Macroeconomics of the labor market

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Agenda

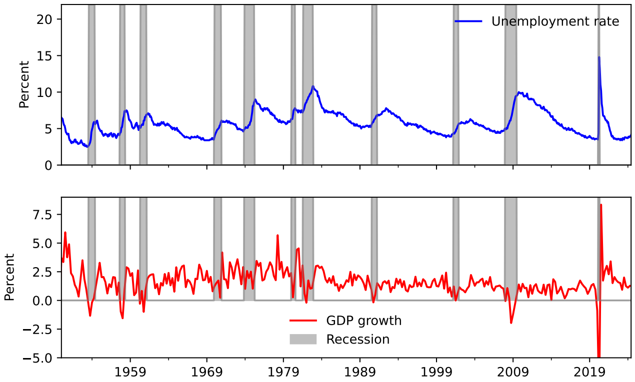
A simple search model

- Recursivity
- Discrete choice (accept vs reject)

The Diamond-Mortensen-Pissarides model

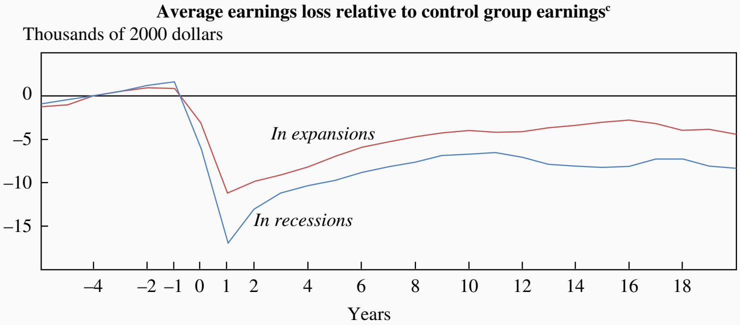
- Matching function
- The Beverdige curve
- Equilibrium
- Welfare

Importance of studying the labor market



- Unemployment rate is a major recession indicator

Importance of studying the labor market



- Unemployment has persistent effects on earnings (Davis & van Wachter, 2011)

Importance of studying the labor market

Additional reasons

- Unemployment risk is the most salient risk most people fear
- European governments spend around 1-2% of their GDPs on unempl. insurance
- Some countries have short-time work schemes to keep employment alive
- Some countries may introduce minimum wages

Simplified model

Consumer problem without capital (assets exogenously given)

$$\max_{c_t,l_t} \sum_{t=0}^{\infty} \beta^t \left(u(c_t) - v(h_t)\right) \quad s.t. \quad c_t + a_{t+1} = w_t h_t + (1 + r_t) a_t$$

First order conditions

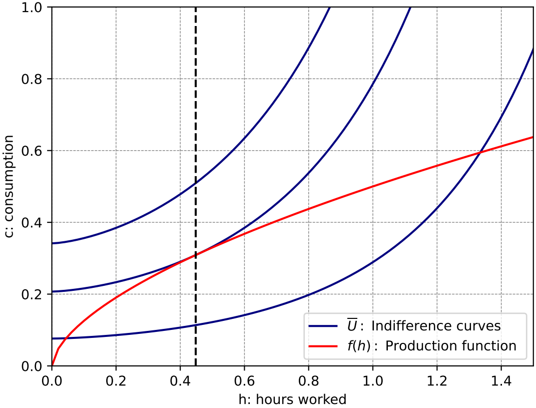
$$\begin{aligned} c_t : && u'(c_t) &= \lambda_t \\ l_t : && v'(h_t) &= \lambda_t w_t \\ a_{t+1} : && \lambda_t &= \beta(1 + r_{t+1})\lambda_{t+1} \end{aligned}$$

Firm problem

$$\max_{h_t} f(h_t) - w_t h_t$$

Optimality

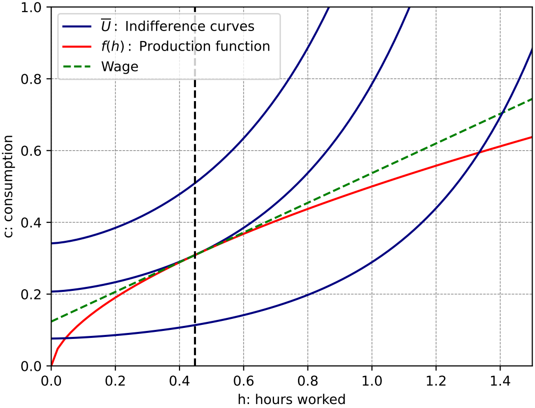
$$f'(h_t) = w_t; \qquad \frac{v'(h_t)}{u'(c_t)} = w_t$$



