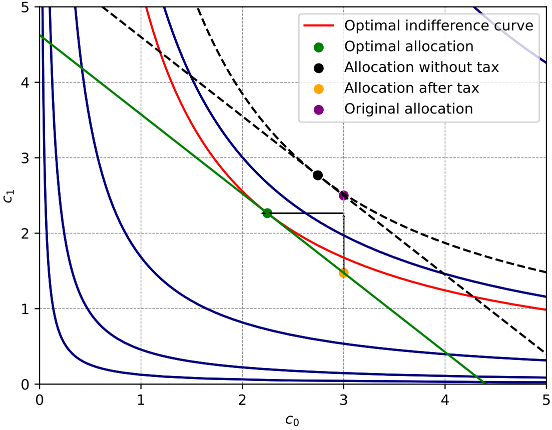
- If $\tau_1 \uparrow$, $\tau_0 \downarrow$ by slightly less, if $r > 0$
- Households can shift consumption at the same price $(1+r)$
- If they don't like the temporal allocation, they can reverse it

Graphical analysis



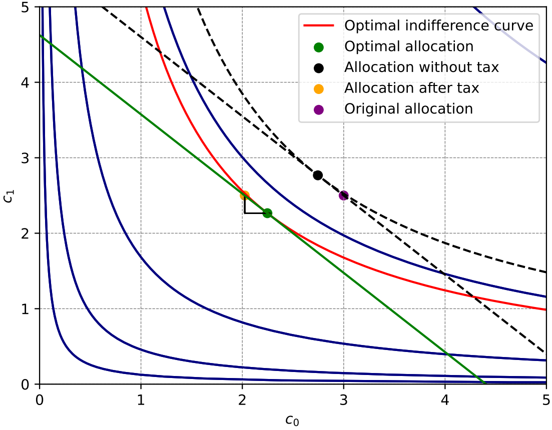
- Government taxes such that $g_0 = \tau_0, g_1 = \tau_1$

Graphical analysis



- Government taxes such that $\tau_0 = 0$

Graphical analysis



- Government taxes such that $\tau_1 = 0$

Conclusion from 2 period model

Ricardian equivalence

- **When** the government taxes the agent, for a **given path** of government spending, does not affect consumption choices (under the assumptions we made)

Intuition

- Permanent income hypothesis $\implies$ consumers only care about present value of lifetime income
- Taxation lowers the present value of income $\implies$ lower consumption
- **But** if $\{G_t\}_0^T$ given, then $PV(G)$ is given and thus, $PV(\tau)$ is given

This turns out to hold for the infinite-horizon model, too.

Important caveat

Breaking Ricardian equivalence is very easy

- Result relies on many assumptions (discuss more below)
- One of them: household's $r =$ government's r_g

Experiment

- Assume households pay r, but governments pay $r_G = 0$
- Government intertemporal budget: $G_0 + G_1 = \tau_0 + \tau_1$
- When do households prefer to be taxed?

Important caveat

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Experiment

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- When do households prefer to be taxed?

Households prefer taxation in period 1

$$c_0 + \frac{c_1}{1+r} = w_0 + \frac{w_1}{1+r} - \left(\tau_0 + \frac{\tau_1}{1+r}\right) = w_0 + \frac{w_1}{1+r} - \left(G_0 + G_1 - \frac{r}{1+r}\tau_1\right)$$

- Waiting benefits HHs more than then government
- HHs save to pay future debt, government doesn't pay interest on it

Taxation in the Ramsey model

Consumers and firms

Households

$$\max_{c_t, a_{t+1} \forall t} U = \sum_{t=0}^{\infty} \beta^t u(c_t)$$

$$\text{subject to} \quad c_t + a_{t+1} = w_t + (1 + r_t - \delta) a_t - \tau_t$$

$$\implies u'(c_t) = \beta (1 + r_{t+1} - \delta) u'(c_{t+1})$$

$$\sum_{t=0}^{T} \left(\prod_{s=0}^{t} \frac{1}{1 - \delta + r_s} \right) (w_t - \tau_t) = \sum_{t=0}^{T} \left(\prod_{s=0}^{t} \frac{1}{1 - \delta + r_s} \right) c_t$$

