The Real Business Cycle model

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Agenda

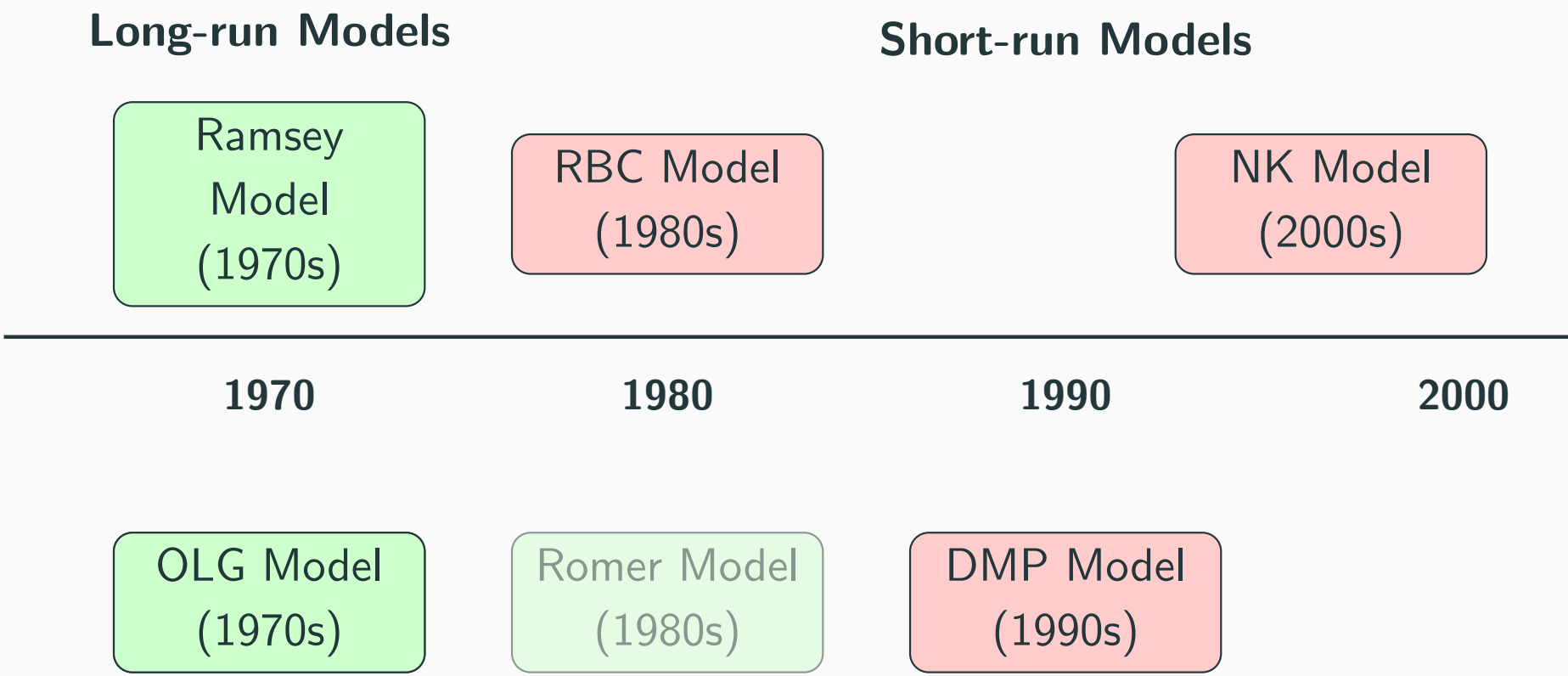
Business cycles

- What are business cycles?
- Another history of microfoundations
- Why should we care about business cycles

The RBC model

- Productivity shocks
- Ramsey, put with shorter periods
- Calibration
- Impulse responses

Agenda


Long-run Models
Short-run Models
Ramsey Model (1970s)
RBC Model (1980s)
NK Model (2000s)
1970
1980
1990
2000
OLG Model (1970s)
Romer Model (1980s)
DMP Model (1990s)

Business cycles

Long-run vs short-run

- There is persistent long-run growth, Solow and Ramsey make sense of this
- But what about short run fluctuations?
- What are short run fluctuations?

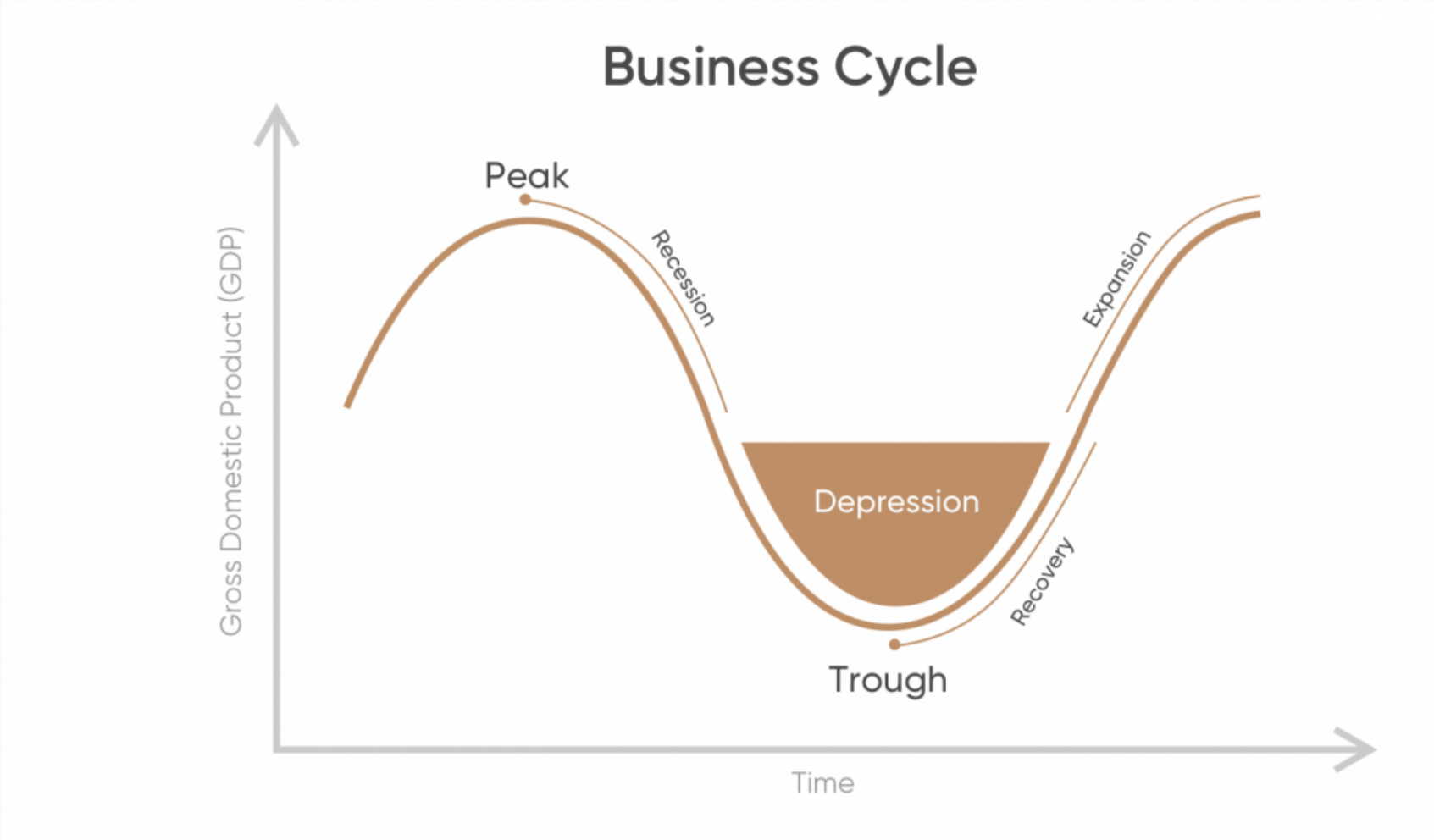
Quick aside about growth rates

$$\ln(Y_t) - \ln(Y_{t-1}) \approx \frac{Y_t - Y_{t-1}}{Y_{t-1}} - 1$$

$$\underbrace{\ln(Y_t) - \ln(Y_{t-1}) + \ln(Y_{t-1}) - \ln(Y_{t-2})}_{=\ln(Y_t)-\ln(Y_{t-2})}; \underbrace{\frac{Y_t - Y_{t-1}}{Y_{t-1}} + \frac{Y_{t-1} - Y_{t-2}}{Y_{t-2}}}_{\neq \frac{Y_t - Y_{t-2}}{Y_{t-2}} - 1}$$

- For small changes in Y_t, Δlog is similar to Δpct
  $\ln(1.02) - \ln(1) = 0.0198$ (Now is a good time to brush up $\ln$-rules)
- log-changes are additive, percentages are not

Terminology



Detrending GDP

Distinguish long- and short-run variation

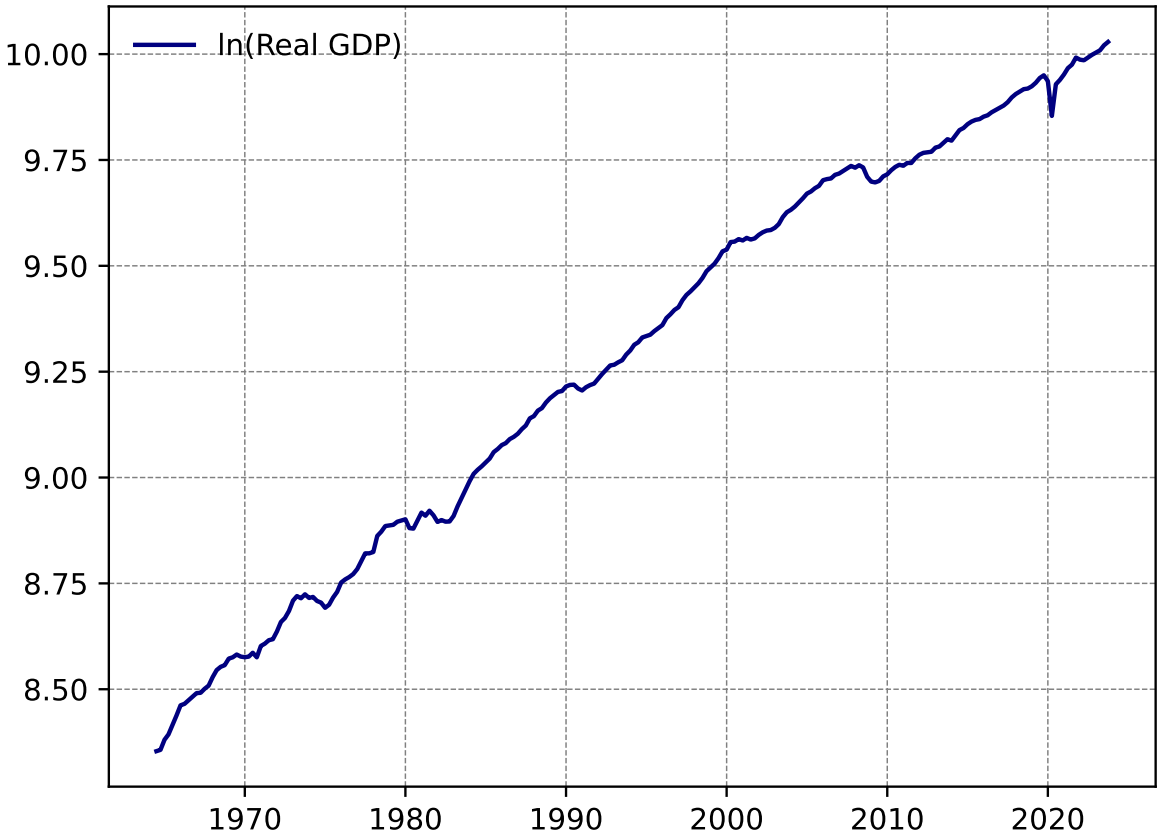
- What should the model hit?
- How does a "normal" business cycle look?