Optimality

Optimality

$$f'(h_t) = w_t; \qquad \frac{v'(h_t)}{u'(c_t)} = w_t$$



Voluntary unemployment

Optimality

- Agents optimally work a share l_t of their time budget
- Wages are always equal to the MRS
- Non-work time is leisure
- At given w, h can only change due to preferences
- Involuntary unemployment does not exist

Realism

- There is no “unemployment rate”, only non-worked hours
- This is not how “normal people” think about unemployment
- Understanding unemployment **important** for economists

Standard model of the labor market

What we want

- Employment is a binary state: $0, 1$
- Some agents "actively searched for work but could not find it"

Solution

- Frictional search models $\implies$ can never attain the first best
- Firms and workers might "miss each other"

Many processes are search processes

- Apartment hunting
- Tinder swiping
- School choice

The model of Diamond, Mortensen & Pissarides

The 2010 "Nobel Prize" in economics

The Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel 2010

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Peter A. Diamond

Prize share: 1/3

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Dale T. Mortensen

Prize share: 1/3

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Christopher A. Pissarides

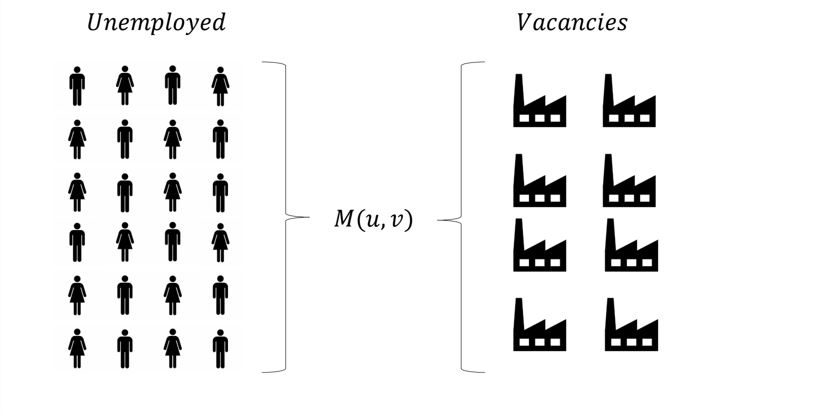
Prize share: 1/3

The Sveriges Riksbank Prize in Economic Sciences in Memory of Alfred Nobel 2010 was awarded jointly to Peter A. Diamond, Dale T. Mortensen and Christopher A. Pissarides "for their analysis of markets with search frictions"

The Diamond-Mortensen-Pissarides model

Crucial feature

- Matching between employers and employees is not frictionless



Starting points

Assumptions

- There is a mass 1 of workers, they are all the same
- There is a mass of firms (how many is endogenous)
- Both live forever and discount the future at $\frac{1}{1+r}(=\beta)$
- Worker utility is linear
- Production happens when a firm meets a worker and they start a match
- Search is not frictionless
- Posting vacancies is costly for firms

Outcomes

- Wage w
- Unemployment u
- Vacancies v, i.e., number of job-postings

The matching function

Even if all unemployed want to work, $M(u, v) < \min(u, v)$, due to a friction

Cobb-Douglas formulation

$$M(u, v) = A_m u^\gamma v^{1-\gamma}$$

- Constant returns to scale
- $u = \frac{N_u}{N_u + N_e}$ is the unemployment rate
- $v = \frac{N_v}{N_v + N_e}$ is the vacancy rate
- Note: We assumed $N_v + N_e = 1$

Matching probabilities

By dividing the number of matches by the #unemployed or the #vacancies, we recover important probabilities

Job-finding probability $f(\theta)$

$$\frac{M(u,v)}{u} = A_m u^{\gamma-1} v^{1-\gamma} = A_m \theta^{1-\gamma} \text{ where } \theta = \frac{v}{u}$$

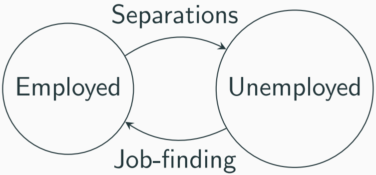
- θ is "market tightness", $\theta \uparrow \rightarrow$ more vacancies per unemployed
- $f(\theta)$ is increasing in $\theta \rightarrow$ more vacancies per searcher $\rightarrow$ higher chances of finding a match

Vacancy-filling probability $\eta(\theta)$

$$\frac{M(u,v)}{v} = A_m u^{\gamma} v^{-\gamma} = A_m \theta^{-\gamma}$$

- $\eta(\theta)$ is decreasing in θ more vacancies per searcher $\rightarrow$ more competition among firms $\rightarrow$ lower chances of finding a match