- Because **non-distortionary** taxation, Euler equation is unchanged
- Last equation by substitution of a_t, see last lecture

Firms

$$\max_{K_t, L_t} F(K_t, L_t) - r_t K_t - w_t L_t$$

$$\implies f_k'(k_t, 1) = r_t$$

$$f_l'(k_t, 1) = w_t$$

The government budget

$$b_{t+1} = b_t(1 - \delta + r_t) + g_t - \tau_t$$

Budget

- The government can access financial markets just like the household
- The budget does not need to balance each period (i.e., $b_t \neq 0$
- **No-Ponzi game condition** just as for households

Intertemporal government budget

- Solve equation forwards (as for household in last lecture)
- Present value of taxes = present value of spending

$$\underbrace{\left(\prod_{t=0}^{T} \frac{1}{(1 - \delta + r_t)}\right) b_{T+1}}_{\text{No-Ponzi as } T \to \infty} = 0 = \sum_{t=0}^{T} \left(\prod_{s=0}^{t} \frac{1}{(1 - \delta + r_t)}\right) (g_t - \tau_t)$$

Ricardian equivalence in Ramsey

Combine household and government budgets

$$\sum_{t=0}^{T}\left(\prod_{s=0}^{t}\frac{1}{1-\delta+r_s}\right)(w_t-g_t)=\sum_{t=0}^{T}\left(\prod_{s=0}^{t}\frac{1}{1-\delta+r_s}\right)c_t$$

Budget

- The path of G_t is set, **no matter what it is**
- Taxes τ do not appear in the budget once G_t is known

What is going on?

- Upon announcement of G_t path, households deduce its present value
- Households are poorer, but still smooth consumption
- “The government doesn’t tell me **when** to spend my money”

Crucial change

How is it possible that the τ-path doesn't matter?

- Case a) PV of spending is taxed immediately
- Case b) $g_t = \tau_t$

Don't HHs have to decrease k_t to pay all the new tax burden?

Crucial change

How is it possible that the τ-path doesn't matter?

- Case a) PV of spending is taxed immediately
- Case b) $g_t = \tau_t$

Don't HHs have to decrease k_t to pay all the new tax burden?

NO! Because of new market clearing condition

$$k_{t+1} = a_{t+1} + b_{t+1}$$

- If the government taxes everything in advance, HHs borrow $a_{t+1} \downarrow$
- Gov't needs to save the same amount, spend in the future $b_{t+1} \uparrow$

Necessary assumptions for Ricardian equivalence

- Interest rates must be the same for households and governments
- Perfectly informed and rational agents
- Path of government spending must not change
- Everyone must be able to borrow and lend at the same rate (PS)
- Taxation must be lump sum (non-distortionary)
- No default risk for borrowers
- Unproductive government spending

Ricardian equivalence does not imply that G_t is useless, or that k_t will remain constant, just that timing of financing does not affect consumption choices of the household.

Full model

Optimality conditions in steady state

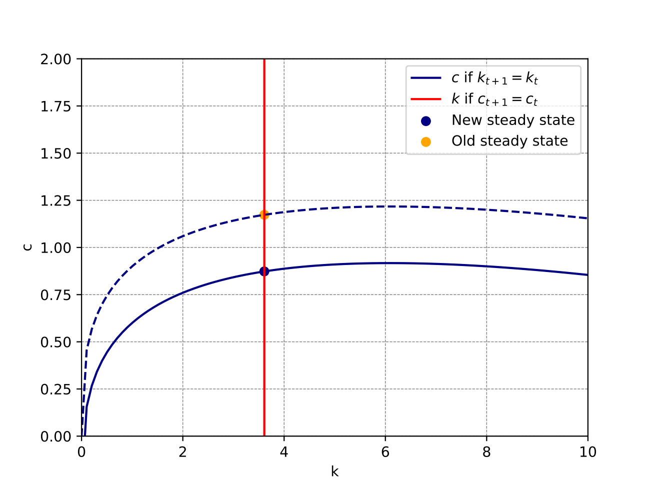
$$1 = \beta(1 - \delta + f'(k))$$
$$c = f(k) - \delta k - g$$

