Problem Set I, Macroeconomics III
University of Copenhagen, 2024

John Kramer
Department of Economics, University of Copenhagen

February 2024

Exercise 1 – General equilibrium

Assume an agent lives for two periods, maximizing the utility function

$$U = \ln(c_0) + \beta \ln(c_1)$$

Each period, she receives an endowment y_t. She can save some of her period 1 endowment until period 2, or borrow against her period 2 endowment (she must always be able to pay it back). Hence, her dynamic budget constraints are

$$c_0 + a_0 = y_0$$
$$c_1 = y_1 + (1+r)a_0$$

Assume β and r are exogenously given initially.

1. Combine the two budget constaints and set up the Lagrangian for the maximization problem. Solve for the first order conditions by maximizing the Lagrangian with respect to c_0, c_1 and λ.

2. Obtain an Euler equation and give an intuition for it.

3. Solve for the values of c_0,c_1 and a_0 as a function of the primitives of the model (i.e. r, β, y_0, y_1).

4. What is the marginal propensity to consume out of income changes at time 0, i.e., if y_0 changes, how much does the agent consume immediately, how much is saved?

5. Assume that $y_1 = (1+g)y_0$, implying that $(1+g)$ represents the growth rate of the endowment. How does the demand for assets a_0 depend on this growth rate? Give an intuitive explanation.

6. Now, solve for the **general equilibrium** interest rate of this problem. The government collects no taxes, implying it cannot provide bonds for the agent to save in. Thus, $a_0 = 0$. Solve for the interest rate that must prevail in order for markets to clear, i.e., for the agent to be **indifferent** between holding bonds or not. *(Hint: at this interest rate, the Euler equation holds, but the agent optimally chooses $a_0 = 0$.)*

7. Compare the results of the previous exercise for two cases: a) The economy is in a boom today and will return to it's steady state tomorrow, i.e., $g < 0$; and (b) the agent is very optimistic about the future, i.e., $g > 0$. What are the interest rates in each of the two cases. Give an intuitive explanation.

Exercise 2 – CCRA and Log utility

1. Consider the following Constant Relative Risk Aversion (CRRA) utility function

$$u(c) = \frac{c^{1-\sigma} - 1}{1 - \sigma}, \sigma > 0; \sigma \neq 1.$$

   Calculate the coefficient of relative risk aversion

$$\varepsilon \equiv -\frac{du'(c)}{dc}\frac{c}{u'(c)} = -u''(c)\frac{c}{u'(c)}$$

2. Calculate the coefficient of relative risk aversion for the following utility function

$$u(c) = \ln(c)$$

3. According to L'Hopital's rule, we have that when $f(x_0) = g(x_0) = 0$ and $g'(x_0) \neq 0$, then

$$\lim_{x \to x_0} \frac{f(x)}{g(x)} = \frac{f'(x_0)}{g'(x_0)}.$$

   Use this to calculate

$$\lim_{\sigma \to 1} \frac{c^{1-\sigma} - 1}{1 - \sigma}$$

   Hint: Remember that $\frac{d}{dx}a^u = a^u \ln(a)\frac{du}{dx}$