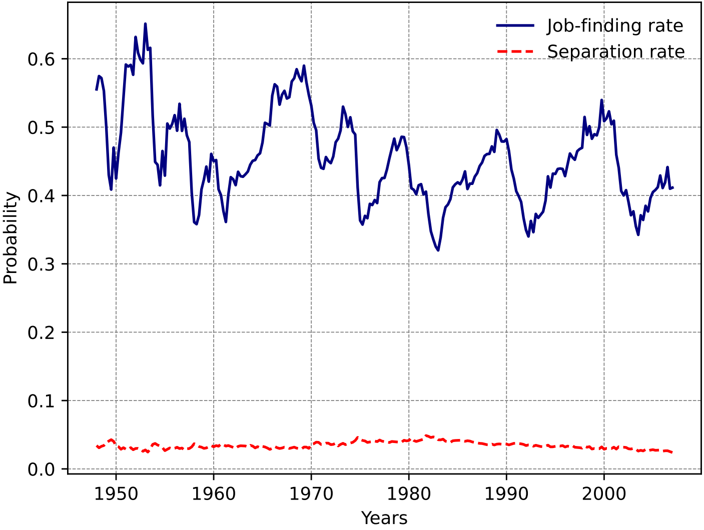
Stocks and flows



- The number (or rate) of employed and unemployed are a stock
- Separations (job-loss) or job-finding are flows

  In steady state, constant flows mean constant stocks

Stocks and flows



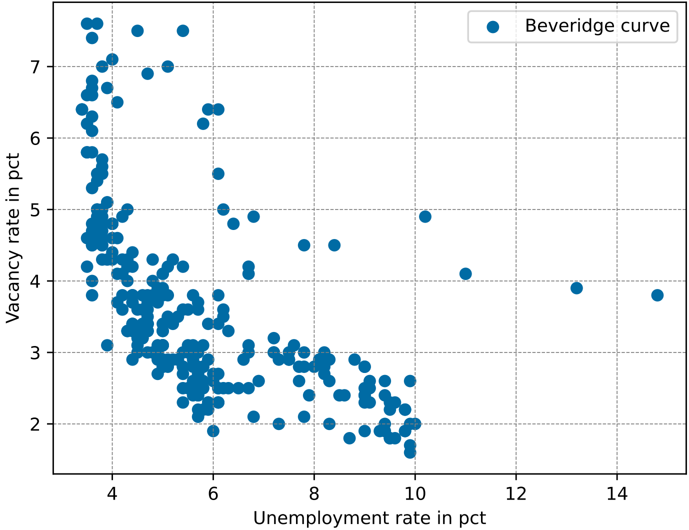
Aggregate unemployment

The stock of unemployed workers is constant in steady state

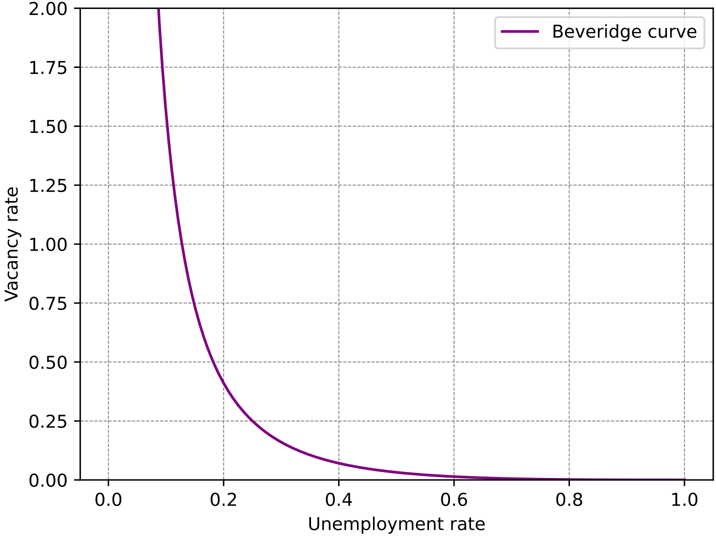
$$\begin{aligned} u &= \underbrace{\delta e}_{\text{Separations}} + \underbrace{(1 - f(\theta))u}_{\text{Didn't find a job}} \\ &= \delta(1-u) + (1 - f(\theta))u \\ u &= \frac{\delta}{\delta + f(\theta)} \end{aligned}$$

- The population is normalized to 1, hence $1 - u = e$
- Jobs (i.e., matches) can be destroyed with probability δ (exogenous)
- Remember: $\theta = v/u$, therefore, this equation gives a relationship between u and v
- This relationship is known as the "Beveridge curve"