Hodrick-Prescott filter

- Separate "wiggles" and "trend"
- Smooth out any data series
- As $\lambda \uparrow$, trend growth rate varies less
- $\lambda \to \infty$, the trend is a straight line (for quarterly data, $\lambda = 1600$)

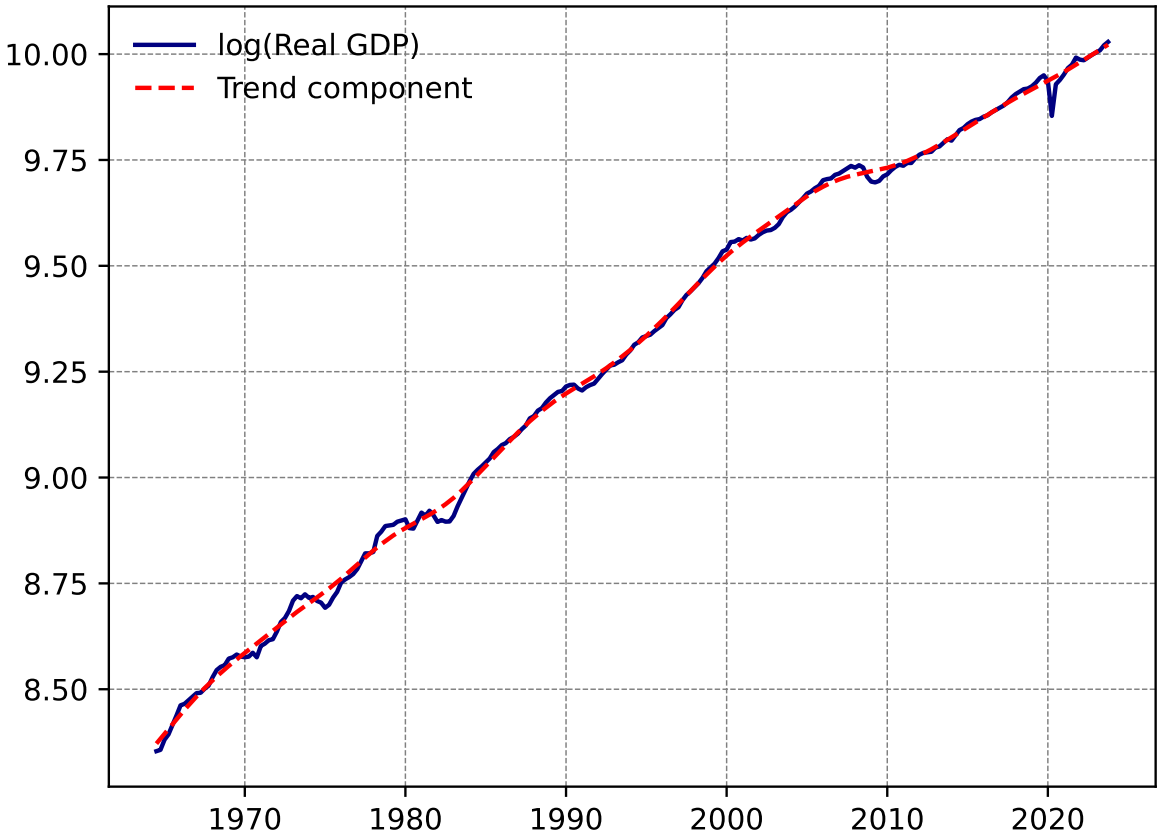
$$\min_{\{\tau_t\}_1^T} \sum_{t=1}^{T} (y_t - \tau_t)^2 + \lambda \left[\underbrace{(\tau_{t+1} - \tau_t)}_{g_\tau(t)} - \underbrace{(\tau_t - \tau_{t-1})}_{g_\tau(t-1)} \right]^2$$

US business cycles DK DE PT



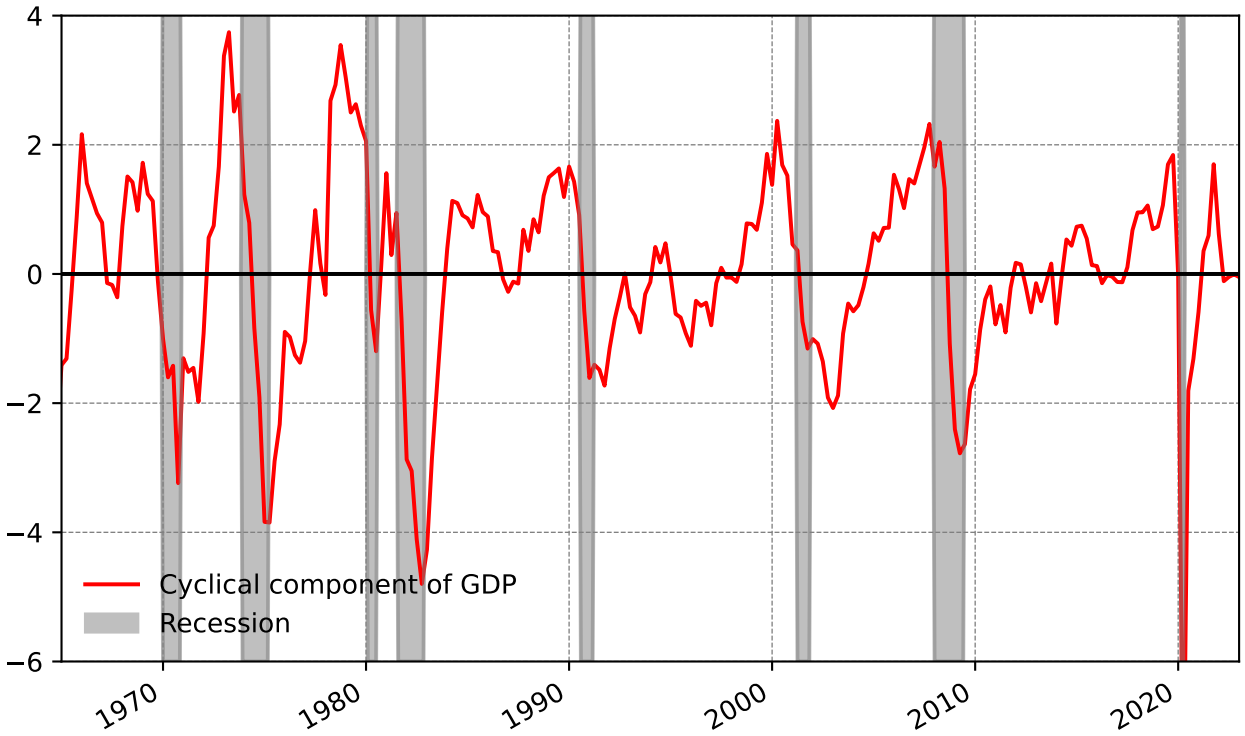
- GDP increases, but not smoothly → distinguish SR & LR

US business cycles



- Many periods of deviation from trend

US business cycles



- Business cycles

Other aggregates

How do other economic aggregates behave over the business cycle?

- GDP (Real Gross Domestic Product)
- Investment (Real Gross Private Domestic Investment)
- Consumption (Real Personal Consumption Expenditures)
- Hours (Nonfarm Business Sector: Hours Worked for All Workers)
- Unemployment rate

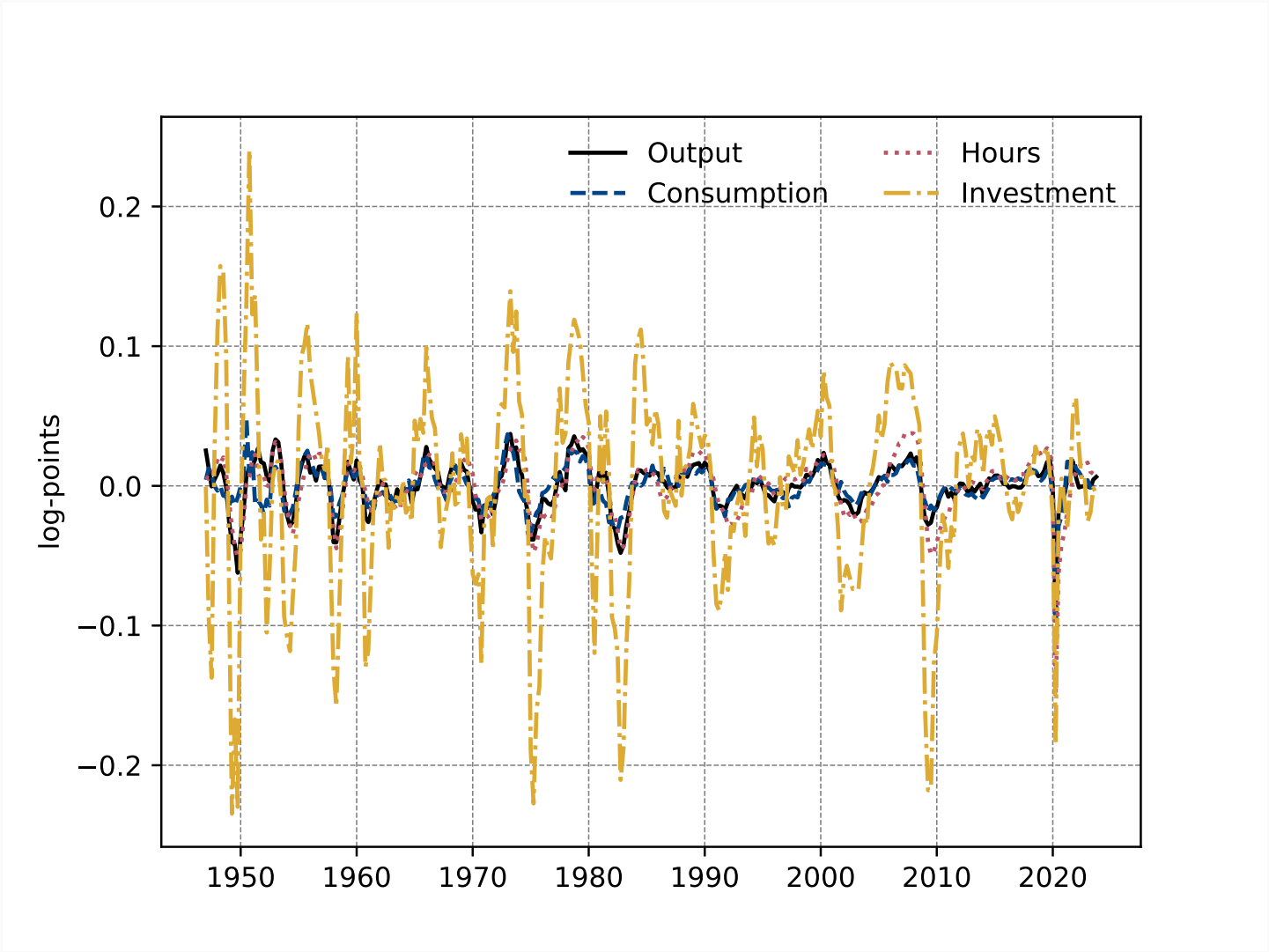
All quarterly

Sources for the US (and many countries): Fred

Sources for European countries: Eurostat National accounts data

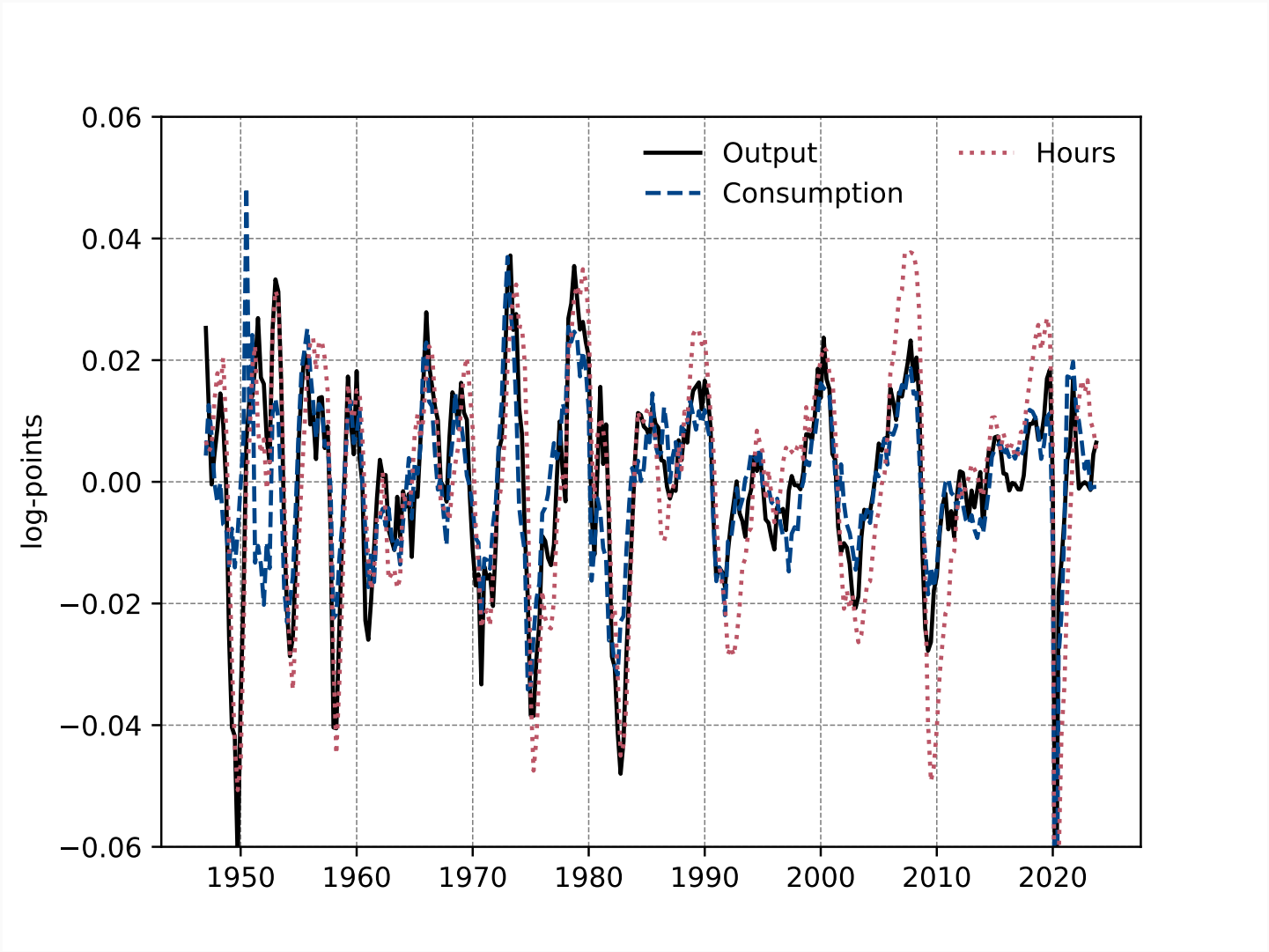
Sources for most countries: OECD

The cyclical relationship of aggregates (HP-filtered)



