PHYS 225 HW 7

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1. First get the space time cordinates in ground frame and then transform them into different frames. Frist, the system is synchronized at E_1. Meaning that E_1 happens at $(0,0)$ in all 3 frames. Meaning that we can have build a table like this:

	Ground Frame		Train's back Frame		C's Frame	
Event	t	x	t'	x'	t_c	x_c
E_1	0	0	0	0	0	0
E_2	?	?	?	?	?	0

$$\Delta v = \frac{4}{5}c - \frac{3}{5}c = \frac{1}{5}c \qquad t = \frac{L}{v} = \frac{L}{\frac{1}{5}c} = \frac{5L}{c}$$

$$x = v \times t = \frac{4}{5}c \times \frac{5L}{c} = 4L = L + \frac{3}{5}c \times \frac{5L}{c}$$

Thus, the space time coordinate of E_1 is $\left(\frac{5L}{c}, 4L\right)$. Apply lorentz transformation on this, first to solve for the event in train's frame:

$$\begin{bmatrix} \gamma & \gamma\beta \\ \gamma\beta & \gamma \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} & \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\frac{v}{c} \\ \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\frac{v}{c} & \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \end{bmatrix}$$
$$= \begin{bmatrix} \frac{5}{4} & -\frac{3}{4} \\ -\frac{3}{4} & \frac{5}{4} \end{bmatrix}$$
$$\begin{bmatrix} \gamma & \gamma\beta \\ \gamma\beta & \gamma \end{bmatrix}\begin{bmatrix} ct \\ x \end{bmatrix} = \begin{bmatrix} \frac{5}{4} & -\frac{3}{4} \\ -\frac{3}{4} & \frac{5}{4} \end{bmatrix}\begin{bmatrix} 5L \\ 4L \end{bmatrix}$$
$$= \begin{bmatrix} \frac{13}{4}L \\ \frac{5}{4}L \end{bmatrix}$$

Thus, the space time coordinate of E_2 at train's back frame is $\left(\frac{13L}{4c}, \frac{5}{4}L\right)$

Now compute the coordinate in C's frame:

$$\begin{bmatrix} \gamma & \gamma\beta \\ \gamma\beta & \gamma \end{bmatrix} = \begin{bmatrix} \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} & \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\frac{v}{c} \\ \frac{1}{\sqrt{1-\frac{v^2}{c^2}}}\frac{v}{c} & \frac{1}{\sqrt{1-\frac{v^2}{c^2}}} \end{bmatrix}$$
$$= \begin{bmatrix} \frac{5}{3} & -\frac{4}{3} \\ -\frac{4}{3} & \frac{5}{3} \end{bmatrix}$$
$$\begin{bmatrix} \gamma & \gamma\beta \\ \gamma\beta & \gamma \end{bmatrix} \begin{bmatrix} ct \\ x \end{bmatrix} = \begin{bmatrix} \frac{5}{3} & -\frac{4}{3} \\ -\frac{4}{3} & \frac{5}{3} \end{bmatrix} \begin{bmatrix} 5L \\ 4L \end{bmatrix}$$
$$= \begin{bmatrix} 3L \\ 0 \end{bmatrix}$$

Thus, the table becomes:

	Ground Frame		Train's back Frame		C's Frame	
Event	t	x	t'	x'	t_c	x_c
E_1	0	0	0	0	0	0
E_2	$\frac{5L}{c}$	$4L$	$\frac{13L}{4c}$	$\frac{5}{4}L$	$\frac{3L}{c}$	0

Now, check for values of $c^2(\Delta t)^2 - (\Delta x)^2$

$$c^2(\Delta t)^2 - (\Delta x)^2 = c^2\left(\frac{5L}{c}\right) - (4L)^2 = 9L^2$$
$$= c^2\left(\frac{13L}{4c}\right) - \left(\frac{5}{4}L\right)^2 = \frac{13^2 - 5^2}{16}L^2 = 9L^2$$
$$= c^2\left(\frac{3L}{c}\right) - 0 = 9L^2$$

2. a) Synchronize two system at E_1 when the left side of pole passing left side of barn. Label the space time event in barn's frame as S, poles as S', coordinate of left of barn as B_l, and right side as B_r, for pole, it is P_l and P_r.

Label		S		S'	
		t	x	t'	x'
E_1	B_l	0	0	0	0
	P_l	0	0	0	0
E_2	B_r	t_1	L	t_2	L
	P_r	t_1	L	t_2	L

Now solve for t_1, t_2:

$$\begin{bmatrix} \frac{5}{4} & -\frac{3}{4} \\ -\frac{3}{4} & \frac{5}{4} \end{bmatrix} \begin{bmatrix} ct_1 \\ L \end{bmatrix} = \begin{bmatrix} ct_2 \\ L \end{bmatrix}$$

$$\begin{cases} \frac{5}{4}ct_1 - \frac{3}{4}L = ct_2 \\ -\frac{3}{4}ct_1 + \frac{5}{4}L = L \end{cases}$$

$$\begin{cases} t_1 = \frac{L}{3c} \\ t_2 = -\frac{L}{3c} \end{cases}$$

Thus, $\Delta t = t_1 - 0 = \frac{L}{3c}$, $\Delta x = L - 0 = L$

b) In the red frame or double prime frame, the 2 events are all on x-axis, meaning they are happening at the same time of $t = 0$