The Beveridge curve



The model's Beveridge curve



Next steps

Beveridge curve

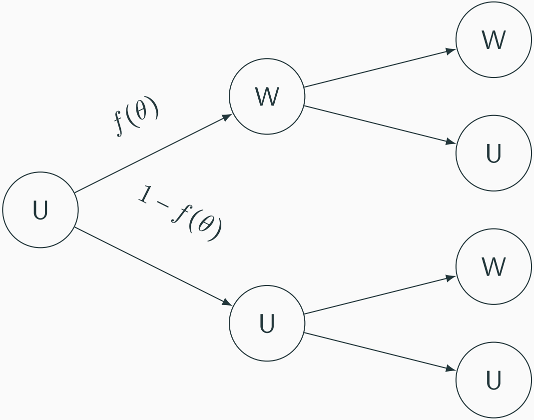
- The Beveridge curve establishes one part of the equilibrium
- Keep in mind that any point on the curve also represents a value of tightness θ

Next:

- Workers and firms determine the equilibrium wage
- Solve the steady state of the model

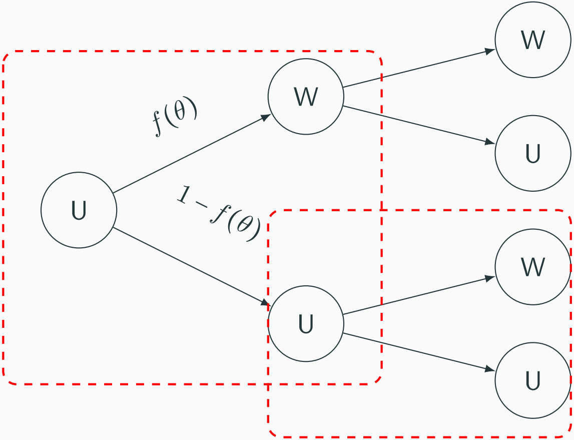
The unemployed worker's problem

Unemployed worker



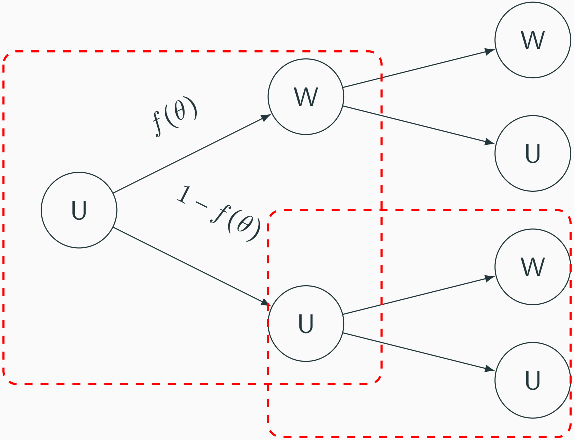
The unemployed worker's problem

Unemployed worker



The unemployed worker's problem

Unemployed worker



$$U = b + \frac{1}{1+r}\left[f(\theta)W(w) + (1 - f(\theta))U\right]$$

The workers' problems I

Unemployed worker

$$U = b + \frac{1}{1+r}\left[f(\theta)W(w) + (1 - f(\theta))U\right]$$

- Unemployed workers receive unemployment benefit b
- $f(\theta)$ is the endogenous job-finding probability

Employed worker

$$W(w) = w + \frac{1}{1+r}\left[(1-\delta)W(w) + \delta U\right]$$

- Employed workers receive endogenous wage $w > b$
- Separation probability δ is exogenous

Worker's surplus

$$S_w = W(w) - U = (w - b)\frac{1+r}{r + \delta + f(\theta)}$$

The firm's problem

Matched firm

$$J(w) = y - w + \frac{1}{1+r}\left[(1-\delta)J(w) + \delta V\right]$$

- A firm that hires a worker produces y and pays wage w
- If the match doesn't separate $(1-\delta)$, it continues
- Otherwise: the firm can post a vacancy V next period

Vacant firms

$$V = \max\left\{\underbrace{-\kappa + \frac{1}{1+r}\left[\eta(\theta)J(w) + (1-\eta(\theta))V\right]}_{V_{\text{post}}}, \underbrace{0 + \frac{1}{1+r}V}_{V_{\text{don't post}}}\right\}$$

- Posting a vacancy costs the firm κ, it is only active for one period
- The firm finds a worker with probability $\eta(\theta)$
- Technically, the firm can decide not to post a vacancy

