# MATH 461 Homework 2

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## Part 1

17. $\dfrac{8!}{\binom{64}{8}}$

18. $\dfrac{\binom{4}{1}\binom{16}{1}}{\binom{52}{2}}$

20. $n_{tot} = \binom{52}{2}\binom{50}{2} = 1624350,$

$n_{both} = \binom{4}{1}\binom{16}{1}\binom{3}{1}\binom{15}{1} = 2880$

$n_1 = 2 \times \binom{4}{1}\binom{16}{1}\left(\binom{3}{2} + \binom{3}{1}\binom{32}{1} + \binom{47}{2}\right) = 151040$

$P = 1 - \dfrac{n_1 + n_{both}}{n_{tot}} = 1 - \dfrac{151040 + 2880}{1624350} = 90.5242\%$

21. a) $P(i=1) = \frac{4}{20} = 20\%,\ P(i=2) = \frac{8}{20} = 40\%,$
$P(i=3) = \frac{5}{20} = 25\%,\ P(i=4) = \frac{2}{20} = 10\%,$
$P(i=5) = \frac{1}{20} = 5\%$

b) $n_c = 4 + 2\times 8 + 3\times 5 + 4\times 2 + 5\times 1 = 48$
$P(i=1) = \frac{4}{48} = 8.33\%,\ P(i=2) = \frac{16}{48} = 33.3\%$
$P(i=3) = \frac{15}{48} = 31.25\%, P(i=4) = \frac{8}{48} = 16.7\%$
$P(i=5) = \frac{5}{48} = 10.4\%$

25. $n_5 = |\{(1,4),(2,3),(3,2),(4,1)\}| = 4$
$n_7 = |\{(1,6),(2,5),(3,4),(4,3),(5,2),(6,1)\}| = 6$
Thus, for rolling a sum of 5 at the $\mathrm{n}^{\mathrm{th}}$ turn while not rolling any sum of 7 before is $\dfrac{4}{36}\left(\dfrac{36-4-6}{36}\right)^{n-1}$.

$P(5 \text{ before } 7) = \displaystyle\sum_{n=1}^{\infty} \frac{4}{36}\left(\frac{26}{36}\right)^{n-1} = \frac{4}{36}\sum_{n=1}^{\infty}\left(\frac{26}{36}\right)^{n-1}$

$= \dfrac{1}{9} \times \left(\dfrac{\frac{26}{36}^{\infty} - 1}{\frac{26}{36} - 1}\right) = \dfrac{1}{9} \times \dfrac{18}{5} = 40\%$

By getting the simulations going to infinity, the series should be the probability we want.

27. $P(\text{A's first ball is red}) = \dfrac{3}{10} = 0.3$
$P(\text{A's second ball is red}) = \dfrac{7}{10}\dfrac{6}{9}\dfrac{3}{8} = 0.175$
$P(\text{A's third ball is red}) = \dfrac{7}{10}\dfrac{6}{9}\dfrac{5}{8}\dfrac{4}{7}\dfrac{3}{6} = 0.08333$
$P(\text{A's fourth ball is red}) = \dfrac{7}{10}\dfrac{6}{9}\dfrac{5}{8}\dfrac{4}{7}\dfrac{3}{6}\dfrac{2}{5}\dfrac{3}{4} = 0.025$
$P(\text{A gets red ball}) = 58.33\%$

28. a) $P(\text{get 3 red}) = \left(\dfrac{5}{19}\right)^3$
$P(\text{get 3 blue}) = \left(\dfrac{6}{19}\right)^3$
$P(\text{get 3 green}) = \left(\dfrac{8}{19}\right)^3$
$P(\text{3 same color}) = \left(\dfrac{5}{19}\right)^3 + \left(\dfrac{6}{19}\right)^3 + \left(\dfrac{8}{19}\right)^3 = 12.4362\%$
b) $P = 3! \times \left(\dfrac{8}{19} \cdot \dfrac{6}{19} \cdot \dfrac{5}{19}\right) = 20.99\%$

32. since each girl is different, then to first put a girl on the i$^{\text{th}}$ location there will be $g$ ways, then permute the rest people we have a answer of:
$\dfrac{g(g+b-1)!}{(g+b)!}$

37. a) $\frac{\binom{7}{5}}{\binom{10}{5}} = 8.33\%$
b) $\frac{\binom{7}{4}\binom{3}{1}+\binom{7}{5}}{\binom{10}{5}} = 50\%$

43. a) $\frac{2\times(N-1!)}{N!}$
b) $\frac{2\times(n-1-1)!}{(n-1)!}$

50. $P = \frac{\binom{13}{5}\binom{39}{8}\binom{31}{5}\binom{8}{8}}{\binom{52}{13}\binom{31}{13}}$

53. $P(A) = P(\text{At least 1 couple together}) = \dfrac{\binom{4}{1} \times (8-1)! \times 2}{8!} =$

$P(B) = P(\text{At least 2 couple together}) = \dfrac{\binom{4}{2} \times (8-2)! \times 4}{8!} =$

$P(C) = P(\text{At least 3 couple together}) = \dfrac{\binom{4}{3} \times (8-3)! \times 8}{8!} =$

$P(D) = P(\text{At least 4 couple together}) = \dfrac{\binom{4}{4} \times (8-4)! \times 16}{8!} =$

