3.2 Velocity-based motion model

In the remainder of this chapter we will describe two probabilistic motion models for planar movement: the **velocity motion model** and the **odometry motion model**, the former being the main topic of this section. Remember that when a movement command is given to a robot, there are different factors that affect such movement (*e.g.* wheel slippage, unequal floor, inaccurate calibration, motors response, etc.), adding uncertainty to the actual move done. This results in a need for characterizing the robot motion in *probabilistic terms*, that is:

$$p(x_t|u_t, x_{t-1})$$

being:

- x_t the robot pose at time instant t,
- u_t the motion command (also called control action) at t, and
- x_{t-1} the robot pose at the previous time instant $t-1$.

So basically this probability models the probability distribution over robot poses when executing the motion command u_t, having the robot the previous pose x_{t-1}. In other words, we are considering a function $g(\cdot)$ that performs $x_t = g(x_{t-1}, u_t)$ and outputs $x_t \sim p(x_t|u_t, x_{t-1})$.

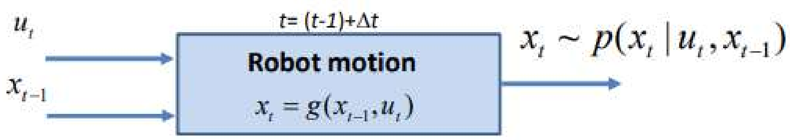


Fig. 1: Inputs and outputs of a probabilistic motion model.

Different definitions for the $g(\cdot)$ function lead to different probabilistic motion models, like the velocity motion model explored here.

3.2.1 The model

Usage: The ***velocity motion model*** is mainly used for motion planning, where the details of the robot's movement are of importance and odometry information is not available (*e.g.* no wheel encoders are available).

This motion model is characterized by the use of two velocities to control the robot's movement: **linear velocity** v and **angular velocity** w. Therefore, during the following sections, the movement commands will be of the form:

$$u_t = \begin{bmatrix} v_t \\ w_t \end{bmatrix}, \quad u_t \sim N(\overline{u}, \Sigma_{u_t})$$

The velocity motion model defines the function $g(\cdot)$ as:

$$g(x_{t-1}, u_t) = x_{t-1} \oplus \Delta x_t, \quad x_{t-1} \sim N(\overline{x}_{t-1}, \Sigma_{x_{t-1}})$$

being $\Delta_{x_t} = [\Delta_{x_t}, \Delta_{y_t}, \Delta_{\theta_t}]$ (assuming w and v constant):

- $\Delta x_t = \frac{v}{w}\sin(w\Delta t)$
- $\Delta y_t = \frac{v}{w}[1 - \cos(w\Delta t)]$
- $\Delta\theta_t = w\Delta t$

Note that $g(x_{t-1}, u_t) = x_{t-1} \oplus \Delta x_t$ **is not a linear operation!**

In this way, this motion model is characterized by the following equations, depending on the value of the angular velocity w (note that a division by zero would appear in the first case with $w = 0$):

- If $w \neq 0$:

$$\begin{bmatrix} x_t \\ y_t \\ \theta_t \end{bmatrix} = \begin{bmatrix} x_{t-1} \\ y_{t-1} \\ \theta_{t-1} \end{bmatrix} + \begin{bmatrix} -R\sin\theta_{t-1} + R\sin(\theta_{t-1} + \Delta\theta) \\ R\cos\theta_{t-1} - R\cos(\theta_{t-1} + \Delta\theta) \\ \Delta\theta \end{bmatrix}$$

- If $w = 0$:

$$\begin{bmatrix} x_t \\ y_t \\ \theta_t \end{bmatrix} = \begin{bmatrix} x_{t-1} \\ y_{t-1} \\ \theta_{t-1} \end{bmatrix} + v \cdot \Delta t \begin{bmatrix} \cos\theta_{t-1} \\ \sin\theta_{t-1} \\ 0 \end{bmatrix}$$

with:

- $v = w \cdot R$ *(R is also called the curvature radius)*
- $\Delta\theta = w \cdot \Delta t$

In [1]:
```
%matplotlib widget

# IMPORTS
import numpy as np
from numpy import random
import matplotlib.pyplot as plt
from IPython.display import display, clear_output
import time


import sys
sys.path.append("..")
from utils.DrawRobot import DrawRobot
from utils.PlotEllipse import PlotEllipse
```

ASSIGNMENT 1: The model in action

Modify the following `next_pose()` function, used in the `VelocityRobot` class below, which computes the next pose x_t of a robot given:

- its previous pose x_{t-1},

- the velocity movement command $u = [v, w]^T$, and
- a lapse of time Δt.