The free entry condition

Free entry

- Firms will enter (i.e., post vacancies) until they are indifferent
- Hence, $V_{\text{post}} = V_{\text{don't post}} \implies V = 0$ (see below)

$$V = \max\{V_{\text{post}}, V_{\text{don't post}}\} = \max\{V_{\text{don't post}}, V_{\text{don't post}}\} = V_{\text{don't post}} = \frac{1}{1+r}V$$

$V = 0$ since $\beta > 0$

Firm's values updated from previous slide

$$J(w) = (y - w)\frac{1+r}{r+\delta}$$
$$J(w) = \frac{\kappa(1+r)}{\eta(\theta)}$$

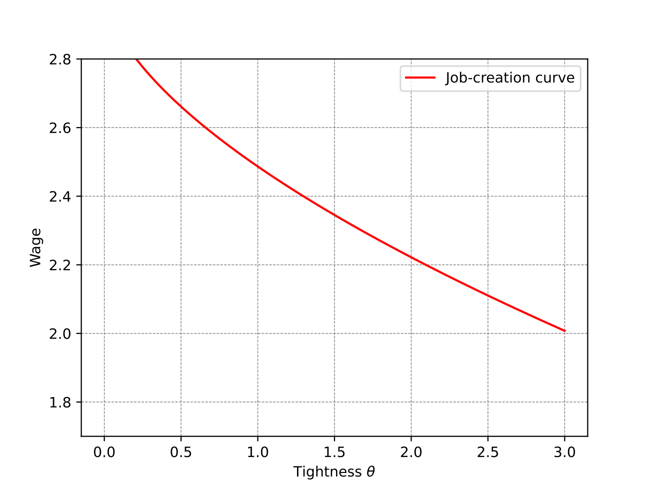
The job-creation curve

$$w = y - \frac{r + \delta}{\eta(\theta)}\kappa$$

Break-even wage

- In expectation, firms that post vacancies break even
- This relationship defines, for each θ, a possible w
- To pin down the wage, we need another restriction

The job-creation curve – graphically



- Demand curve: high wage (price) → few vacancies ($\theta \downarrow$)

Wage determination

Firms

- In case of the match forming, firms get $J(w)$
- If negotiations fall apart, they get 0 and can post again next period

Workers

- If the match forms, they receive $W(w)$
- If no match is formed, they receive U

→ The value of the match to the worker is $S_w = W(w) - U$

Total surplus

$$\Omega = \underbrace{[W(w) - U]}_{\text{Worker surplus}} + \underbrace{J(w)}_{\text{Firm surplus}}$$

- Negotiations divide total surplus, according to bargaining power

Nash bargaining

Process

- Workers and firms that meet bargain over the wage
- Both can threaten to walk away
- All information is known
- Workers have bargaining power ξ

Nash bargaining

$$w^* = \arg\max_w S_w(w)^\xi J(w)^{1-\xi}$$

Solution

$$J(w^*) = (1-\xi)\Omega$$
$$S_w(w^*) = \xi\Omega$$

- If $\xi \to 0$, firms pay workers close to their outside option, i.e., b
- If $\xi \to 1$, firms make less profit

Bargaining outcome

Wage curve Algebra

$$w = \xi\kappa\theta + \xi y + (1-\xi)b$$

- If workers have bargaining power ($\xi \uparrow$), wages are high
- Wages are higher in tight labor markets ($\theta \uparrow$)

Wage is weighted average of outside options

- Workers' outside option: unemployment (b)
- Firms outside option: no production (y) and post new vacancy (κ)

→ In extreme case, firm pays the worker more than productivity y

→ Operational losses (mainly a concern in dynamic setting)

Model summary

Step 1: Equilibrium wage and tightness

- Wage curve + job-creation curve pin down w and θ
- Not necessarily solvable by hand, but unique solution

→ Outcomes of firm and worker interactions

Step 2: Equilibrium unemployment and vacancies

- Job-creation curve and Beveridge curve pin down u and v
- JC: $w^* = y - \frac{r+\delta}{\eta(\theta^*)}\kappa$ can be solved for $v(u)$

→ What interactions mean for aggregates

Equilibrium

Wage curve and job-creation curve

$$w = \xi\kappa\theta + \xi y + (1-\xi)b \quad \text{and} \quad w = y - \frac{r+\delta}{\eta(\theta)}\kappa$$

$$\implies (y-b)(1-\xi) = \kappa\frac{r+\delta}{\eta(\theta)} + \kappa\theta\xi$$