- Derivation in last lecture (except for g)
- Government purchases need to be produced (resource constraint)
- Assume that $g = \tau$ is also in steady state

"MIT shock"

- $g = 0$ as starting point (steady state), agent does not anticipate $g > 0$
- At $t = 0$, $g > 0$ is announced, forever
- Agent immediately reoptimizes, only c_0 can "jump", k_0 is given

Full model



- Consumption falls 1-for-1 with g

New steady state

Dynamics

- Capital is still in steady state: $1 = \beta(1 - \delta + f'(k))$
- Steady state level of k is unchanged

  $\implies k_0 = k$
- Upon announcement, only c_0 can jump
- Only one c_0 possible: $c_0 = c$

$\implies$ jump to new steady state immediately upon announcement

Crowding out

- Permanent government consumption crowds out private consumption
- Capital stays constant, hence output $f(k)$ does, too

Temporary government programs

Example: Stimulus programs

- Temporary spending to boost output
- "Spend now, pay later"

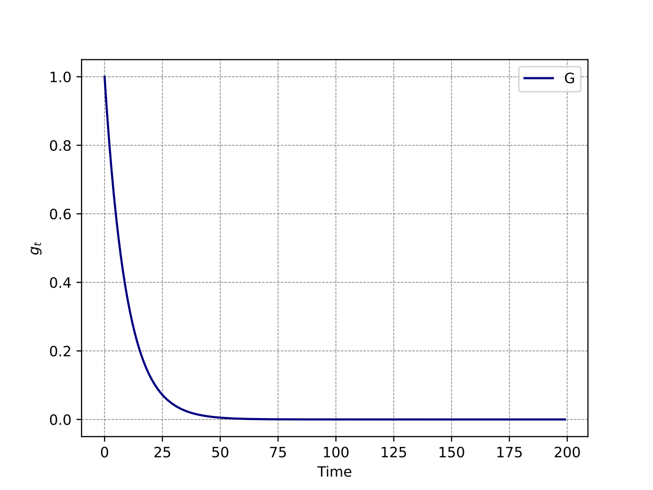
In the model

- At period 0, g rises unexpectedly, then falls back to 0
- As before: c_0 jumps, k only moves little each time
- $g_{t+1} = 0.9 * g_t$
- The laws governing the transition change each period

$$k_{t+1} = (1 - \delta)k_t + f(k_t) - c_t - g_t$$

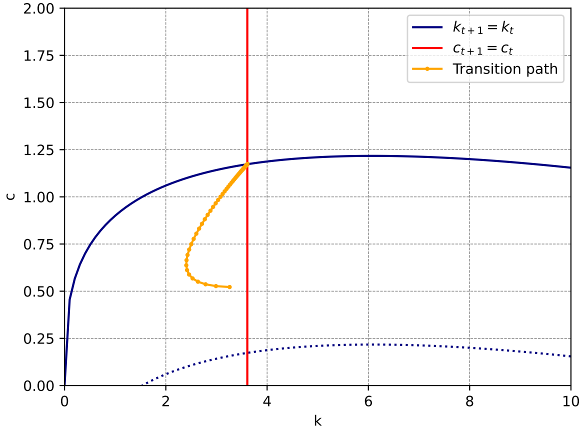
$\Longrightarrow$ **much more complicated adjustment path**

Temporary government programs–graphical



- g jumps up, then falls gradually

Temporary government programs–graphical



- $t = 0$: Don't accept $\Delta g_0 = \Delta c_0$, sacrifice some k, replenish later
- $t > 0$: As $g_t \to 0$, the dynamics of the diagram change each time

Distortionary taxation

Distortionary taxation

Households

$$\max_{c_t, a_{t+1} \forall t} U = \sum_{t=0}^{\infty} \beta^t u(c_t)$$

$$\text{subject to} \quad c_t + a_{t+1} = w_t + (1 - \delta + (1 - \tau_t) r_t) a_t + T_t$$

- $T_t = \tau_t r_t a_t$ represents government transfers to the household
- No wasteful spending, $g = 0$
- This is **distortionary** taxation

Question

- Does this taxation regime affect the model's outcome?

