NPRE200 HW 5

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1. a)

$$\begin{aligned}\frac{\mathrm{d}n}{\mathrm{d}t} &= S + P - A + R \\ &= S + (\nu - 1)\Sigma_f\phi - \Sigma_c\phi + D\nabla^2\phi\end{aligned}$$

b)

$$\begin{aligned}\frac{\mathrm{d}n}{\mathrm{d}t} &= S + P - A + R \\ &= S + (\eta - 1)\Sigma_a\phi + D\nabla^2\phi\end{aligned}$$

c)

$$\begin{aligned}\frac{\mathrm{d}n}{\mathrm{d}t} &= S + P - A + R = 0 \\ 0 &= S + (\eta - 1)\Sigma_a\phi + D\nabla^2\phi\end{aligned}$$

d)

$$\begin{aligned}\frac{\mathrm{d}n}{\mathrm{d}t} &= S + P - A + R = 0 \\ 0 &= S - \Sigma_a\phi + D\nabla^2\phi\end{aligned}$$

2. a)

$$\begin{aligned}\frac{\mathrm{d}n}{\mathrm{d}t} &= S - \Sigma_a\phi + D\nabla^2\phi = 0 \\ 0 &= S - \Sigma_a\phi + D\nabla^2\phi \\ -D\nabla^2\phi &= S - \Sigma_a\phi \\ -\nabla^2\phi + \frac{\Sigma_a}{D}\phi &= \frac{S}{D} \\ -\nabla^2\phi + \frac{1}{L^2}\phi &= \frac{S}{D}\end{aligned}$$

b)

$$\nabla^2\phi - \frac{1}{L^2}\phi = 0$$
$$\nabla^2 = \frac{\mathrm{d}^2}{\mathrm{d}x^2}$$
$$\frac{\mathrm{d}^2\phi}{\mathrm{d}x^2} - \frac{1}{L^2}\phi = 0$$
$$\phi = Ae^{xp} + Ce^{-xp}$$
$$\phi(x) = Ae^{\frac{x}{L}} + Ce^{-\frac{x}{L}}$$
$$\phi(0) = \phi_0$$
$$\phi(\infty) = 0$$
$$0 = Ae^{xp} + Ce^{-xp}$$
$$0 = A\cdot\infty + C\cdot 0$$
$$A = 0$$
$$\phi_0 = Ae^{xp} + Ce^{-xp}$$
$$\phi_0 = C\cdot 1$$
$$C = \phi_0$$
$$\phi(x) = \phi_0 e^{-\frac{x}{L}}$$

3.

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}x^2} - \frac{1}{L^2}\phi = -\frac{S}{D}$$
$$\phi_g(x) = Ae^{\frac{x}{L}} + Ce^{-\frac{x}{L}}$$

Take a particular solution ϕ_{pi}

$$\frac{\mathrm{d}^2\phi}{\mathrm{d}x^2}p^2\phi = Q$$
$$\phi_{pi} = \frac{Q}{p^2}$$
$$= \frac{S/D}{1/L^2}$$
$$= \frac{L^2}{D}\cdot S$$
$$= \frac{1}{D/L^2}\cdot S$$
$$= \frac{S}{\Sigma_a}\phi(x) = Ae^{\frac{x}{L}} + Ce^{-\frac{x}{L}} + \frac{S}{\Sigma_a}$$

4. a)

$$
\begin{aligned}
\phi &= \phi_0 e^{-\frac{x}{L}} \\
J_x(x) &= J_x^+(x) - J_x^-(x) \\
J_x^\pm(x) &= \frac{1}{4}\phi(x) \mp \frac{1}{2}D\frac{\mathrm{d}}{\mathrm{d}x}\phi(x) \\
J_x^\pm(0) &= \frac{1}{4}\phi(0) \mp \frac{1}{2}D\frac{\mathrm{d}}{\mathrm{d}x}\phi(x) \\
&= \frac{1}{4}\phi_0 \mp \frac{1}{2}D\left.\left[\frac{\mathrm{d}}{\mathrm{d}x}\phi_0 e^{-\frac{x}{L}}\right]\right|_{x=0} \\
&= \frac{1}{4}\phi_0 \pm \frac{1}{2}\frac{D}{L}\phi_0 \\
\phi_0 &= J_x^+(0)\left(\frac{1}{4} + \frac{D}{2L}\right)^{-1} \\
\phi &= \phi_0 e^{-\frac{x}{L}} \\
\phi &= J_x^+(0)\left(\frac{1}{4} + \frac{D}{2L}\right)^{-1} e^{-\frac{x}{L}}
\end{aligned}
$$

b)

$$
\begin{aligned}
\alpha &= \frac{J_x^-(0)}{J_x^+(0)} \\
&= \frac{\frac{1}{4} + \frac{D}{2L}}{\frac{1}{4} - \frac{D}{2L}} \\
&= \frac{L - 2D}{L + 2D}
\end{aligned}
$$