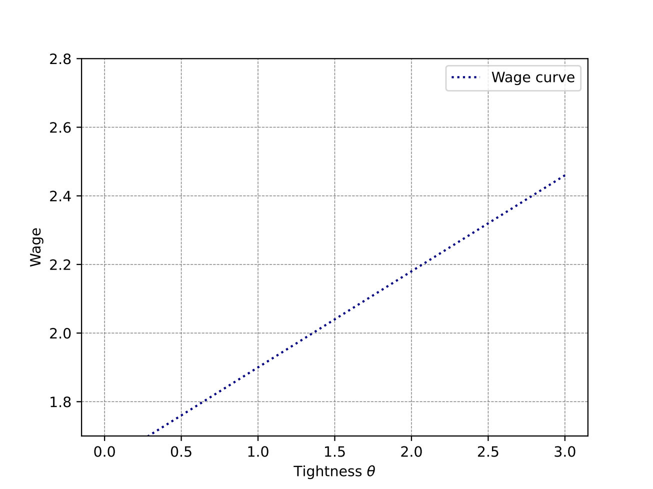
- Both relate the wage w to labor market tightness θ
- Intersection identifies equilibrium (not solvable by hand)
- Keep this second equation in mind, it may reappear

Beveridge curve

$$u = \frac{\delta}{\delta + f(\theta)}$$

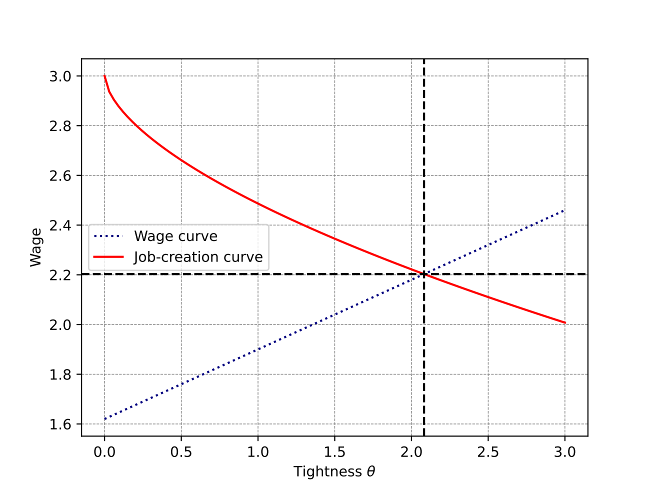
- Identifies u and v separately, given θ

Wage curve and equilibrium



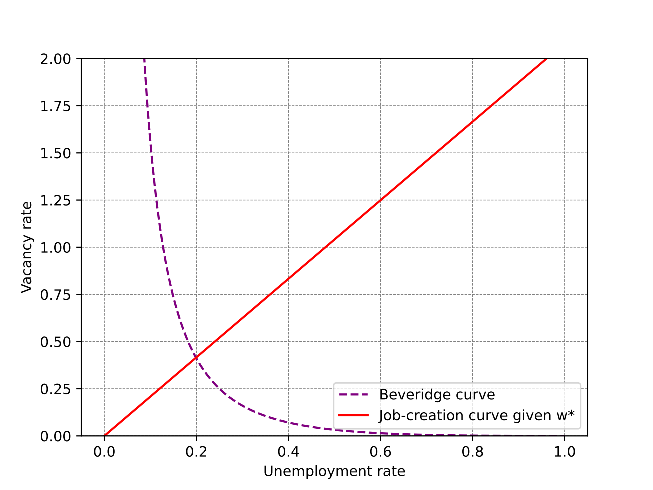
- Supply curve: wage low → few ($\theta \downarrow$)