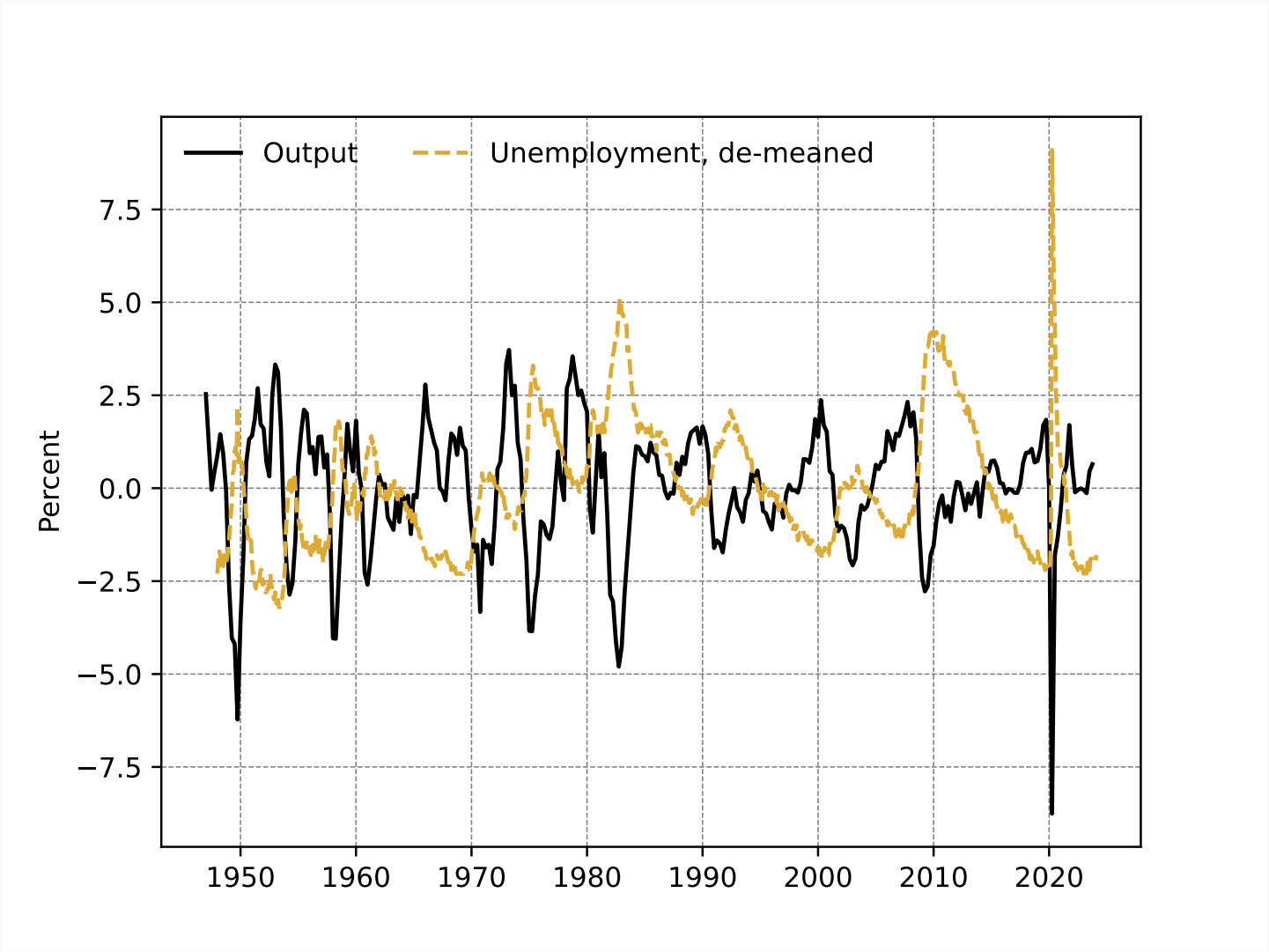
- Investment is **much** more volatile than output, and pro-cyclical

The cyclical relationship of aggregates (HP-filtered)



- Consumption and hours are pro-cyclical, C is less volatile than output

The cyclical relationship of aggregates (HP-filtered)



- The unemployment rate is **counter**-cyclical (bankruptcies, too)

Important facts about business cycles

Descriptive statistics of business cycles

	$\sigma(x_t)$	$\sigma(x_t)/\sigma(Y_t)$	$\rho(x_t, x_{t-1})$	$\rho(x_t, Y_t)$
Y	1.64	1.00	0.78	1.00
C	1.37	0.84	0.71	0.79
I	7.15	4.37	0.78	0.82
H	2.11	1.29	0.81	0.86

- Business cycle fluctuations in all major macroeconomic variables
- Labor is about as volatile as output, I much more, C less
- There is a lot of persistence $\rho(x_t, x_{t-1})$ is the data
- All variables are highly correlated with output

History

Business cycle macro – historical perspective I

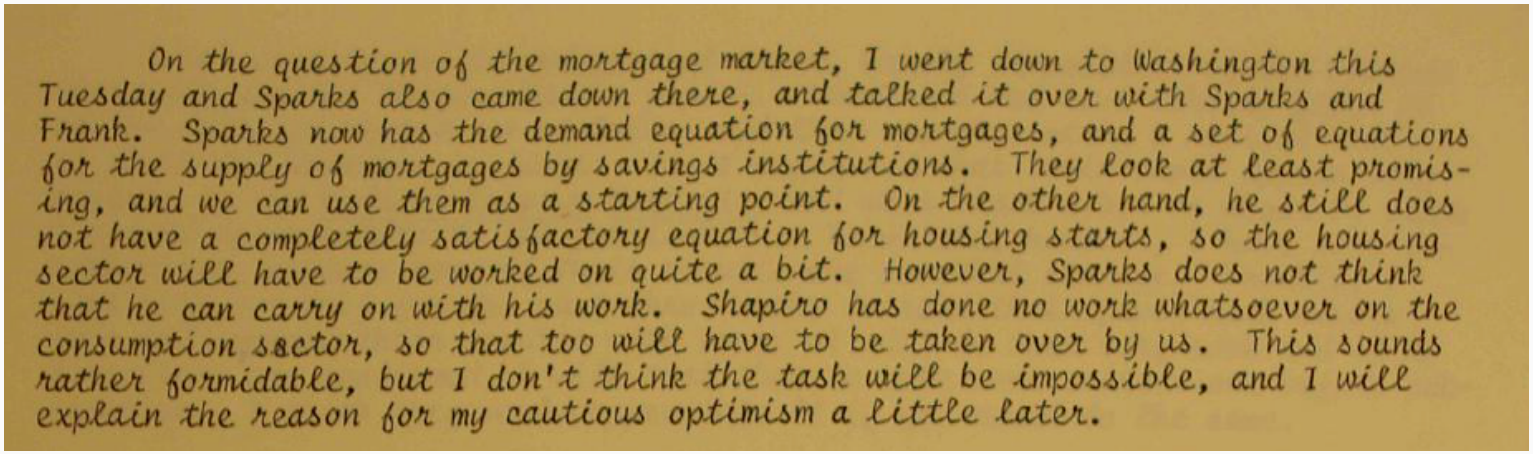
1930s – Keynes

- Countercyclical fiscal policy, stimulus in recessions (IS-LM, Hicks)

Macro econometrics before 1970s Source

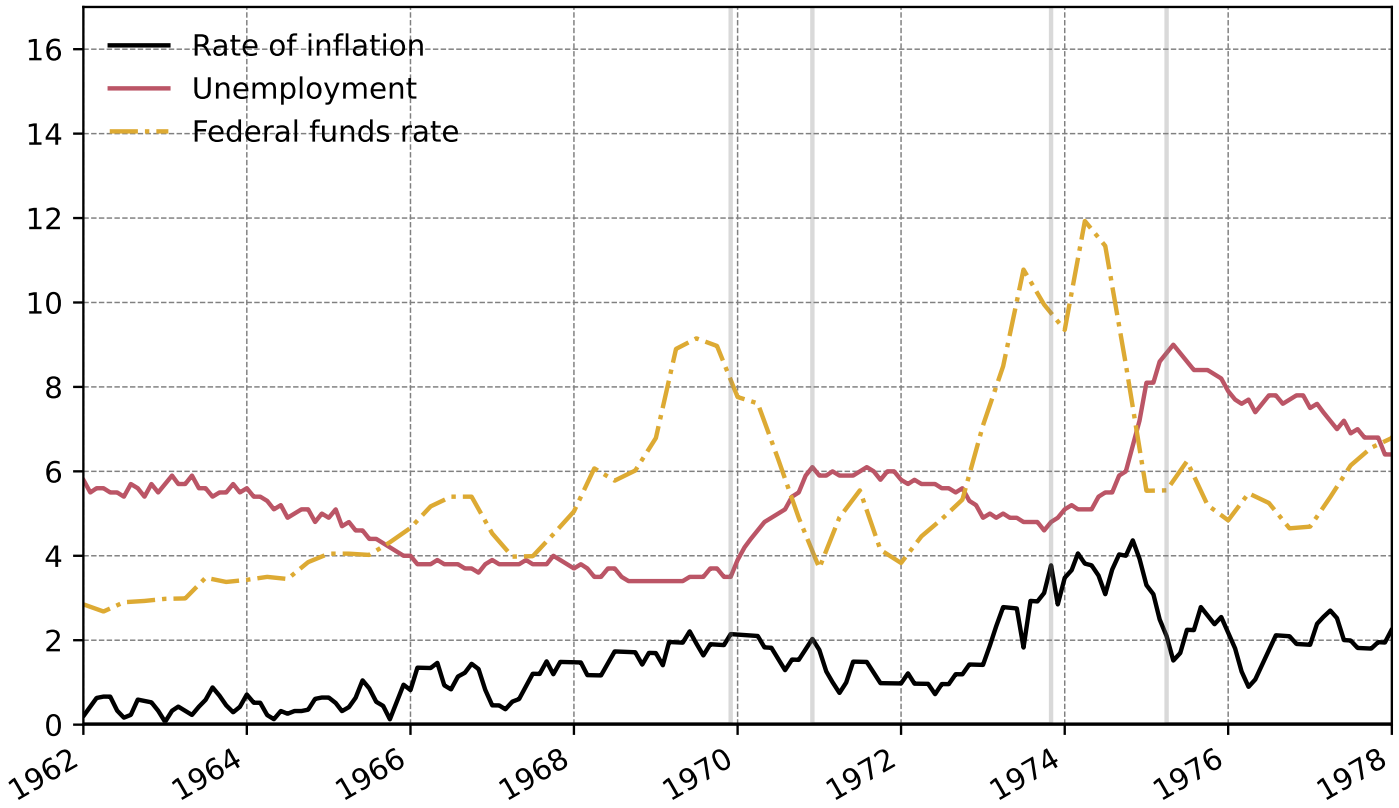
- Write down equations that may explain the economy
- Estimate the equations using time series data (scientific revolution!)
- Worked well until the 1970s (no major recessions)

Letter to Franco Mondigliani on Fed's model:



On the question of the mortgage market, I went down to Washington this Tuesday and Sparks also came down there, and talked it over with Sparks and Frank. Sparks now has the demand equation for mortgages, and a set of equations for the supply of mortgages by savings institutions. They look at least promising, and we can use them as a starting point. On the other hand, he still does not have a completely satisfactory equation for housing starts, so the housing sector will have to be worked on quite a bit. However, Sparks does not think that he can carry on with his work. Shapiro has done no work whatsoever on the consumption sector, so that too will have to be taken over by us. This sounds rather formidable, but I don't think the task will be impossible, and I will explain the reason for my cautious optimism a little later.

The US macroeconomy in the 1970s



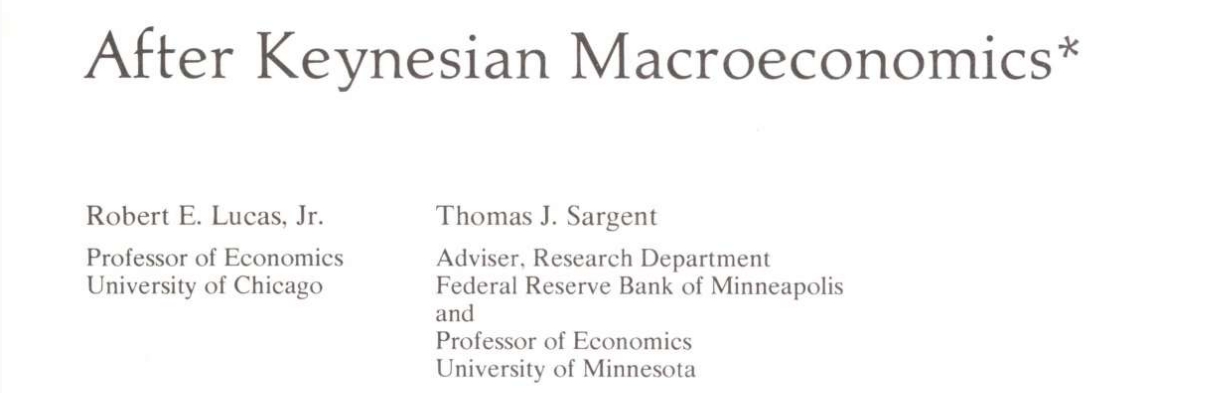
Business cycle macro – historical perspective II

1970s – What now?