Concretely you have to complete the if-else statement that takes into account when the robot moves in an straight line so $w = 0$. *Note: you don't have to modify the* `None` *in the function header nor in the* `if cov is not None:` *condition.*

Remark that at this point **we are not taking into account uncertainty in the system**: neither from the initial pose ($\Sigma_{x_{t-1}}$) nor the movement (v, w) (Σ_{u_t}).

Example

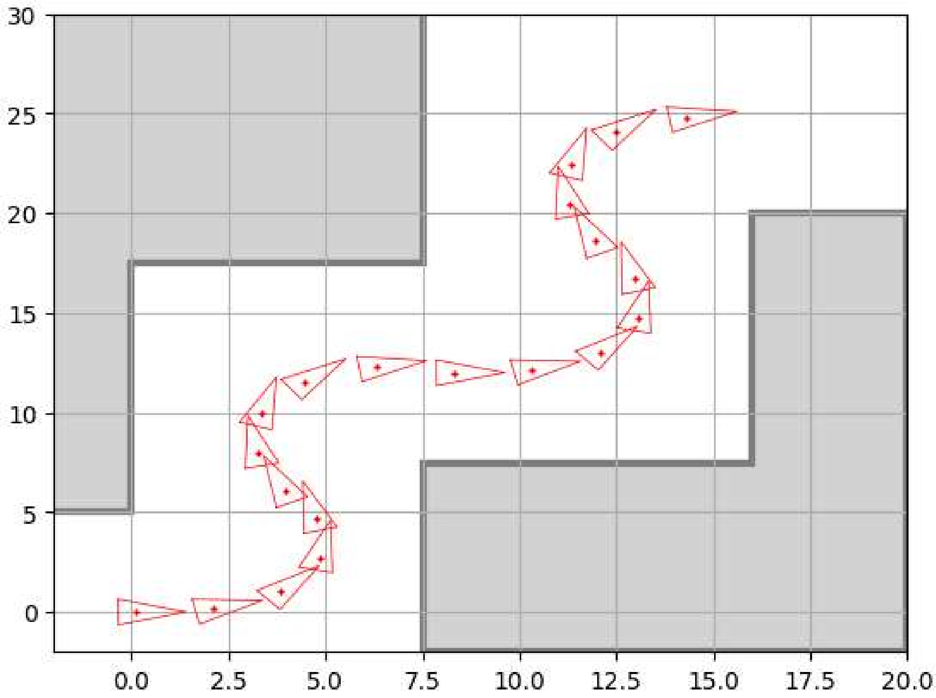


Fig. 2: Route of our robot.

In [2]:
```
def next_pose(x, u, dt, cov=None):
    ''' This function takes pose x and transform it according to the motion u=[v
        applying the differential drive model.

        Args:
            x: current pose
            u: differential command as a vector [v, w]'
            dt: Time interval in which the movement occurs
            cov: covariance of our movement. If not None, then add gaussian nois
    '''
    if cov is not None:
        u += np.sqrt(cov) @ random.randn(2, 1)
        #u = np.random.multivariate_normal(u.flatten(),cov)

    if u[1] == 0: #linear motion w=0
        next_x = np.vstack([x[0] + u[0] * dt * np.cos(x[2]),
                            x[1] + u[0] * dt *np.sin(x[2]),
                            x[2] + 0])
    else: #Non-linear motion w=!0
        R = u[0]/u[1] #v/w=r is the curvature radius
        next_x = np.vstack([x[0] - R * np.sin(x[2]) + R*np.sin(x[2] + dt*u[1]),
                            x[1] + R*np.cos(x[2]) - R*np.cos(x[2] + dt*u[1]),
                            x[2] + dt*u[1]])
```

```python
    return next_x
```