Wage curve and equilibrium



- Wage and tightness are pinned down

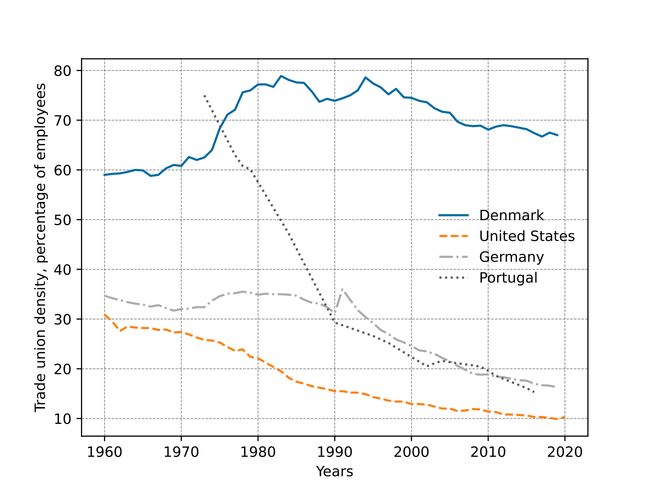
Wage curve and equilibrium



- Unemployment and vacancies are solved

Experiments

Collective bargaining



- The bargaining power of workers differs across countries

Bargaining power in the model

Moving from the US to Denmark $\rightarrow \xi \uparrow$

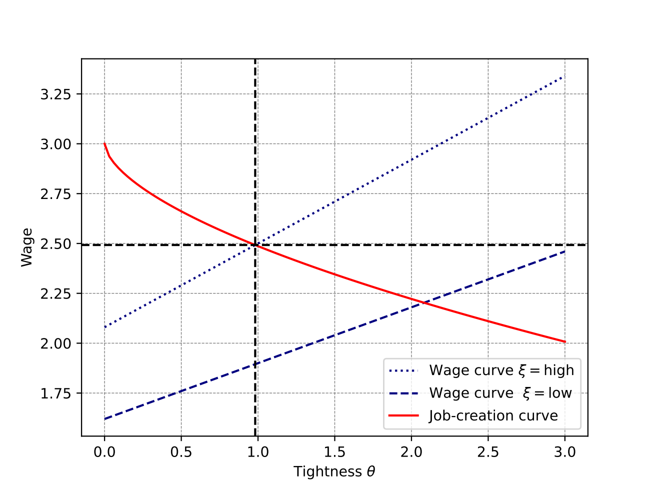
$$
\begin{aligned}
w &= \xi\kappa\theta + \xi y + (1-\xi)b && \implies w \uparrow \\
w &= y - \frac{r+\delta}{\eta(\theta)}\kappa && \implies \theta \downarrow \\
u &= \frac{\delta}{\delta + f(\theta)} && \implies u \uparrow, v \downarrow
\end{aligned}
$$

- As $\xi \uparrow$, wages rise
- But, employment falls

Welfare question

- There's a trade-off between wage and employment

US → Denmark, $\xi \uparrow$



- Higher wages, but lower employment

Welfare

Welfare

What can the planner do?

- Employment seems better than unemployment ($w > b$)
- More employment means higher production → higher welfare

Inefficiency I: Hold-up problem

- The vacancy posting cost κ is sunk for the firm
- Example: if workers had all bargaining power ($\xi = 1$), then $w > y$. No firm would post vacancies ($\theta = 0$). $u = 1$ is inefficient

Inefficiency II: Congestion externality

- Posting a vacancy has a negative effect on all other firms searching
- Example: if firms had all bargaining power ($\xi = 0$), then $w = b$. There is too much entry and no firm finds a worker. $u = 1$ is inefficient

The social planner

Maximization problem

$$\max_{u_t,\theta_t} \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t \left[y(1-u_t) + bu_t - \underbrace{\kappa\theta_t u_t}_{\kappa v_t}\right]$$

$$\text{subject to } u_{t+1} = \delta(1-u_t) + (1-\underbrace{\theta\eta(\theta_t)}_{f(\theta_t)})u_t$$

- Planner doesn't care about w, as it's just a transfer between firm and worker
- $y(1-u)$ is the social value of employment
- bu is the value of leisure for the unemployed
- κv is the social cost of vacancies

Social planner's solution

Optimal level of u and v in steady state Algebra

$$(y-b)\left(1+\frac{\theta}{\eta(\theta)}\frac{\partial\eta(\theta)}{\partial\theta}\right)=\kappa\frac{\delta+r}{\eta(\theta)}-\kappa\theta\left(\frac{\theta}{\eta(\theta)}\frac{\partial\eta(\theta)}{\partial\theta}\right)$$

- The term in red is the elasticity of the vacancy filling probability w.r.t. tightness
- If tightness rises, how much does the probability of finding a worker fall?
- With Cobb-Douglas: $\frac{\theta}{\eta(\theta)}\frac{\partial\eta(\theta)}{\partial\theta}=-\gamma$

Simplified

$$(y-b)(1-\gamma)=\kappa\frac{\delta+r}{\eta(\theta)}+\kappa\theta\gamma$$

Does this equation look familiar?

The Hosios condition

Vacancy posting vs bargaining power

- If possible, the planner would like to equalize $\xi = \gamma$
- This result is due to Hosios (1990)

What does this solve?

