MATH 461 Homework 8

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6.48 a)

$$\begin{aligned} P(\min(X_1, X_2, X_3, X_4, X_5) \le a) &= 1 - P(\min(X_1, X_2, X_3, X_4, X_5) > a) \\ &= 1 - (e^{-(a\lambda)})^5 \end{aligned}$$

b)

$$\begin{aligned} P(\max(X_1, X_2, X_3, X_4, X_5) \le a) &= 1 - P(X_1 \le a, X_2 \le a, X_3 \le a, X_4 \le a, X_5 \le a) \\ &= (e^{-(a\lambda)})^5 \end{aligned}$$

7.5

$$\begin{aligned} E(|x| + |y|) &= E(|x|) + E(|y|) \\ &= \int_{-1.5}^{1.5} \frac{|x|}{3} dx + \int_{-1.5}^{1.5} \frac{|y|}{3} dy \\ &= 1.5 \end{aligned}$$

7.6

$$\frac{1}{6} \times (1 + 2 + 3 + 4 + 5 + 6) = \frac{7}{2}$$

7.7 a)

$$0.3 \times 0.3 = 0.09$$

$$10 \times 0.09 = 0.9$$

b)

$$10 \times (1 - 0.3)^2 = 4.9$$

c)

$$2 \times (0.3 \times 0.7) \times 10 = 4.2$$

7.8 By noteing in the way, the total table occupied will be: $\sum X_i$,

$$E[X_i] = (1 - p)^{i-1}$$

$$E = \sum_{i=1}^{n} (1 - p)^{i-1}$$

7.11 any change over will have a probability of $2p(1-p)$ as they are landing are different sides. And there are total of $n-1$ slots that are possible for flips, then the total probability is $(n-1)2p(1-p)$

7.18 $52 \times \frac{1}{13} = 4$

7.19 a) Probability of chatching j before getting type 1 is $(1-p_1)^j p_1$, then the $E((1-p_1)^j p_1) = \sum_{j=0}^{\infty} j(1-p_1)^j p_1 = p_1 \frac{1-p_1}{p_1^2} = \frac{1-p_1}{p_1}$

b) It will $\sum_{j=2}^{n} \frac{p_j}{p_j+p_1}$

7.21 a)

$$365 \times \binom{100}{3} \left(\frac{1}{365}\right)^3 \left(\frac{364}{365}\right)^{97}$$

b)

$$365 \times \left(1 - \left(\frac{364}{365}\right)^{100}\right)$$

7.30

$$\begin{aligned} E\left[(X-Y)^2\right] &= E\left[X^2 - 2XY + Y^2\right] \\ &= E\left[X^2\right] - 2E\left[X\right]E\left[Y\right] + E[Y^2] \\ &= (\sigma^2 + \mu^2) - 2\mu\mu + (\sigma^2 + \mu^2) \\ &= \sigma^2 \end{aligned}$$

7.31

$$\begin{aligned} \mathrm{var}(X) &= E[X^2] - (E[X])^2 \\ &= 10 \times \frac{1}{6}(1+4+9+16+25+36) - 10 \times (\frac{1}{6}(1+2+3+4+5+6))^2 \\ &= \frac{175}{6} \end{aligned}$$

7.33 a)

$$\begin{aligned} E\left[(2+X)^2\right] &= E[4 + 4X + X^2] \\ &= 4 + (5+1) + 4 \\ &= 14 \end{aligned}$$

b)

$$\begin{aligned} \mathrm{var}(4+3X) &= 3^2 \mathrm{var}(X) \\ &= 9 \times 5 \\ &= 45 \end{aligned}$$

7.38

$$
\begin{aligned}
E(XY) &= \int_0^\infty \int_0^x xy\frac{2}{x}e^{-2x}dydx \\
&= \frac{1}{4} \\
E(X) &= \int_0^\infty \int_0^x x\frac{2}{x}e^{-2x}dydx \\
&= \frac{1}{2} \\
E(Y) &= \int_0^\infty \int_0^x y\frac{2}{x}e^{-2x}dydx \\
&= \frac{1}{4} \\
\text{cov}(X,Y) &= E(XY) - E(X)E(Y) = \frac{1}{4} - \frac{1}{2}\frac{1}{4} \\
&= \frac{1}{8}
\end{aligned}
$$

7.39

$$
\begin{aligned}
j = 0 &: \text{cov}(Y_n, Y_n) = \sigma^3 \\
j = 1 &: \text{cov}(Y_n, Y_{n+1}) = \sigma^2 \\
j = 2 &: \text{cov}(Y_n, Y_{n+2}) = \sigma \\
j \geq 3 &: \text{cov}(Y_n, Y_{n+j}) = 0
\end{aligned}
$$

7.41 The assumptions made are follows Hyper geometric distribution with m=30,n=20,N=100.

$$
\begin{aligned}
\mu &= 0.3 \times 20 \\
&= 6 \\
\text{var}(X) &= \frac{mn}{N}\left[\frac{(n-1)(m-1)}{N-1} + 1 - \frac{mn}{N}\right] \\
&= \frac{30 \times 20}{100}\left[\frac{(20-1)(30-1)}{100-1} + 1 - \frac{30 \times 20}{100}\right] \\
&= \frac{600}{100}\left[\frac{19 \times 29}{99} + 1 - \frac{600}{100}\right] \\
&= \frac{600}{100}\left[\frac{551}{99} + 1 - \frac{600}{100}\right] \\
&= \frac{600}{100}\left[\frac{56}{99}\right] \\
&= \frac{112}{33}
\end{aligned}
$$

7.42

$$E(X) = \frac{10}{19}\text{var} \qquad\qquad\qquad = \frac{3240}{6137}$$