- Oil crises, major recession
- High inflation rates (→ not too little money)
- Major government deficits (→ not too little spending)

Enter: Minnesota & Chicago macro The manifesto

- Robert Lucas & four horsemen (Prescott, Sims, Sargeant, Wallace)
- Behaviors in models are not policy-invariant
- Past relationships cannot predict the future after major changes to the system



After Keynesian Macroeconomics*

Robert E. Lucas, Jr.
Professor of Economics
University of Chicago

Thomas J. Sargent
Adviser, Research Department
Federal Reserve Bank of Minneapolis
and
Professor of Economics
University of Minnesota

Microfoundations

Focus on the most fundamental things

- Certain relationships may change with policy (Philips curve)
- Structural aspects are permanent (β, σ, ϕ)
- Need a model that can take structural parameters and reproduce aggregate relationships

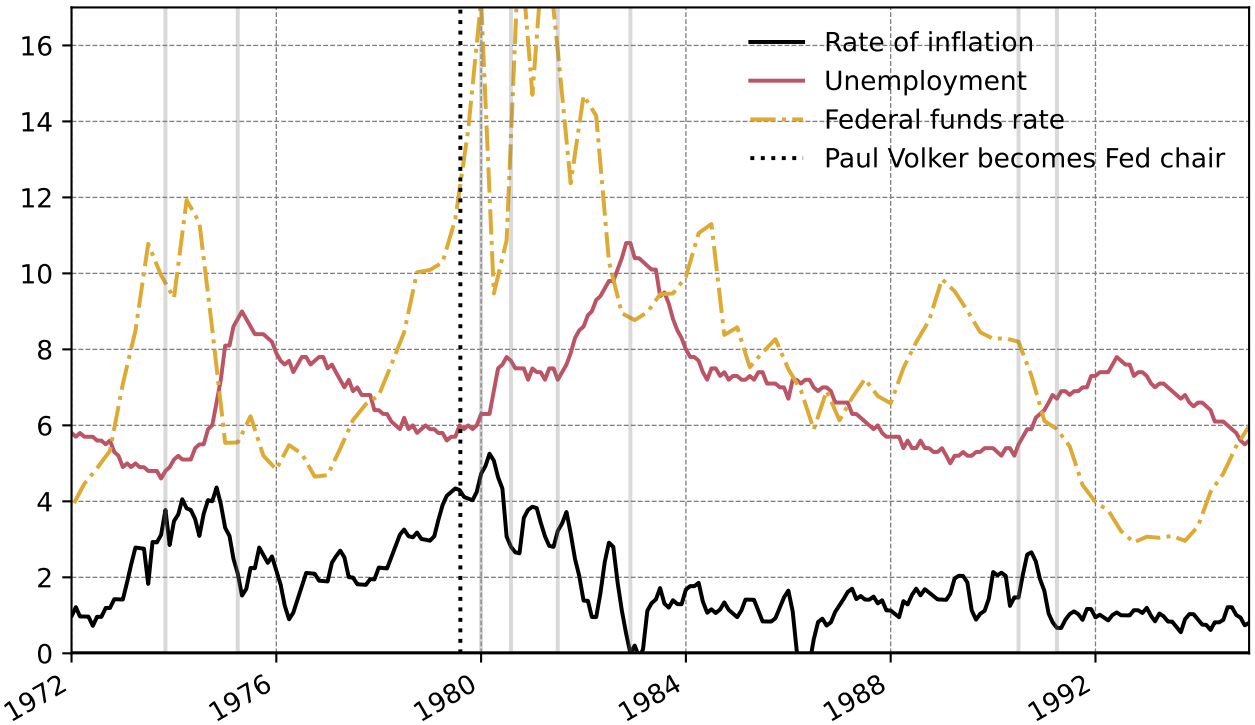
New models based on classical microeconomics

- Preferences (utility function)
- Technology (production function)
- Environment (resource constraint / market clearing)

$\Longrightarrow$ Less focus **what** will happen, more focus on **why** it happens

$\Longrightarrow$ Need a good way of choosing structural parameters (more later)

The US macroeconomy after the 1970s



- Fed chair Paul Volker: "The standard of living of the average American has to decline." [...to reign in inflation]

Rational expectations (independent of microfoundations)

Dynamic models with shocks

- Ramsey & Solow models are deterministic (everything is certain)
- Business cycles are surprising → model is stochastic
- Model agents have to form some expectations ($\mathbb{E}$) for the future

Solution: people know the underlying process of their world

- Very strong assumption, but mathematically convenient
- Agents can still get it wrong, but never systematically

Very controversial, but much less fundamental than microfoundations

The road to the RBC model

Lucas' models make beautiful points but are complicated

- Imperfect information, money, will see in future lectures

Operationalizing microfoundations

- "Kydland & Prescott did for Lucas what Hicks, Mondigliani, and Klein had done (...) for Keynes: they quantified original thinking and made it operational" (Azariadis, 2018)
- Build on Ramsey model (which people know), add technology shocks
- Money doesn't cause business cycles, technology does

Disciplining the model

- Chose parameters through **calibration**
- Don't try to match the macro (like the Keynesians)
- Use micro-estimates for structural parameters

The RBC model

Productivity shocks

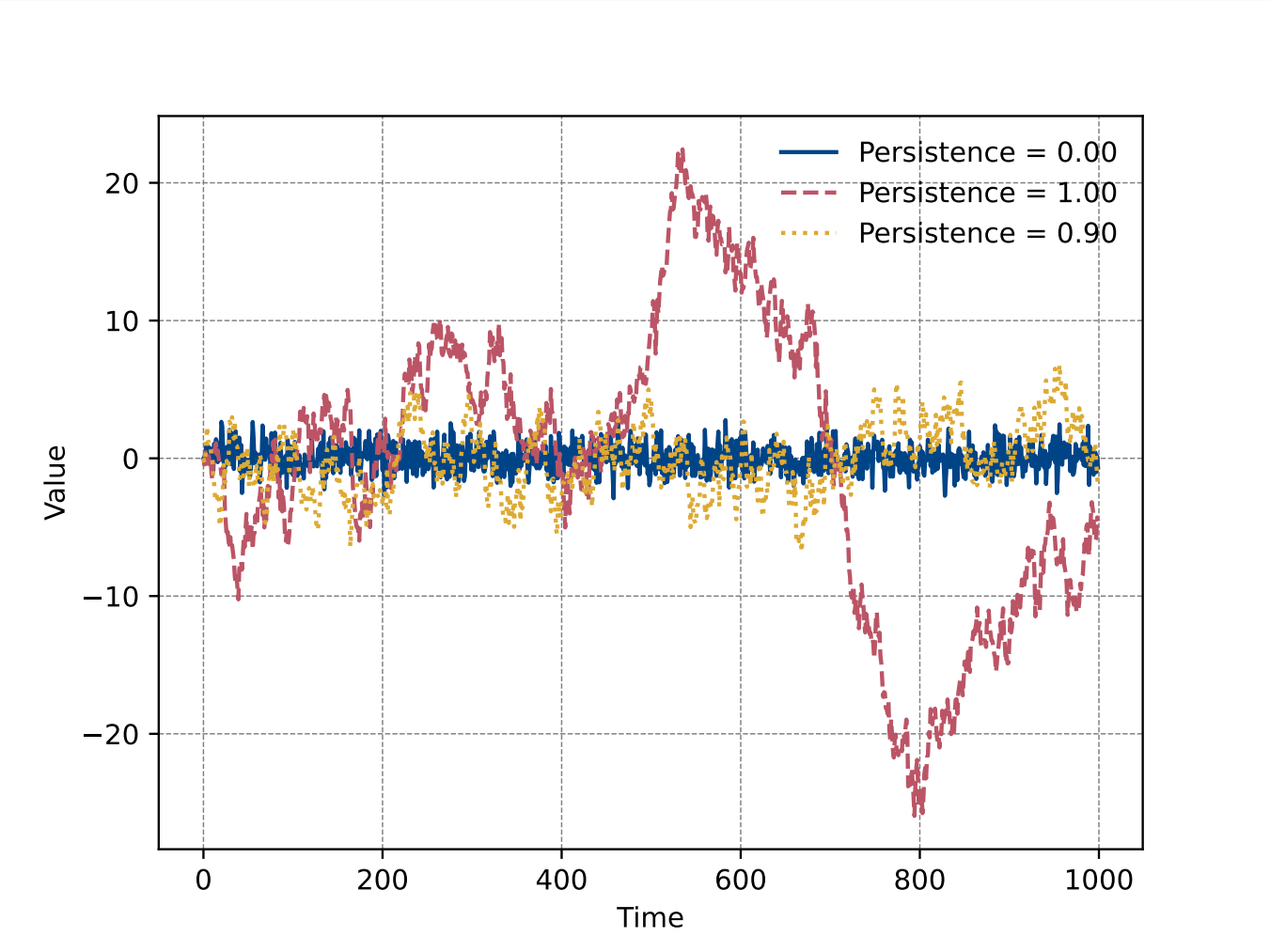
AR(1) process (autoregressive of order 1)

- Ignore growth for now, only short-run fluctuations
- Today's (log) productivity is close to yesterday's
- Productivity evolves independently of k
- Each period, there's a shock that may change the level

$$\ln(z_t) = \rho \ln(z_{t-1}) + \varepsilon_t; \qquad \varepsilon_t \sim N(0, \sigma_\varepsilon)$$

- Shocks are multiplicative & centered at 1 in levels
- Persistence of shocks is governed by ρ ($\rho = 1 \rightarrow$ permanent)
- Shocks are normally distributed with variance σ_ε

AR(1) process – graphical



- Small shocks can lead to big fluctuations if $\rho \uparrow$

The RBC model – firms

This should look very familiar...

Production function

$$f(k,h) = zk^{\alpha}h^{1-\alpha}$$

- Same as before, but not in per-capita terms
- z is the level of technology and moves around over time

Perfectly competitive, profit maximizing firms

$$\max_{\{k_t,h_t\}_0^{\infty}} \Pi = z_t k_t^{\alpha} h_t^{1-\alpha} - r_t k_t - w_t h_t$$

- Firms take rental rate and wages as given

Resource constraint of the economy

$$k_{t+1} = (1-\delta)k_t + f(k_t,h_t) - c_t$$

The RBC model – households

Period utility function

$$u(c, l) = \frac{c^{1-\sigma} - 1}{1 - \sigma} - B\frac{h^{1+\phi}}{1 + \phi}$$

- σ is the parameter for risk aversion and the inverse of the intertemporal elasticity of substitution
- ϕ is the inverse of the Frisch elasticity
- Other functional forms can be used (no need for balanced growth)

Budget constraint

$$a_{t+1} + c_t = h_t w_t + (1 + r_t)a_t$$

- w_t is the "wage rate": work more → earn more
- Agent saves in assets a

Household problem

Optimization

$$\max_{\{c_t, h_t, a_{t+1}\}_0^\infty} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, h_t)$$

$$\text{subject to} \quad a_{t+1} + c_t = h_t w_t + (1 + r_t) a_t$$

- Optimize in expectation, the model is not deterministic anymore
- Agents take expectations over z_t, knowing the AR(1) process

Lagrangian

$$\mathcal{L} = \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t u(c_t, h_t) + \sum_{t=0}^{\infty} \lambda_t \left[h_t w_t + (1 + r_t) a_t - a_{t+1} - c_t \right]$$

- Take derivatives w.r.t. $c_t, h_t, a_{t+1}, \lambda_t$
- Agent knows everything up to period t, but not the future

Household optimality

First order conditions

$$\frac{\partial \mathcal{L}}{\partial c_t}: \qquad c_t^{-\sigma} = \lambda_t$$

$$\frac{\partial \mathcal{L}}{\partial a_{t+1}}: \qquad \beta \mathbb{E}_t\left[(1+r_{t+1})c_{t+1}^{-\sigma}\right] = \mathbb{E}_t\left[\lambda_{t+1}\right]$$

$$\frac{\partial \mathcal{L}}{\partial h_t}: \qquad B h_t^{\varphi} = \lambda_t w_t$$

$$\frac{\partial \mathcal{L}}{\partial \lambda}: \qquad h_t w_t + (1+r_t)a_t = a_{t+1} + c_t$$

Optimality conditions

$$c_t^{-\sigma} = \beta \mathbb{E}_t\left[(1+r_{t+1})c_{t+1}^{-\sigma}\right] \qquad \text{(Intertemporal)}$$

$$B\frac{h_t^{\varphi}}{c^{-\sigma}} = w_t \qquad \text{(Intratemporal)}$$

Equation system describing the RBC model

Equation system

$$c_t^{-\sigma} = \beta \mathbb{E}_t \left[(1 + r_{t+1} - \delta) c_{t+1}^{-\sigma} \right] \qquad \text{(Euler equation)}$$

$$B \frac{h_t^{\phi}}{c_t^{-\sigma}} = w_t \qquad \text{(Labor supply choice)}$$

$$k_{t+1} = (1 - \delta) k_t + z_t k_t^{\alpha} n_t^{1-\alpha} - c_t \qquad \text{(Resource constraint)}$$

$$r_t = \alpha z_t \left(\frac{k_t}{n_t} \right)^{\alpha - 1} \qquad \text{(Interest rate)}$$

$$w_t = (1 - \alpha) z_t \left(\frac{k_t}{n_t} \right)^{\alpha} \qquad \text{(Wage rate)}$$

$$\ln(z_t) = \rho \ln(z_{t-1}) + \varepsilon_t; \qquad \varepsilon_t \sim N(0, \sigma_\varepsilon) \qquad \text{(Shock process)}$$

- Looks just like a Ramsey model, except for $\mathbb{E}_t$ in the Euler equation
- Suppressing No-Ponzi & transversality conditions, k_0 is given

Calibration

Need to pick some values for the model's parameters

- Previously: estimate the model and compare it to the data
- Now: use micro-estimates for structural parameters

Kydland & Prescott: "Reasonable" parameter values

- β: Patience – match interest rate
- $1-\alpha$: Labor income share – empirical labor share
- δ: Depreciation – match empirical depreciation
- B: disutility of labor – match hours worked
- ϕ: Labor supply elasticity – match empirical elasticity (problem)
- σ: Risk aversion – micro-estimates