- When $\gamma \uparrow$, posting a vacancy hurts other firms a lot
- When $\xi \uparrow$, firms are disincentivised from posting

→ The two forces strike a balance to minimize the inefficiency in the model

Last thoughts

DMP extensions

- Unifying RBC and DMP (Merz, 1995) leads to puzzles (Shimer, 2005)
- DMP and sticky wages (Hall, 2005)
- New Keynesian model + DMP (Walsh, 2003)
- RBC+DMP+heterogeneous workers (Krusell et al, 2010)
- Optimal unemployment benefits in ↑ framework (Mitman & Rabinovich, 2015)

Other models of the labor market

- Directed search model (DMP is random search)

Appendix

Algebra

Back

$$\frac{1}{1-\beta}w^* = b + \beta\left\{\frac{w^*-\underline{w}}{\overline{w}-\underline{w}}\frac{1}{1-\beta}w^* + \frac{\overline{w}-w^*}{\overline{w}-\underline{w}}\frac{1}{1-\beta}\mathbb{E}[w'|w' > w\right.$$

$$\left(\frac{\overline{w}-w^*}{\overline{w}-\underline{w}} + \frac{w^*-\underline{w}}{\overline{w}-\underline{w}}\right)\frac{w^*}{1-\beta} = b + \beta\left\{\frac{w^*-\underline{w}}{\overline{w}-\underline{w}}\frac{1}{1-\beta}w^* + \frac{\overline{w}-w^*}{\overline{w}-\underline{w}}\frac{1}{1-\beta}\mathbb{E}[w'|w' > w\right.$$

$$\frac{w^*-\underline{w}}{\overline{w}-\underline{w}}w^* - b = \frac{1}{1-\beta}\left\{\frac{\overline{w}-w^*}{\overline{w}-\underline{w}}\left(\beta\mathbb{E}[w'|w' > w^*] - w^*\right)\right\}$$

$$\frac{w^*-\underline{w}}{\overline{w}-\underline{w}}w^* + \frac{\overline{w}-w^*}{\overline{w}-\underline{w}}w^* - b = \frac{1}{1-\beta}\left\{\frac{\overline{w}-w^*}{\overline{w}-\underline{w}}\left(\beta\mathbb{E}[w'|w' > w^*] - w^*\right)\right\} + \frac{\overline{w}-w^*}{\overline{w}-\underline{w}}w$$

$$\underbrace{w^*-b}_{\text{Cost of search}} = \underbrace{\frac{\beta}{1-\beta}\left\{\frac{w^*-\underline{w}}{\overline{w}-\underline{w}}\left(\mathbb{E}[w'|w' > w^*] - w^*\right)\right\}}_{\text{Benefit of search}}$$

Wage curve algebra

$$
\begin{aligned}
S_w &= \xi(S_w + J(w)) \\
\xi J(w) &= (1-\xi)S_w \\
\xi \frac{y-w}{r+\delta} &= (1-\xi)\frac{w-b}{f(\theta)+\delta+r} \\
\xi(y-w)(f(\theta)+\delta+r) &= (1-\xi)(w-b)(r+\delta) \\
\xi y(f(\theta)+\delta+r) - \xi w f(\theta) &= (r+\delta)w - (1-\xi)b(r+\delta) \\
\xi y f(\theta) + \xi y(\delta+r) - \xi w f(\theta) &= (r+\delta)w - (1-\xi)b(r+\delta) \\
\xi {\color{red}(y-w)} f(\theta) + \xi y(\delta+r) &= (r+\delta)w - (1-\xi)b(r+\delta) \\
\xi \frac{\kappa}{\eta(\theta)}(r+\delta)\theta\eta(\theta) + \xi y(\delta+r) &= (r+\delta)w - (1-\xi)b(r+\delta) \\
w &= \xi\kappa\theta + \xi y + (1-\xi)b
\end{aligned}
$$

from 7th to 8th line, use $\frac{y-w}{r+\delta} = \frac{\kappa}{\eta(\theta)}$ and $f(\theta) = \theta\eta(\theta)$ Back

Social planner algebra

$$\mathcal{L} = \sum_{t=0}^{\infty} \left(\frac{1}{1+r}\right)^t \left[\beta^t (y(1-u_t) + bu_t - \kappa v_t\right] + \sum_{t=0}^{\infty} \lambda_t \left[\delta(1-u_t) + (1-\theta\eta(\theta_t))u_t - u_{t+1}\right]$$

First order conditions

$$\frac{\partial \mathcal{L}}{\partial v_t} : \quad -\kappa\beta^t - \lambda_t \left(\eta(\theta_t) + \theta_t \frac{\partial \eta(\theta_t)}{\partial \theta_t}\right) = 0$$

$$\frac{\partial \mathcal{L}}{\partial u_{t+1}} : \quad \beta^{t+1}(b-y) - \lambda_t + \lambda_{t+1}\left[1 - \delta + \theta_{t+1}^2 \frac{\partial \eta(\theta_{t+1})}{\partial \theta_{t+1}}\right] = 0$$

To arrive at this simplification, one needs to use the facts that $d\theta/du = -v/u^2$, $d\theta/dv = 1/u$. Combine the equations to get the equation on the main slide.

Back