Model optimality

Household optimality

$$u'(c_t) = \beta(1 - \delta + (1 - \tau_{t+1})r_{t+1})u'(c_{t+1})$$

- Saving has become "more expensive"
- More attractive to consume more today
- What happens in the long-run?

Firms

- The law of motion for capital is unchanged: $k_{t+1} = f(k_t) + (1 - \delta)k_t - c_t$
- The interest rate must still represents the MPK: $r_t = f'(k_t)$

Long-run steady state

Steady state equations

$$\underbrace{\frac{1}{\beta} - 1 + \delta}_{\text{unchanged by taxation}} = (1 - \tau) r$$

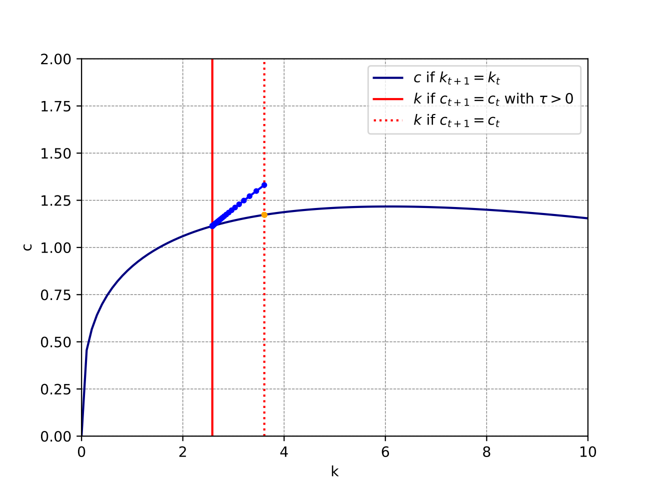
$$c = f(k) - \delta k$$

- In steady state, the Euler equation implies that $(1 - \tau) r$ must be constant, irrespective of τ

What is going on?

- Distortionary capital taxation makes saving more costly
- In equilibrium, there is less capital and consumption is lower
- $\tau \uparrow \rightarrow a \downarrow \rightarrow k \downarrow \rightarrow r \uparrow$

Positive capital taxation – graphical



- Increasing capital taxation decreases capital

Negative taxation?

If positive taxes lower k, then surely, subsidies increase it!

Steady state consumption $c = f'(k) - \delta$

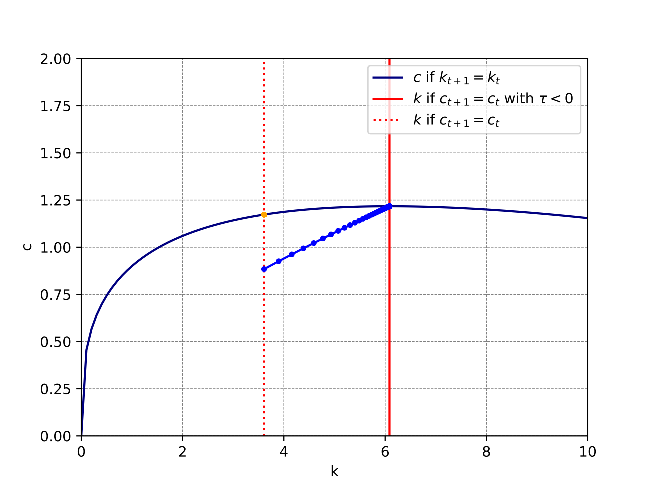
$$\frac{\partial c}{\partial k} = f'(k) - \delta$$

- Incentives for saving can increase aggregate consumption
- Maximum **steady state** consumption is achieved at $r = \delta$

What is going on?