Solving the RBC model

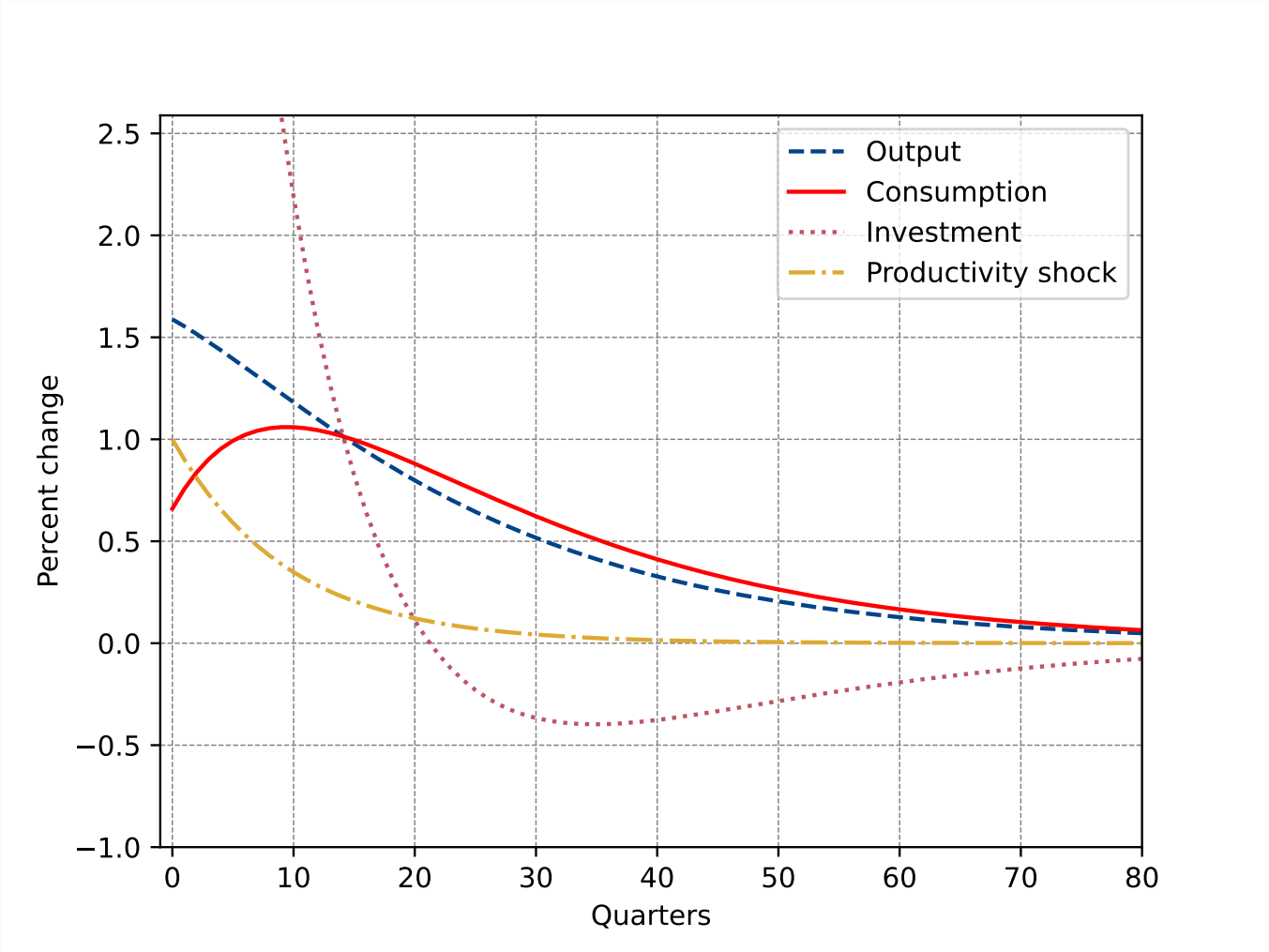
Global solution

- It's possible (but complicated) to solve this model on the computer
- Decision rules for c_t, k_{t+1}, h_t as function of k_t and z_{t-1}

Approximate solution

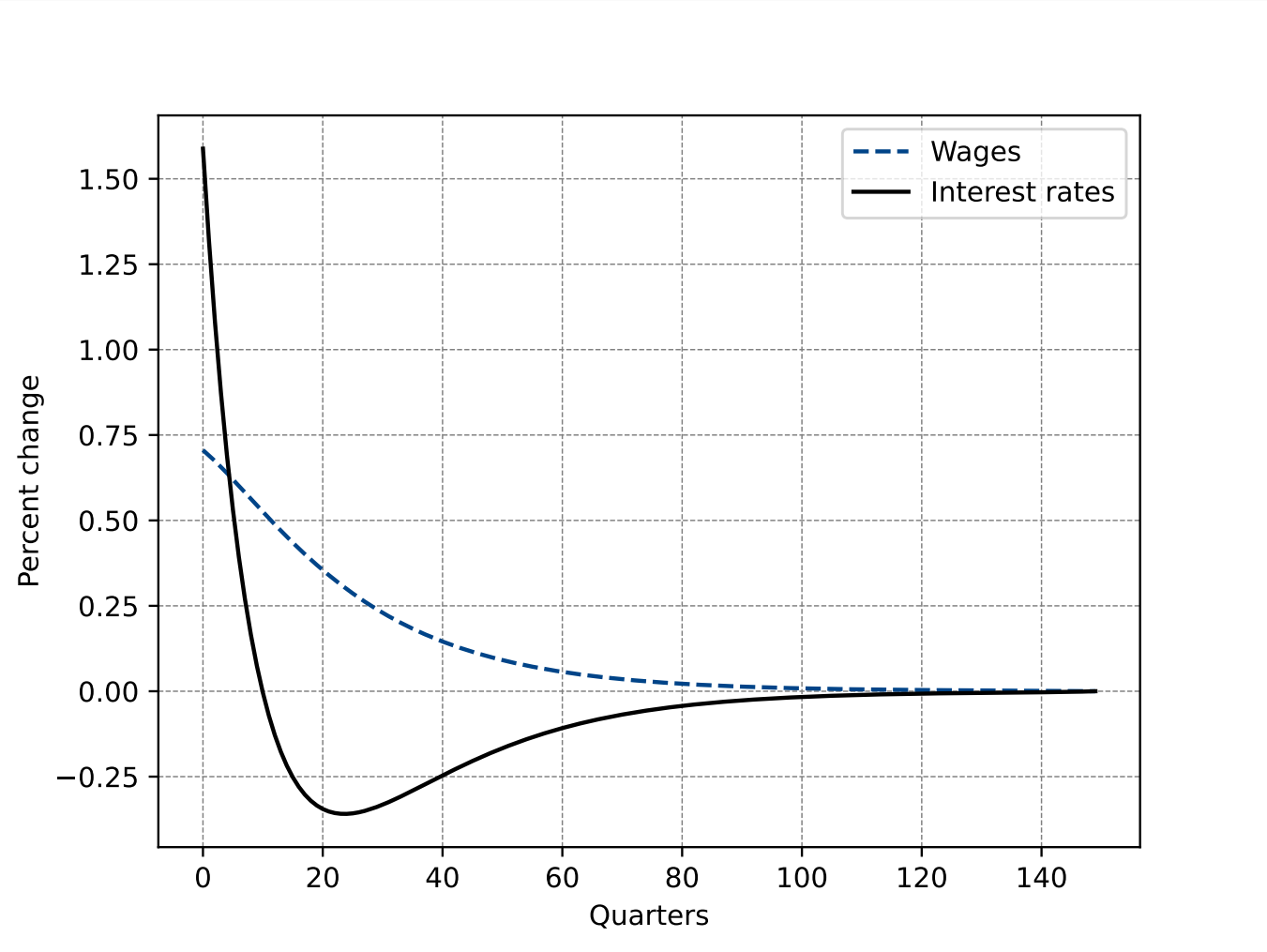
- Argument: technology shocks are small, we're always close to $z_t = 1$
- The uncertainty is so small that we don't have to care about it
- After shock, if we assume there are no further shocks, we're **almost** correct
- Just use MIT shocks: steady state, single shock to z_0, solve impulse response
- Still need a computer, but much easier

Impulse response



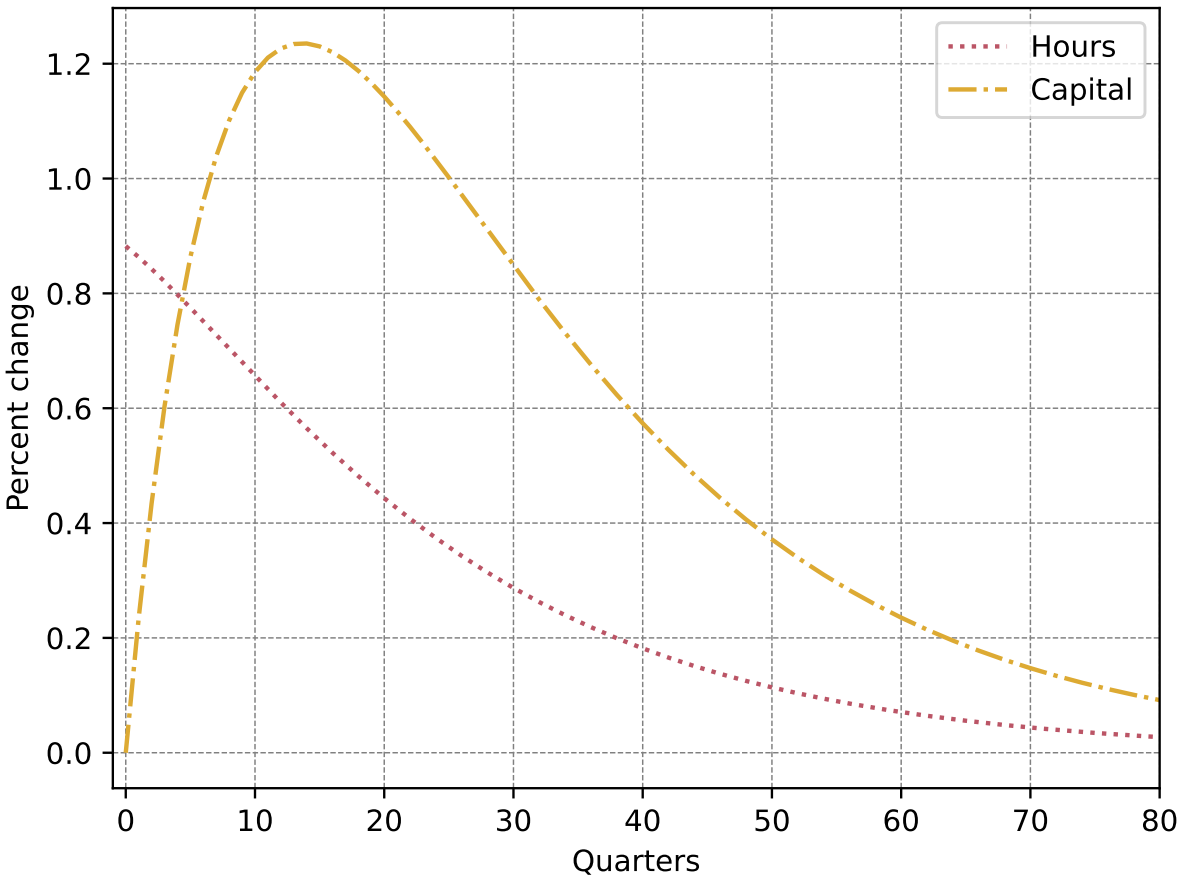
- Output, consumption and investment rise after shock
- The model leads to amplification ($\Delta c > \Delta z$) and persistence

Impulse response



- Mechanism: higher productivity increases wages → work more
- Higher r leads to more savings, capital stock k rises

Impulse response



- Hours move, but not nearly as much as output (miss!)
- Capital moves slowly

Mechanism

Positive productivity shock

- Interest rate (r_t) and wages (w_t) rise
- Higher MPK leads to higher capital accumulation (saving)

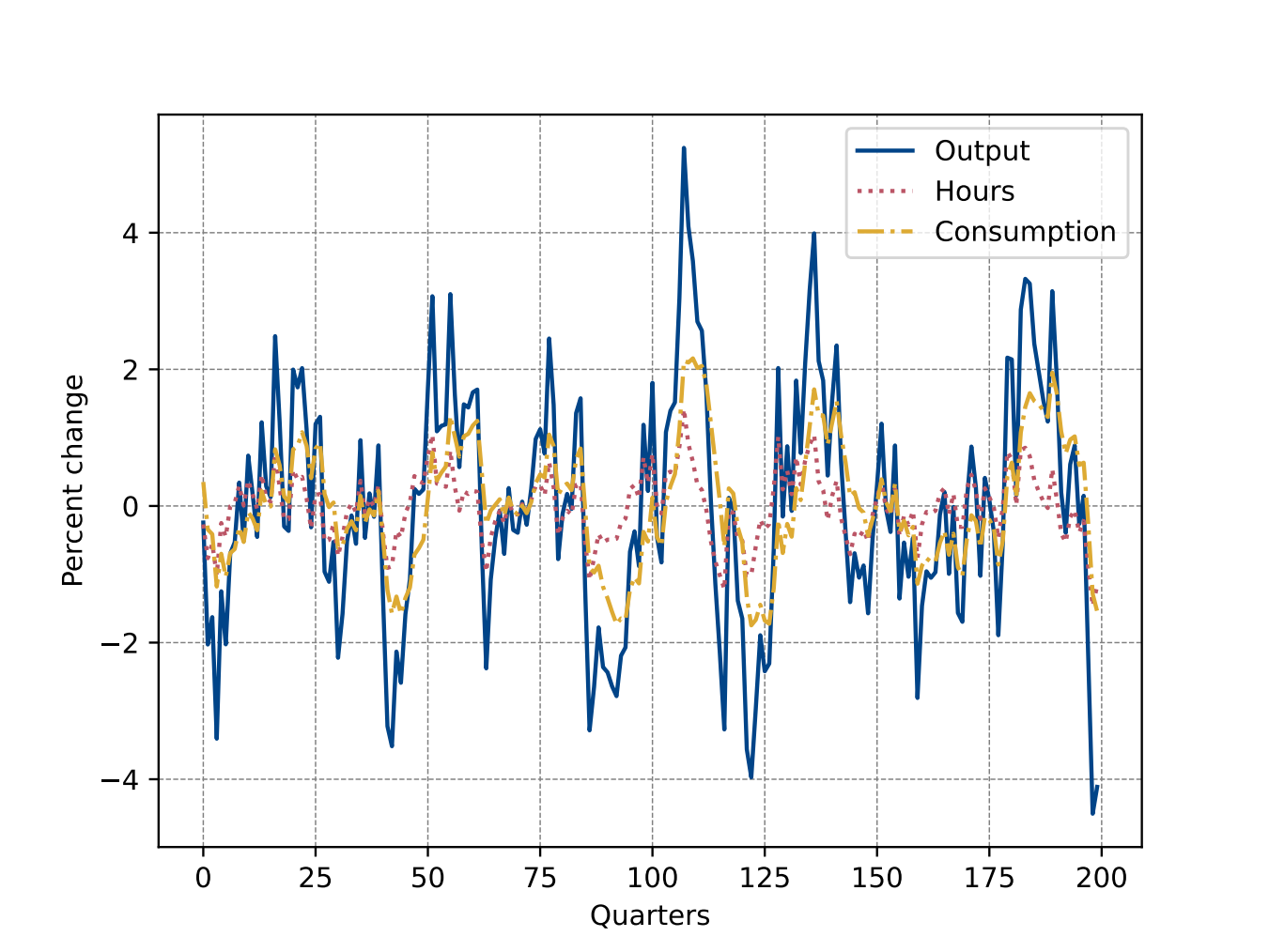
→ Investment rises

- Consumption jumps on impact and remains high ($r \uparrow \rightarrow c_{t+1}/c_t \uparrow$)
- Hours rise as agents work more (w)
- Output rises due to higher k, w and z