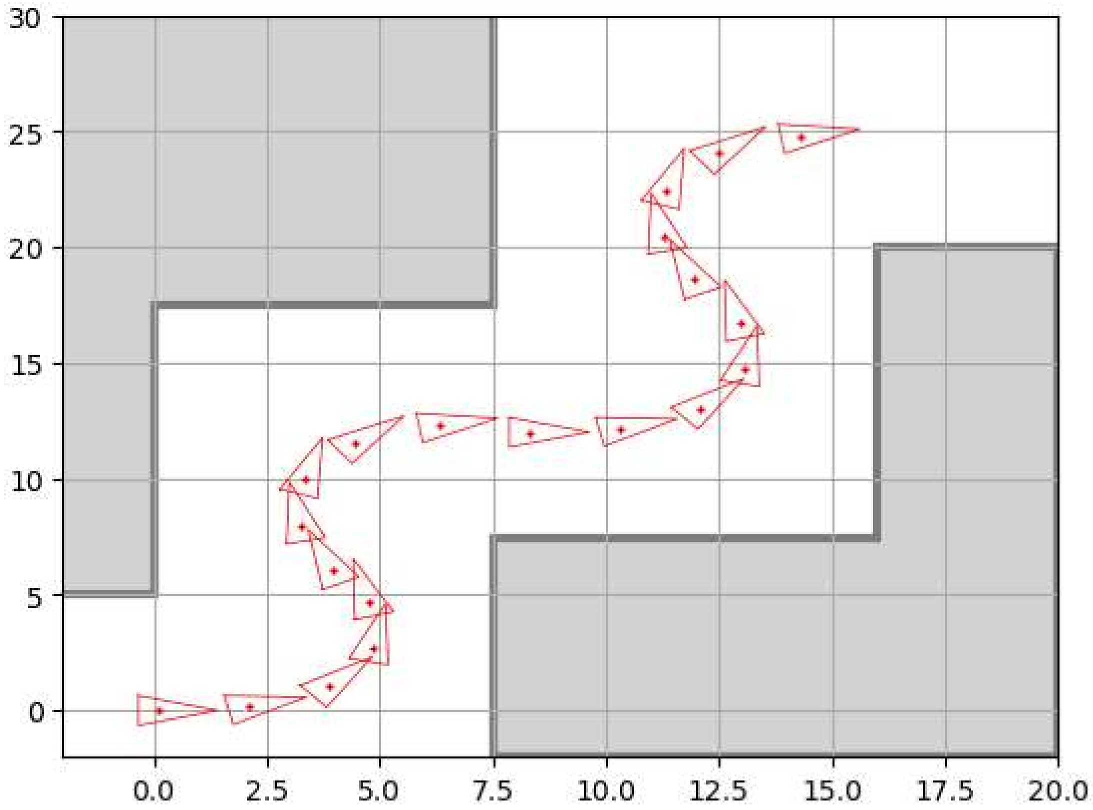
In [3]:
```python
class VelocityRobot(object):
    """ Mobile robot implementation that uses velocity commands.

        Attr:
            pose: expected pose of the robot in the real world (without taking a
            dt: Duration of each step in seconds
    """
    def __init__(self, mean, dt):
        self.pose = mean
        self.dt = dt

    def step(self, u):
        self.pose = next_pose(self.pose, u, self.dt)

    def draw(self, fig, ax):
        DrawRobot(fig, ax, self.pose)
```

Test the movement of your robot using the demo below.

In [4]:
```python
def main(robot, nSteps):

    v = 1 # Linear Velocity
    l = 0.5 #Half the width of the robot

    # MATPLOTLIB
    fig, ax = plt.subplots()
    plt.ion()
    fig.canvas.draw()
    plt.xlim((-2, 20))
    plt.ylim((-2, 30))
    plt.fill([7.5, 7.5, 16, 16, 20, 20],[-2, 7.5, 7.5, 20, 20, -2],
             facecolor='lightgray', edgecolor='gray', linewidth=3)
    plt.fill([-3, 0, 0, 7.5, 7.5, -3],[5, 5, 17.5, 17.5, 32, 32],
             facecolor='lightgray', edgecolor='gray', linewidth=3)

    plt.grid()

    # MAIN LOOP
    for k in range(1, nSteps + 1):
        #control is a wiggle with constant linear velocity
        u = np.vstack((v, np.pi / 10 * np.sin(4 * np.pi * k/nSteps)))

        robot.step(u)

        #draw occasionally
        if (k-1)%20 == 0:
            robot.draw(fig, ax)
            clear_output(wait=True)
            display(fig)
            time.sleep(0.1)

    plt.close()
```

In [5]:
```python
# RUN
dT = 0.1 # time steps size
pose = np.vstack([0., 0., 0.])
```

```
robot = VelocityRobot(pose, dT)
main(robot, nSteps=400)
```



Thinking about it (1)

Now that you have some experience with robot motion and the velocity motion model, **answer the following questions**:

- Why do we need to consider two different cases when applying the $g(\cdot)$ function, that is, calculating the new robot pose?

  Porque el modelo de movimiento diferencia entre movimiento angular (el robot realiza movimiento curvos) y lineal (el robot se mueve en línea recta).

- How many parameters compound the motion command u_t in this model?

  Dos parámetros: la velocidad angular w y la velocidad linear v