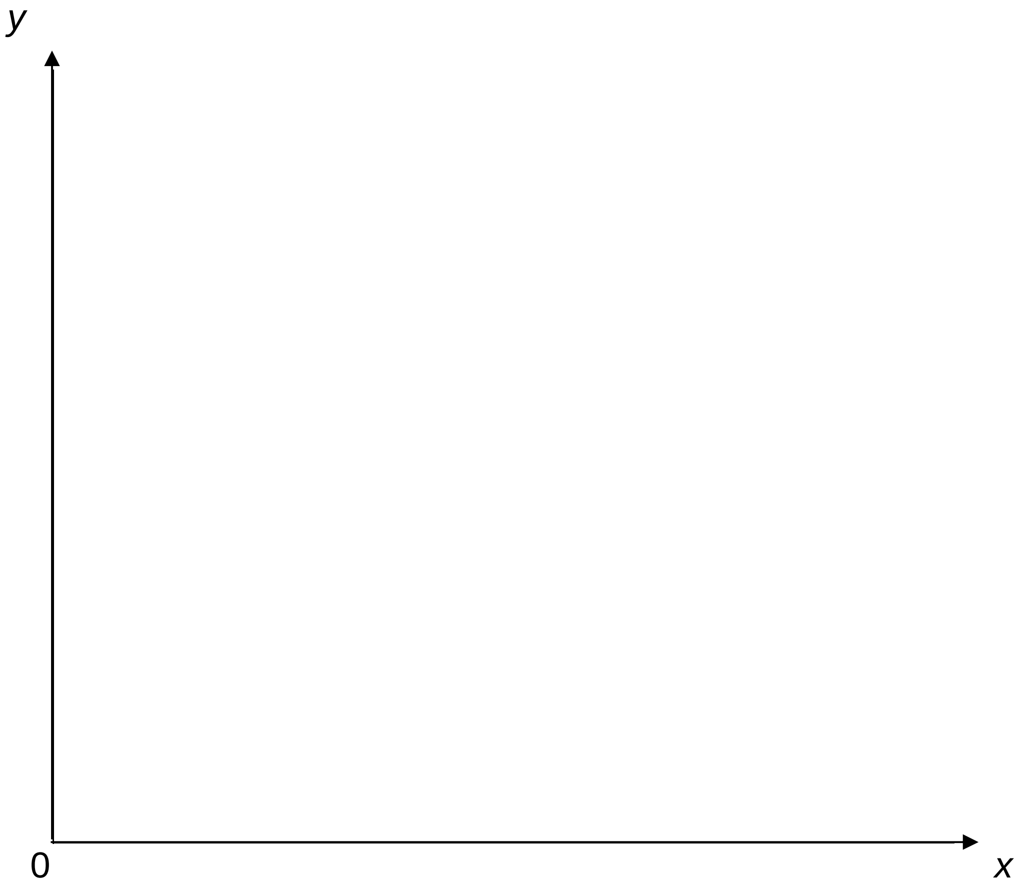
Mathematical Excursus

What you know:

Function with one independent variable (x) and one dependent variable (y):

function: first derivative:

$y = f(x) = x^2$ => $y' = f'(x) = 2x$



How does a change of the independent variable (x) change the dependent variable (y)?

We use the secant for an approximation of the slope:

Sekante jeweils zwischen zwei Punkten
2 Schnittpunkte

how much the value increases if you increase x by 1

If $x = 1$: $\frac{\Delta y}{\Delta x} = \frac{3}{1} = 3$ or $\Delta y = 3 \cdot \Delta x$ difference quotient (secant)

If $x = 2$: $\frac{\Delta y}{\Delta x} = \frac{5}{1} = 5$ or $\Delta y = 5 \cdot \Delta x$ (Differenzenquotient)

For a change within one point, we use the first derivative of the function to find the equation of the tangent and to calculate the slope:

d = difference of x or difference of y

$$\frac{dy}{dx} = f'(x) = 2x \quad \text{or} \quad dy = f'(x) \cdot dx = 2x \cdot dx$$

If $x = 1$: $\frac{dy}{dx} = 2$ or $dy = 2 \cdot dx$ differential quotient (tangent)

If $x = 2$: $\frac{dy}{dx} = 4$ or $dy = 4 \cdot dx$ (Differentialquotient)

What is new?

man hat quasi zwei wie x und eins wie y

Function with two independent variables (x_A *and* x_B) and one dependent variable (y):

function: $y = f(x_A; x_B)$

The change of dependent variable (y) depends on the changes of two variables!

utility depends on apples and bananas

y = f(Xa; Xb) = $X_a^5 - X_b^3$

Because we have two independent variables, we have to find the

TOTAL DIFFERENTIAL.

This is done in the following steps: f(xa;xb) = xa^5 - xb^3

1) How does the value of the function change when you change one of the variables? You have to find the PARTIAL DERIVATIVES: Teilableitung

a) What happens if you change x_A? step by step

Find the Partial Derivative with respect to x_A
by holding all other variables (here: x_B) constant.

$$\frac{dy}{dx_A} = \frac{\partial f(x_A;x_B)}{\partial x_A} \qquad \text{or} \qquad dy = \frac{\partial f(x_A;x_B)}{\partial x_A} \cdot dx_A$$

other variable is treated like a number (e.g. 1)

b) What happens if you change x_B?

Find the Partial Derivative with respect to x_B
by holding all other variables (here: x_A) constant.

$$\frac{dy}{dx_B} = \frac{\partial f(x_A;x_B)}{\partial x_B} \qquad \text{or} \qquad dy = \frac{\partial f(x_A;x_B)}{\partial x_B} \cdot dx_B$$

2) How does the value of the function change when all independent variables are varied simultaneously?

You have to sum up the Partial Derivatives to get the total change.

=> Total Differential:

$$\boxed{dy = df(x_A; x_B) = \underbrace{\frac{\partial f(x_A;x_B)}{\partial x_A} \cdot dx_A}_{\text{change A}} + \underbrace{\frac{\partial f(x_A;x_B)}{\partial x_B} \cdot dx_B}_{\text{change B}}}$$

dxb =

dy = df(xa;xb) = 5x^4 * dxa . (- 3xb^2) * dxb

The same procedure is applied when we have more than two variables:

$$dy = df(x_1; x_2; x_3; \ldots; x_n) = \frac{\partial f}{\partial x_1} \cdot dx_1 + \frac{\partial f}{\partial x_2} \cdot dx_2 + \frac{\partial f}{\partial x_3} \cdot dx_3 + \cdots + \frac{\partial f}{\partial x_n} \cdot dx_n$$

Applying this mathematical concept to our utility function:

$$dU = dU(x_A; x_B) = \frac{\partial U(x_A; x_B)}{\partial x_A} \cdot dx_A + \frac{\partial U(x_A; x_B)}{\partial x_B} \cdot dx_B$$

total change in utility = marginal utility with respect to x_A * change of x_A + marginal utility with respect to x_B * change of x_B

change of cons. of apples

change of cons. of banans

how steep is the hill

how far do we go

total dervative = Summe der Teilableitungen