Medium term

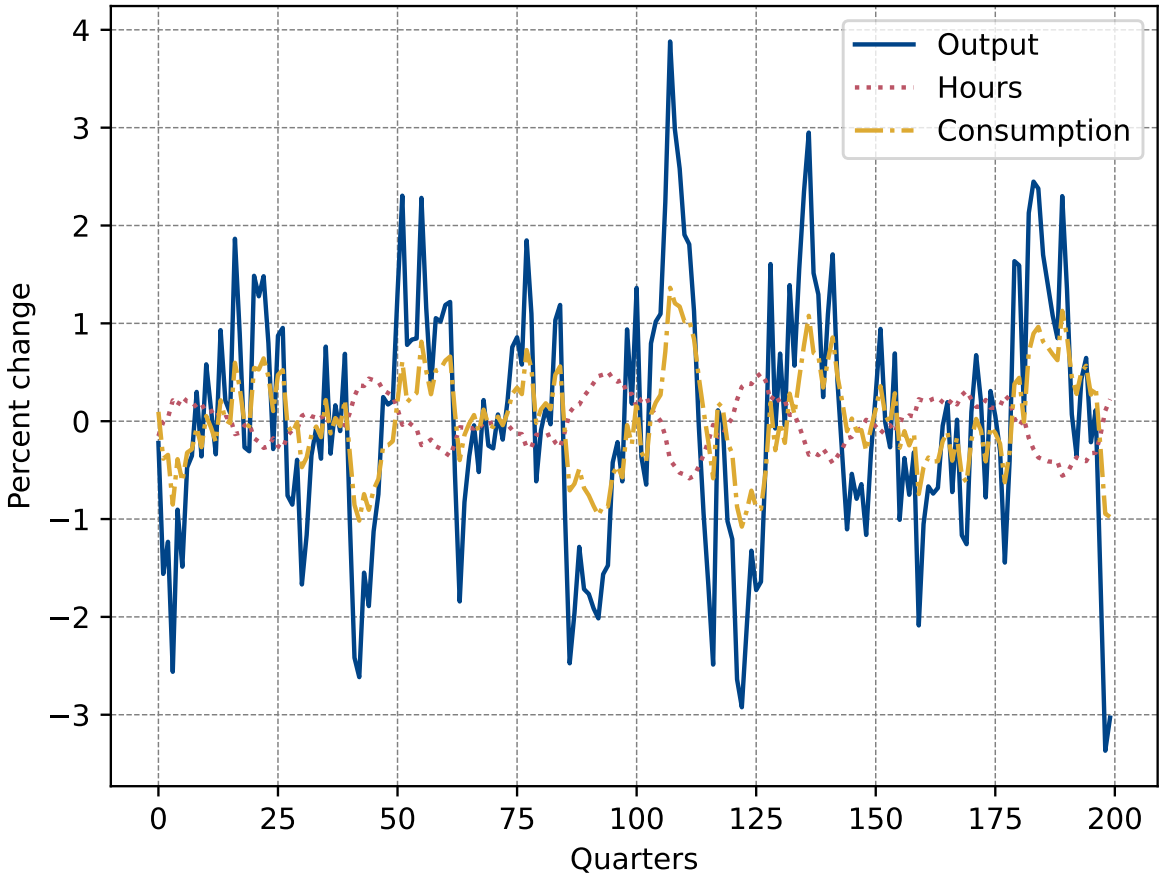
- Consumption remains elevated long after the shock has died out
- Agents use up savings
- Output remains high due to accumulated capital

Simulation



- The model does produce something that looks like a business cycle
- However, consumption moves too much and hours too little

Simulation



- If $\sigma \uparrow$, consumption stabilizes
- If $\phi \uparrow$, hours less sensitive to wages, h countercyclical?!

Why is this Nobel material?

Successes of the model

- Cyclicality looks right
- Only need productivity shocks, all other variables move **endogenously**
- Firm and household decisions are microfounded

Disciplining the model – calibration

- Targeting the micro, not the macro
- Getting answers right, for the right reasons

The model vs the data

Data

	$\sigma(x_t)$	$\sigma(x_t)/\sigma(Y_t)$	$\rho(x_t, x_{t-1})$	$\rho(x_t, Y_t)$
Y	1.64	1.00	0.78	1.00
C	1.37	0.84	0.71	0.79
I	7.15	4.37	0.78	0.82
H	2.11	1.29	0.81	0.86

Model

	$\sigma(x_t)$	$\sigma(x_t)/\sigma(Y_t)$	$\rho(x_t, x_{t-1})$	$\rho(x_t, Y_t)$
Y	1.61	1.00	0.71	1.00
C	0.76	0.47	0.86	0.86
I	9.83	6.09	0.67	0.95
H	0.47	0.29	0.67	0.93

- Treat model like the data: HP-filter output and compare
- Autocorrelations matched quite well
- C and H are not volatile enough, I too much

Quick fixes to RBC's problems

Labor is not volatile enough GHH preferences

- With current preferences, income effect is too strong

→ People don't want to work more when $w \uparrow$

Fix: Different preferences, see problem set

Investment is too volatile

- As $r \uparrow$, savings demand jumps and k_t rises quickly

Fix: Capital adjustment costs → dampens volatility

Policy implications

Welfare

- Fundamentally, the RBC model is a Ramsey model

  $\Longrightarrow$ the first welfare theorem holds

  $\Longrightarrow$ choices and outcomes are efficient, welfare cannot be increased

Recessions

- The economy is (exogenously) less productive
- MPK and MPL are low $\rightarrow$ should work and invest less
- Government intervention can only decrease welfare

Caveat

- Very strong implication that can easily be overturned

Criticisms

What is a negative productivity shock?

- We don't forget how to produce things over night
- The shocks drive most of the model, isn't that cheating?

The model cannot reproduce the volatility of hours

- Need implausible values for labor supply elasticity ϕ
- No unemployment in the model (Great Recession = Great Vacation)

Miscellaneous

- Representative agent, rational expectations, full information
- Perfect credit markets

Main innovations since the 90s

Heterogeneous agents (firms and households) History

- No more representative agent
- Many agents with different incomes/exposures to the cycle
- Inequality

→ Business cycles much more costly than Lucas suggested

Frictions and other additions

- Search-and-matching labor markets (Merz, 1995)
- Borrowing constraints (Aiyagari, 1994)
- Asset markets (Mehra & Prescott, 2003)
- Pricing frictions (Smets & Wouters, 2003) (→ New Keynesian model)

Appendix

Danish business cycles

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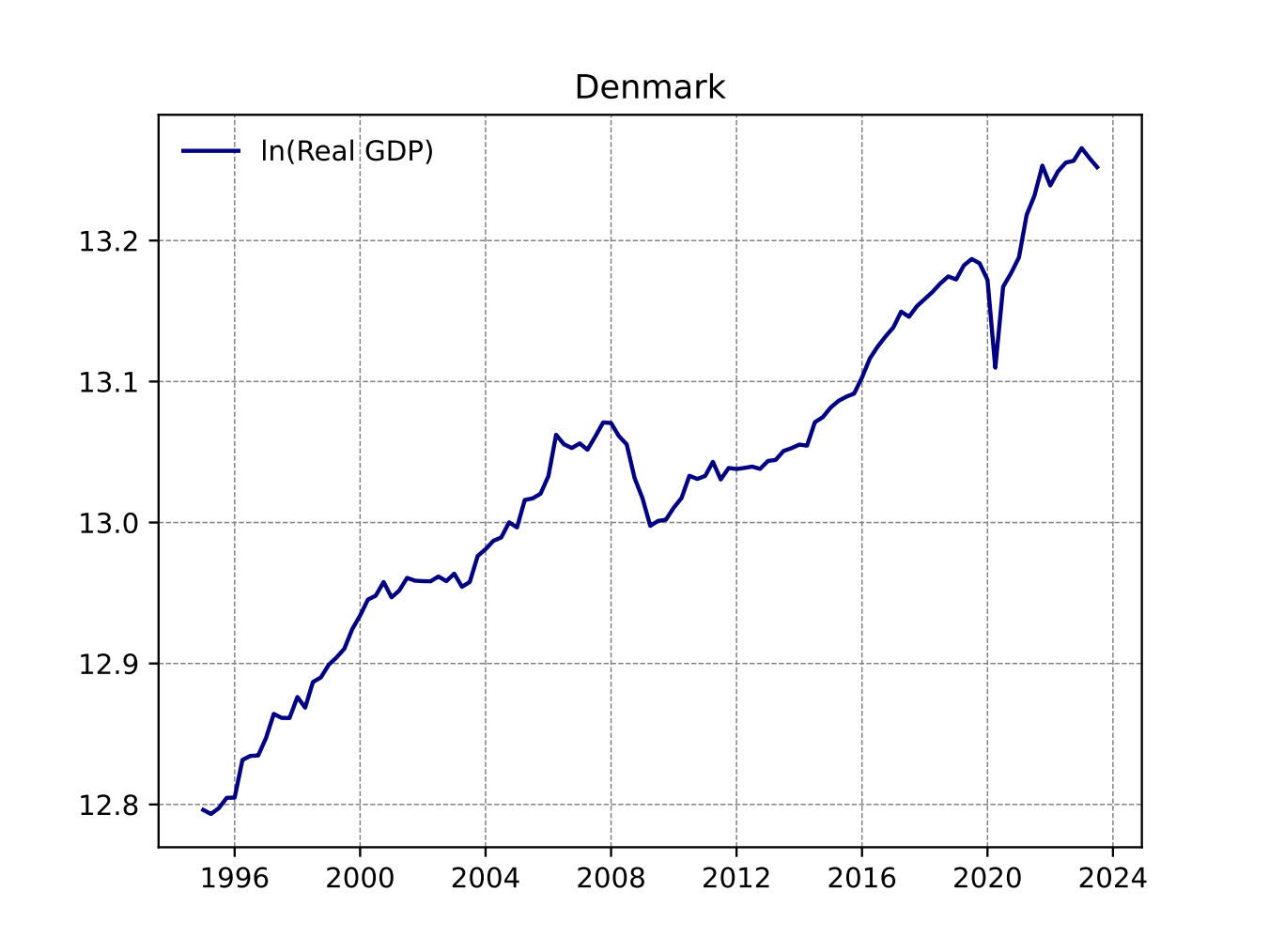

- GDP increases, but not smoothly → distinguish SR & LR

Danish business cycles

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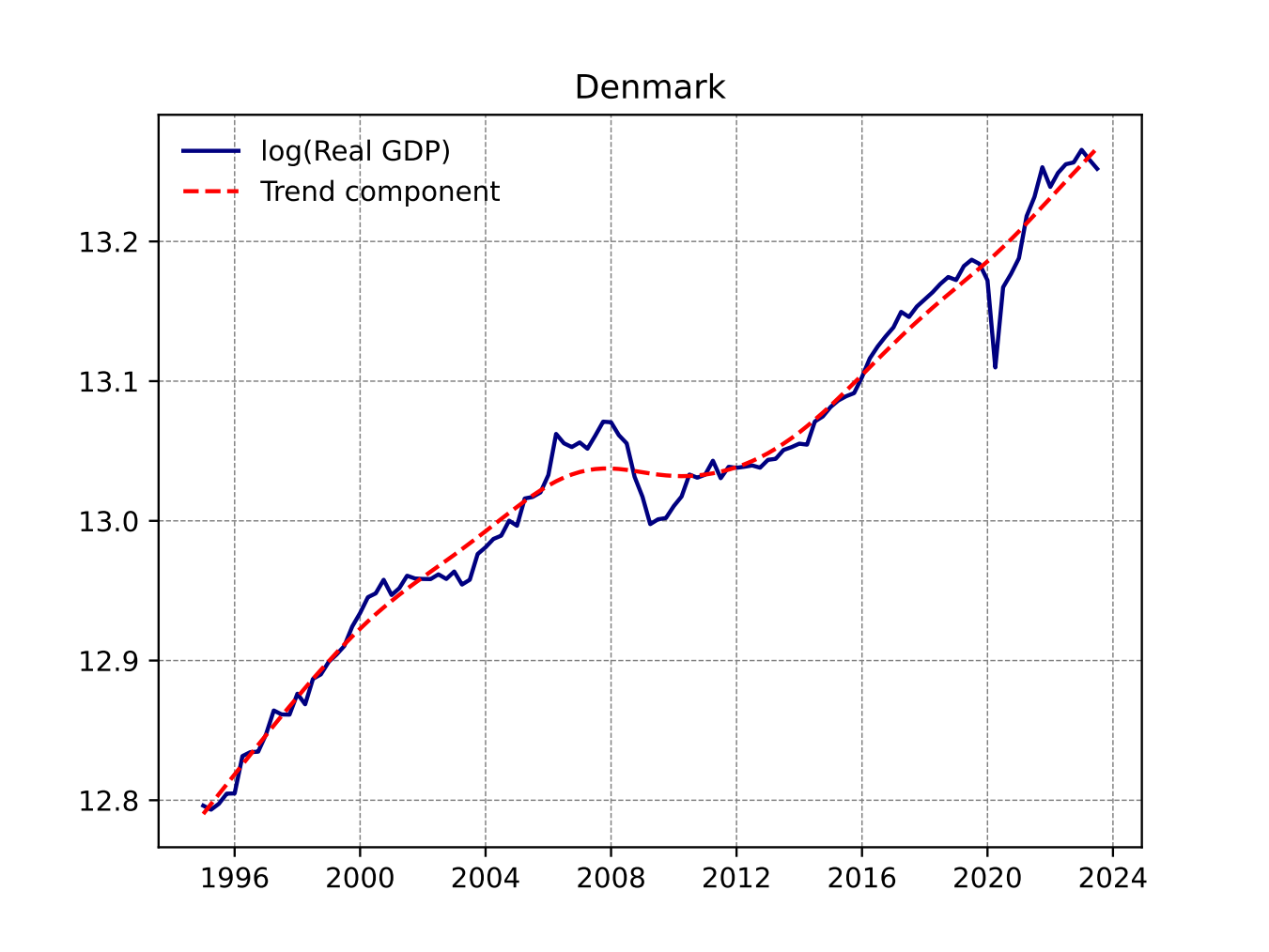