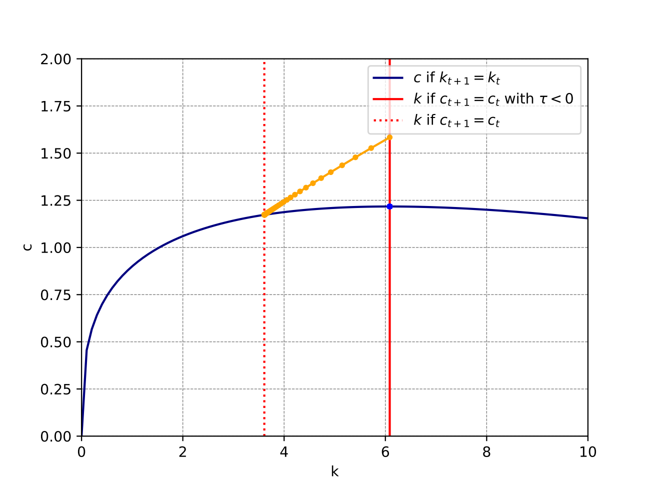
- The undistorted economy is Pareto efficient → leave $\tau = 0$
- Above equation implies that steady state → $c_{\tau<0} > c_{\tau=0}$

Negative taxation – graphical



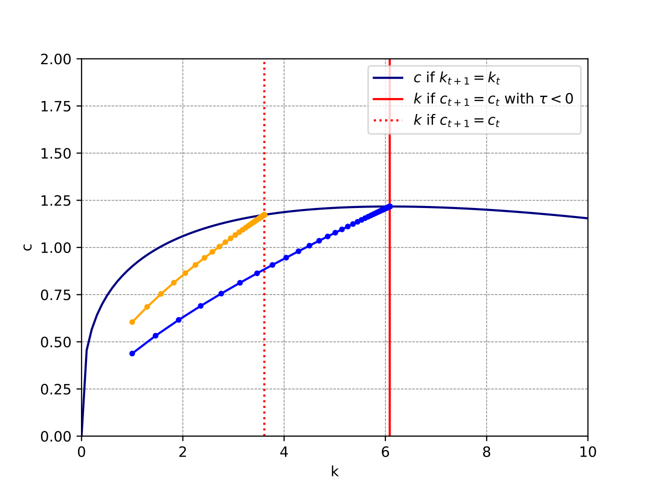
- From the old steady state, moving to the new one is costly

Negative taxation – graphical



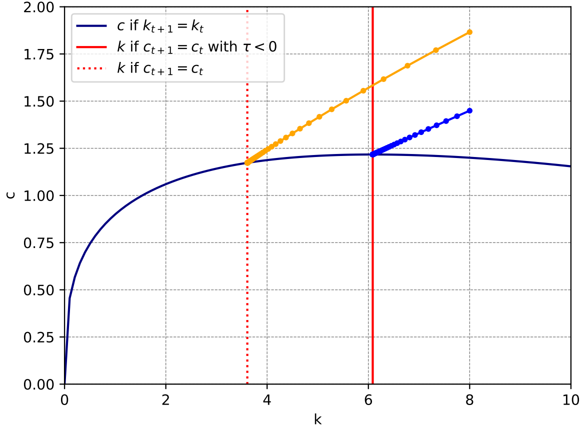
- Starting at k which maximizes c, old steady state more attractive

Negative taxation – graphical



- Starting from any k_0, $\tau = 0$ transition is always preferred

Negative taxation – graphical



- Starting from any k_0, $\tau = 0$ transition is always preferred

Final thoughts

Ricardian equivalence

- Huge literature estimates "fiscal multipliers" > 1
- Models that replicate them have many bells an whistles
- Starting point: Ramsey model
- Key ingredient: borrowing constraints ($+$ risk)