$P(\text{no couple together}) =$

$1 - \dfrac{\binom{4}{1} \times (8-1)! \times 2 - \binom{4}{2} \times (8-2)! \times 4 + \binom{4}{3} \times (8-3)! \times 8 - \binom{4}{4} \times (8-4)! \times 16}{8!}$

$= \dfrac{12}{35}$

54. $N(\text{void in one}) = 4 \times \dfrac{\binom{39}{13}}{\binom{52}{13}} - 6 \times \dfrac{\binom{26}{13}}{\binom{52}{13}} + 4 \times \dfrac{\binom{13}{13}}{\binom{52}{13}} - 0 = 5.11\%$

# Part 2

3.1 $P(\text{diff}) = \dfrac{36-6}{36}$

$P(\text{diff} \cap \text{one } 6) = \dfrac{10}{36}$

$P(\text{one } 6|\text{diff}) = \dfrac{P(\text{diff} \cap \text{one } 6)}{P(\text{diff})} = \dfrac{1}{3}$

3.5 $P = \dfrac{6}{15}\dfrac{5}{14}\dfrac{9}{13}\dfrac{8}{12} = \dfrac{6}{91} = 6.59\%$

3.6 a) **Without** replacement: 50%

b) **With** replacement:50%

3.9 $P(2W) = \dfrac{1}{3}\dfrac{2}{3}\dfrac{3}{4} + \dfrac{2}{3}\dfrac{2}{3}\dfrac{1}{4} + \dfrac{1}{3}\dfrac{1}{3}\dfrac{1}{4} = \dfrac{11}{36} = 30.56\%$

$P(2W \cap AW) = \dfrac{1}{3}\dfrac{2}{3}\dfrac{3}{4} + \dfrac{1}{3}\dfrac{1}{3}\dfrac{1}{4} = \dfrac{7}{36} = 19.44\%$

$P = P(2W \cap AW)/P(2W) = \dfrac{7}{11} = 63.64\%$

3.10 $P(L2S) = \dfrac{13 \times 12 \times 11 + 39 \times 13 \times 12}{\binom{52}{3}} = \dfrac{6}{17} = 35.29\%$

$P(F1S \cap L2S) = \dfrac{13 \times 12 \times 11}{\binom{52}{3}} = \dfrac{33}{425} = 7.76\%$

$P = \dfrac{33}{425} / \dfrac{6}{17} = 22\%$

## Part 3

3.57 a) $P(2S) = 2 \times p(1-p)$

b) $P(3I) = 3 \times p^2(1-p)$

c) $P = \dfrac{2p^2(1-p)}{3p^2(1-p)} = \dfrac{2}{3}$

3.59 a) $P = p^4$

b) $P = p^3(1-p)$

c) $1 - p^4$ as something else appear in first 4 toses which means at least a T is produced

3.64 a) $P_1 = P(C) = p$

b) $P_2 = P(C) = p^2 + 0.5 \times 2p(1-p)$

$P_2 - P_1 = p^2 + p(1-p) - p = 0$, thus both strategy shall have same probability of wining.

3.66 a) $P = P(C1C2C \cup C3C4C5) = (p_1p_2 + p_3p_3 - p_1p_2p_3p_4)p_5$

b)

$$\begin{aligned}
P(E) &= P(C_1C_4 \cup C_2C_5 \cup C_3C_1C_5 \cup C_3C_2C_4) \\
&= P\left[C_3^c(C_1C_4 \cup C_2C_5) \cup C_3(C_1C_4 \cup C_2C_5 \cup C_1C_5 \cup C_2C_4)\right] \\
&= P(C_3^c)P(C_1C_4 \cup C_2C_5) + P(C_3)P(C_1C_4 \cup C_2C_5 \cup C_1C_5 \cup C_2C_4) \\
&= p_1p_4 + p_2p_5 + p_3(p_1p_5 + p_2p_4) - (p_1p_2p_3p_4 + p_1p_2p_3p_5 \\
&+ p_1p_3p_4p_5 + p_2p_3p_4p_5) + 2p_1p_2p_3p_4p_5
\end{aligned}$$

3.78 a) $P = 2 \times p^3(1-p) + 2 \times p(1-p)^3$

b) $P = \frac{p^2}{p^2+(1-p)^2}$

3.81 $P = \dfrac{0.55^{15}}{0.45^{15} + 0.55^{15}} = 95.30\%$

3.83 a) $P = 0.5 \times \frac{4}{6} + 0.5 \times \frac{2}{6} = \frac{3}{6} = \frac{1}{2}$

b) $P = \frac{3}{5}$

c) $P = \frac{4}{5}$

3.84 a) $P(A) = \dfrac{1}{3}\displaystyle\sum_{i=1}^{\infty}\left(\dfrac{2}{3}\right)^{3(i-1)} = \dfrac{1}{3} \times \dfrac{-1}{\frac{8}{27} - 1} = \dfrac{9}{19}$

$P(B) = \dfrac{2}{9}\displaystyle\sum_{i=1}^{\infty}\left(\dfrac{2}{3}\right)^{3(i-1)} = \dfrac{2}{9} \times \dfrac{-1}{\frac{8}{27} - 1} = \dfrac{6}{19}$

$P(C) = \dfrac{19 - 9 - 6}{19} = \dfrac{4}{19}$

b) $P(A) = P(A1) + P(A2) + P(A3) = \frac{7}{15}$

$P(B) = P(B1) + P(B2) + P(A3) = \frac{68}{165}$

$P(C) = 1 - P(A) - P(B) = \frac{4}{33}$