- Many periods of deviation from trend

Danish business cycles

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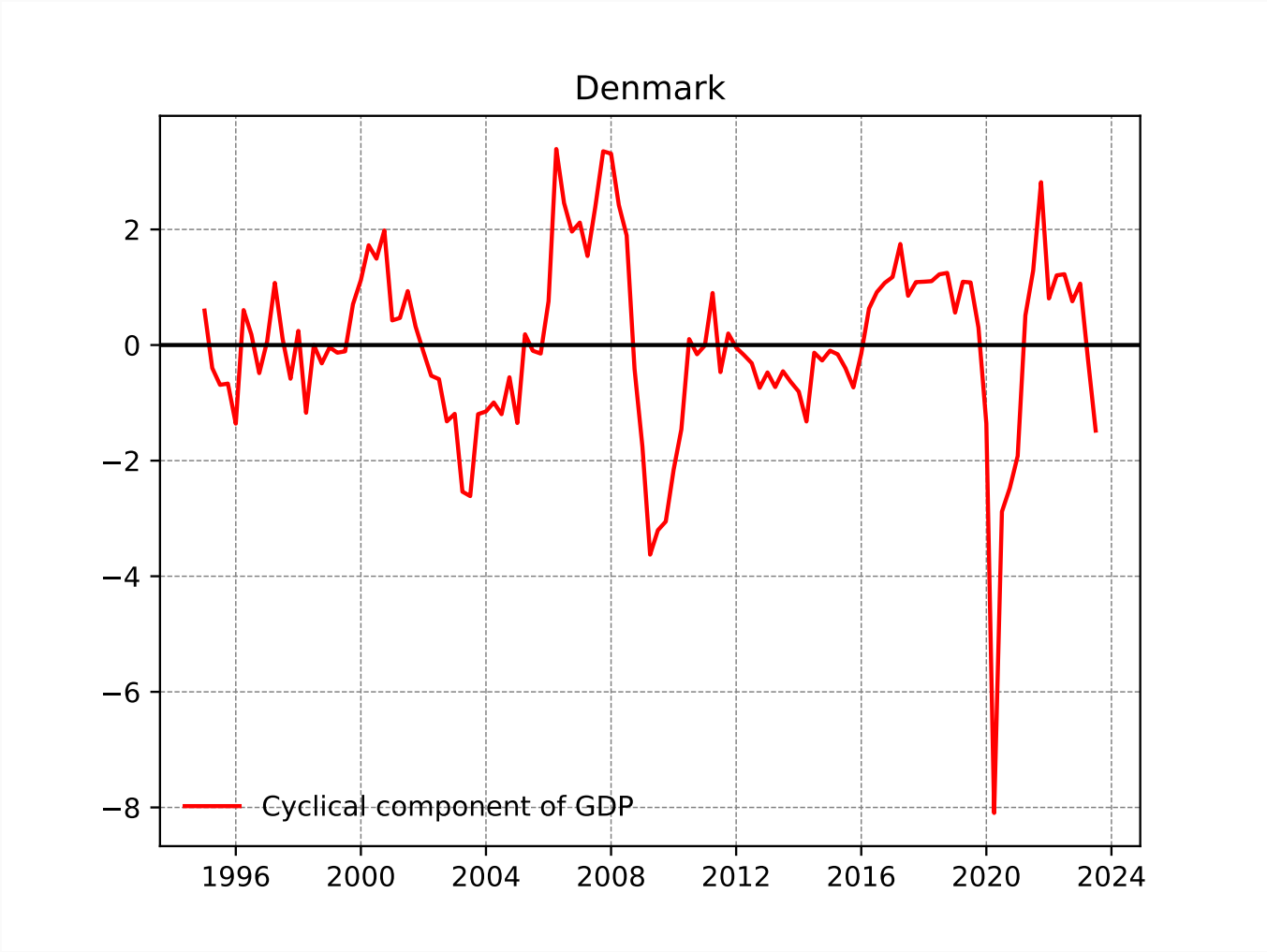

- Business cycles ($CLVMNACSCAB1GQDK$)

German business cycles

Back

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German business cycles

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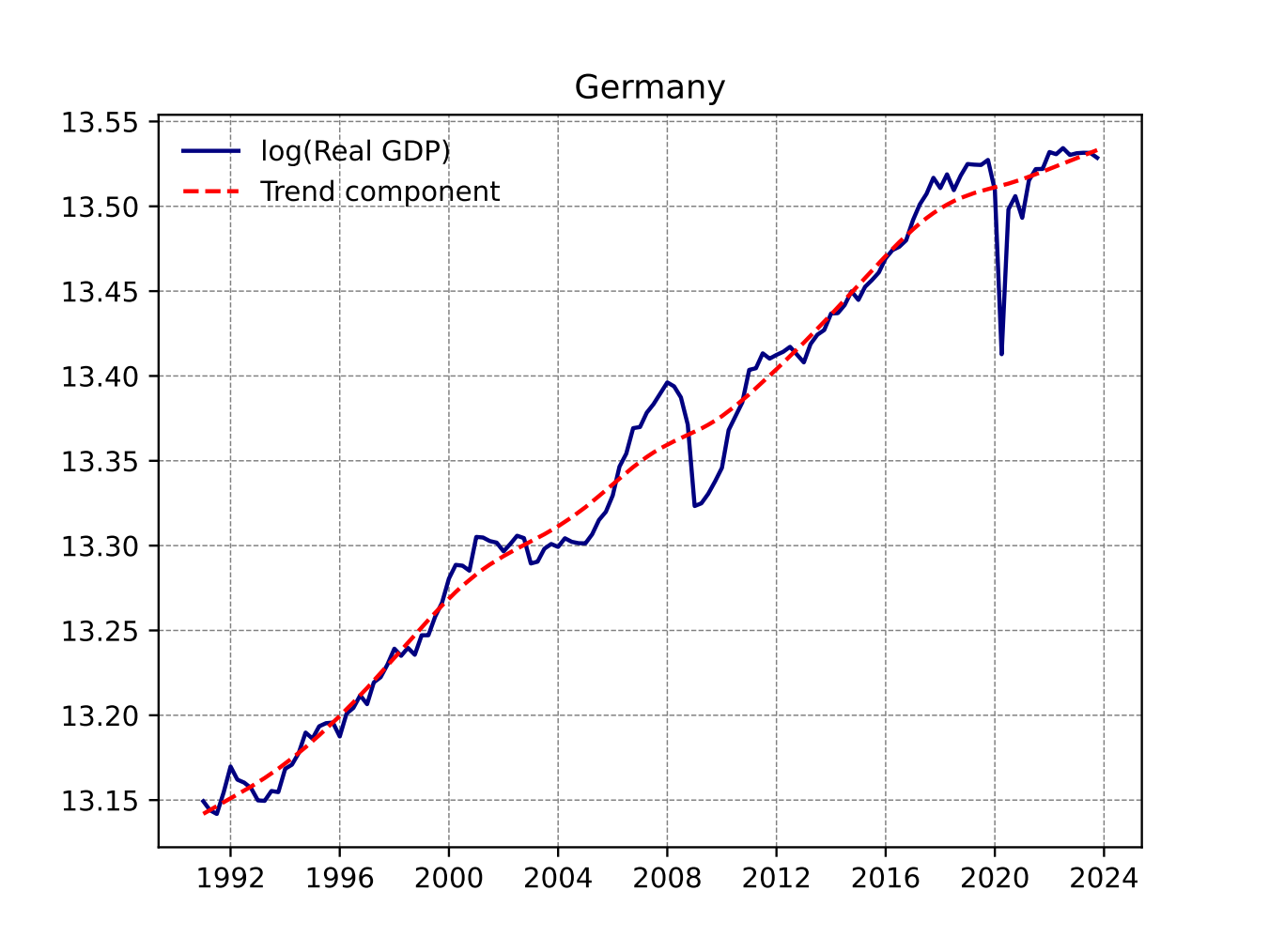

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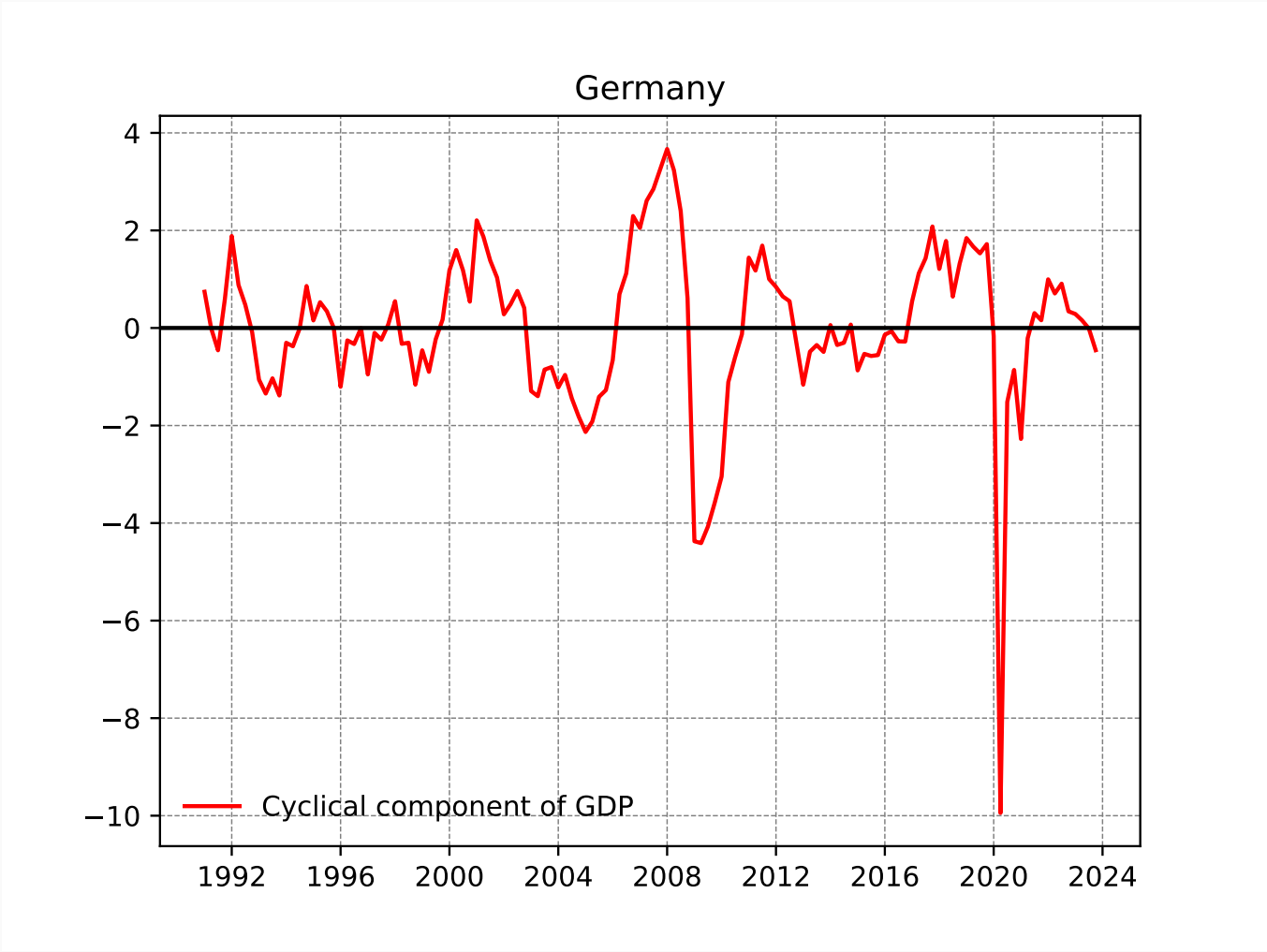

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Portuguese business cycles

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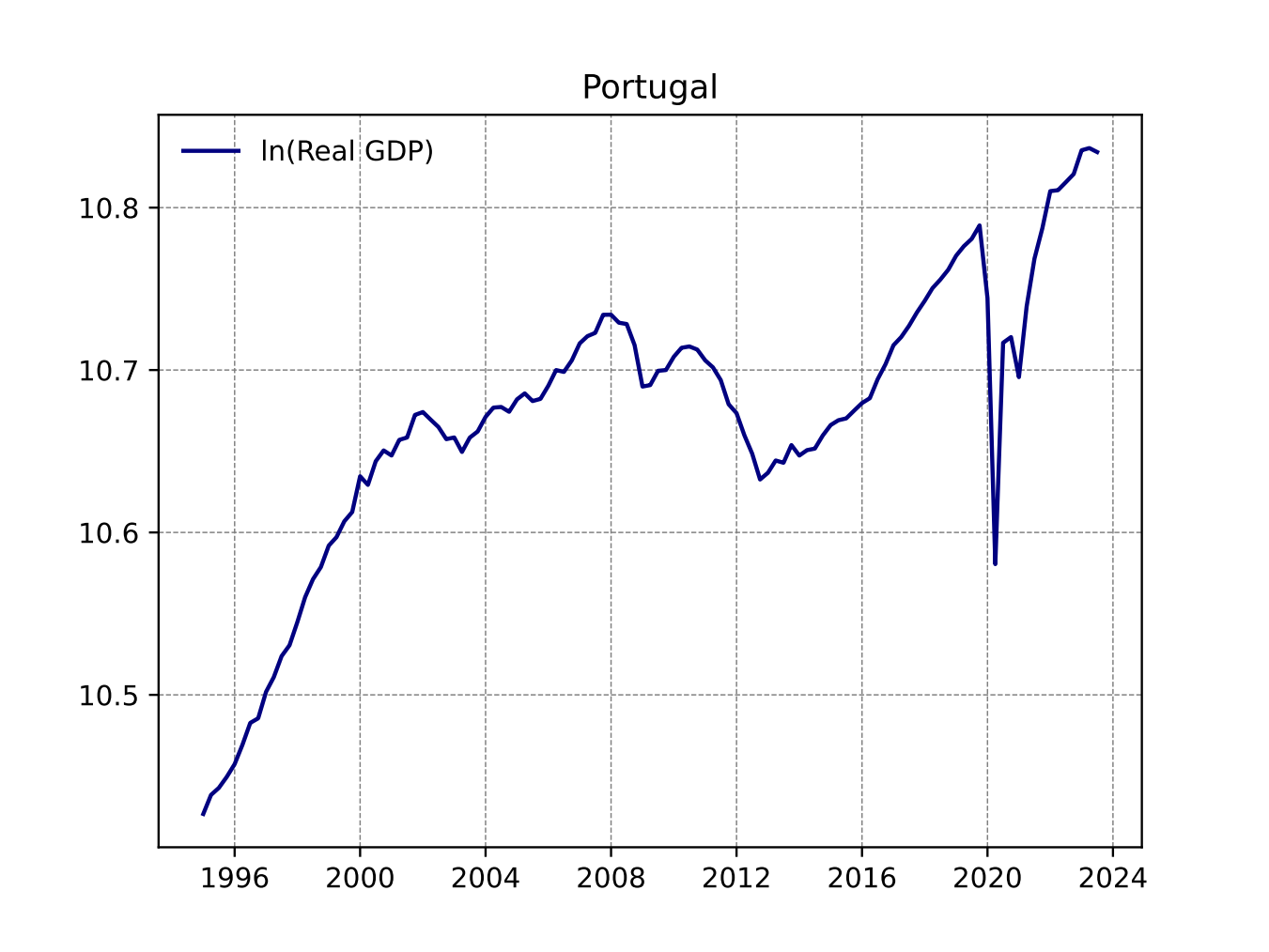

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Portuguese business cycles

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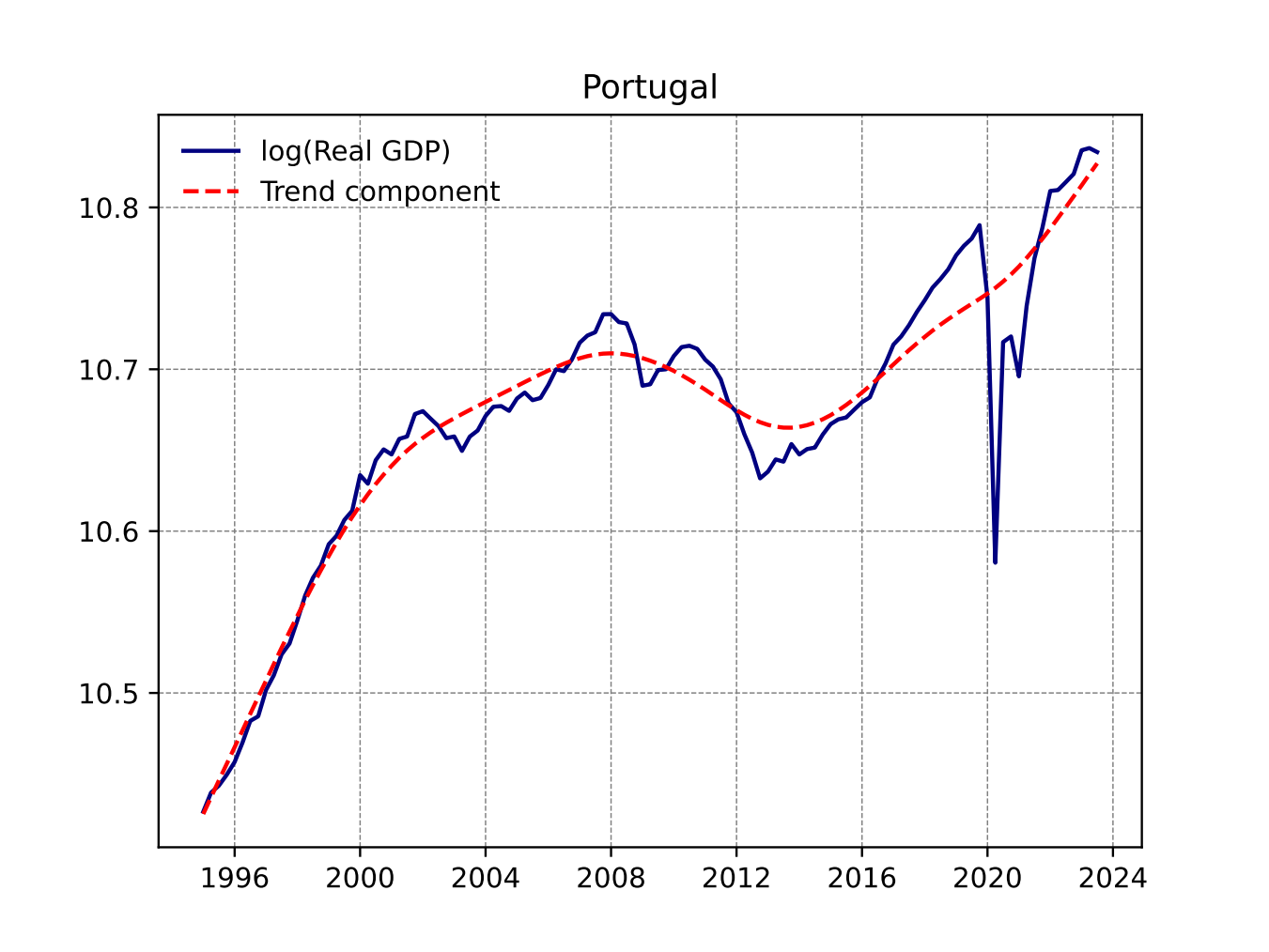

- Many periods of deviation from trend

Portuguese business cycles

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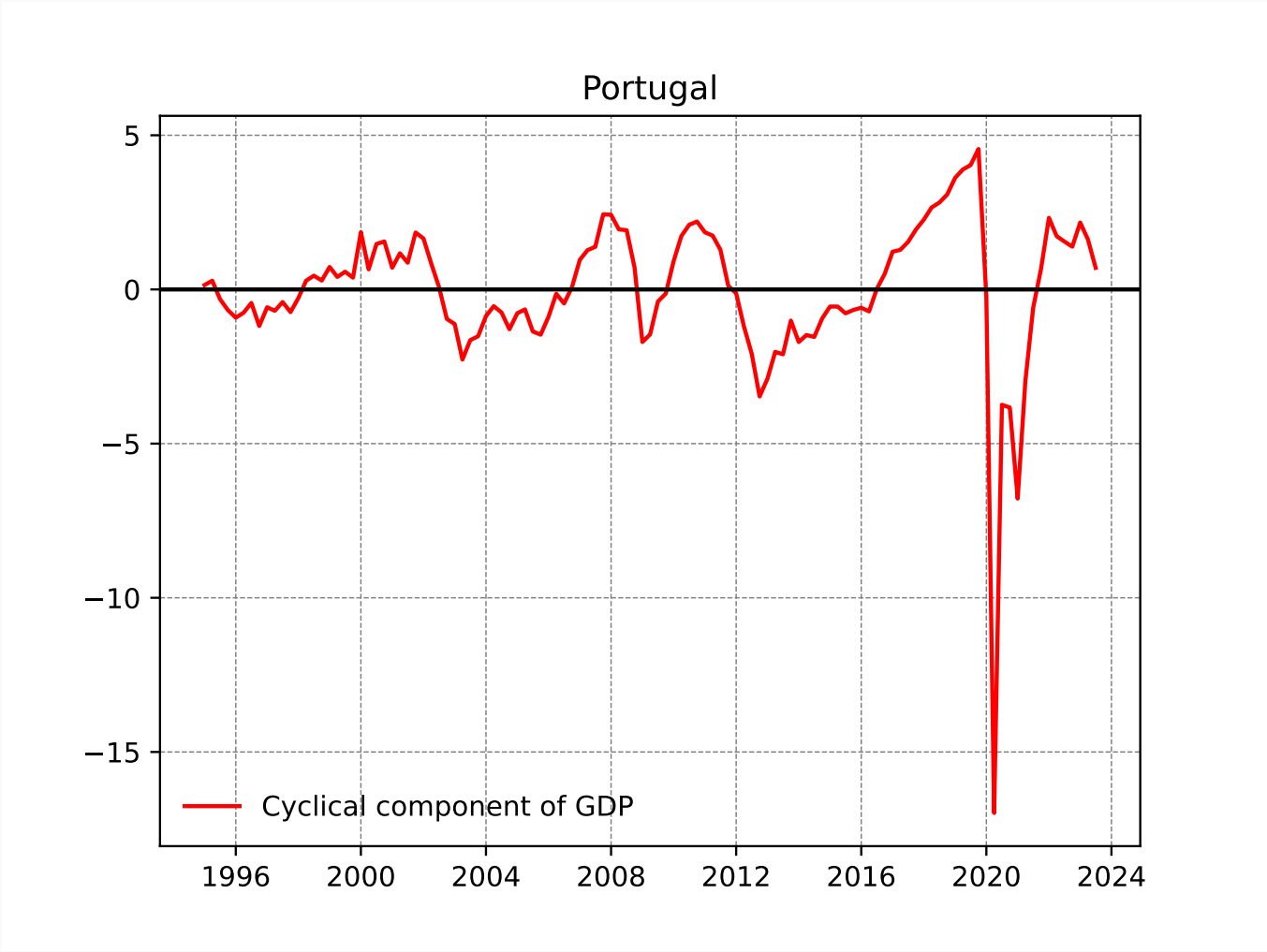
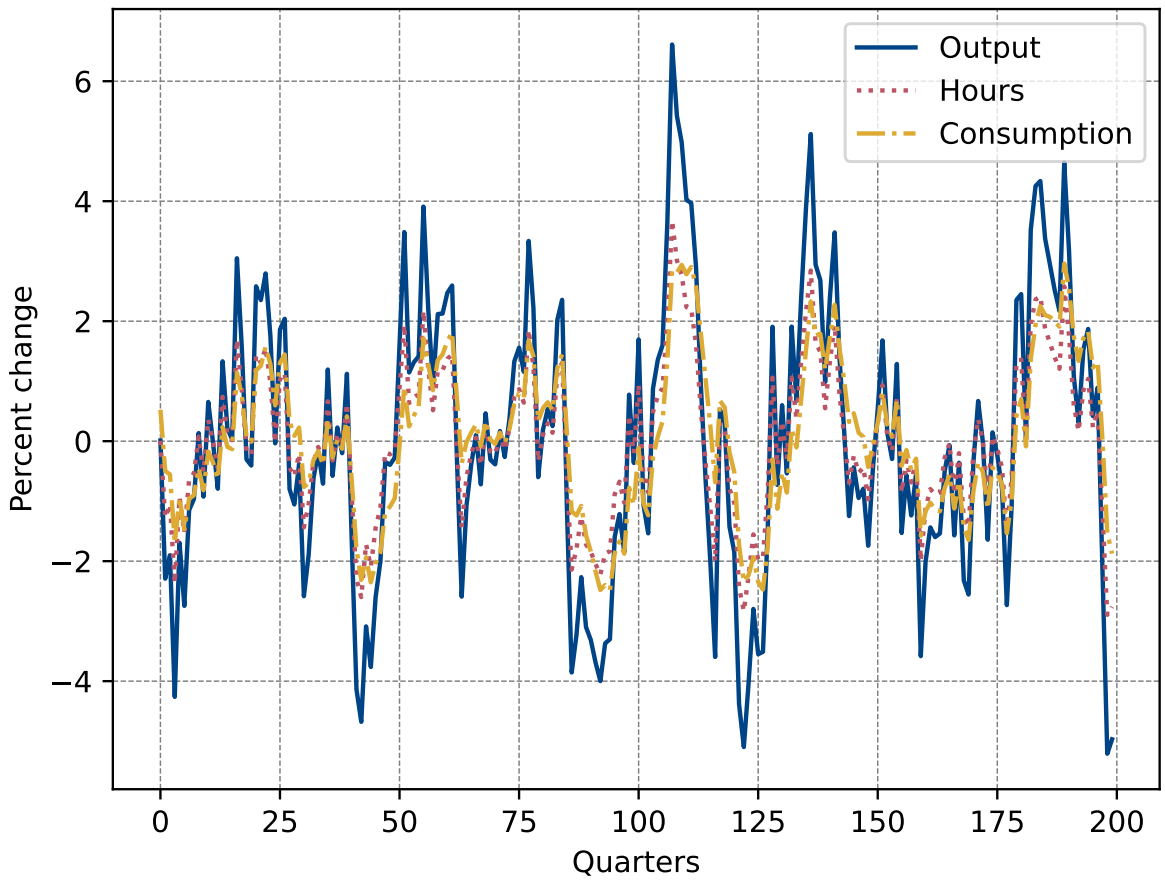


- Business cycles ($CLVMNACSCAB1GQPT$)

Simulation – GHH preferences



- The model does produce something that looks like a business cycle
- However, consumption moves too much and hours too little

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The model vs the data – GHH preferences

Data

	$\sigma(x_t)$	$\sigma(x_t)/\sigma(Y_t)$	$\rho(x_t, x_{t-1})$	$\rho(x_t, Y_t)$
Y	1.64	1.00	0.78	1.00
C	1.37	0.84	0.71	0.79
I	7.15	4.37	0.78	0.82
H	2.11	1.29	0.81	0.86

Model

	$\sigma(x_t)$	$\sigma(x_t)/\sigma(Y_t)$	$\rho(x_t, x_{t-1})$	$\rho(x_t, Y_t)$
Y	2.08	1.00	0.73	1.00
C	1.09	0.52	0.84	0.91
I	11.48	5.50	0.69	0.95
H	1.16	0.56	0.73	1.00

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