ME 200 Homework 6

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1. a)

$$
\begin{aligned}
\eta_1 &= 1 - \frac{Q}{Q_H} \\
Q_H &= \frac{Q}{1-\eta_1} \\
\eta_2 &= 1 - \frac{Q_C}{Q} \\
Q_C &= Q - Q\eta_2 \\
\eta_{tot} &= 1 - \frac{Q_C}{Q_H} \\
&= 1 - \frac{Q - Q\eta_2}{\frac{Q}{1-\eta_1}} \\
&= 1 - Q(1-\eta_2) \times \frac{1-\eta_1}{Q} \\
&= 1 - (1-\eta_1)(1-\eta_2) \\
&= 1 - (1 - \eta_1 - \eta_2 + \eta_1\eta_2) \\
&= \eta_1 + \eta_2 - \eta_1\eta_2
\end{aligned}
$$

b)

$$
\begin{aligned}
\eta_1 &= 1 - \frac{T}{T_H} \\
\eta_2 &= 1 - \frac{T_C}{T}
\end{aligned}
$$

plug them back in a)

$$
\begin{aligned}
\eta_{tot} &= 1 - \frac{T}{T_H} + 1 - \frac{T_C}{T} - (1 - \frac{T}{T_H})(1 - \frac{T_C}{T}) \\
&= 1 - \frac{T_C}{T_H}
\end{aligned}
$$

c)

$$\begin{aligned}
\eta_1 &= \eta_2 \\
1 - \frac{T}{T_H} &= 1 - \frac{T_C}{T} \\
\frac{T}{T_H} &= \frac{T_C}{T} \\
T^2 &= T_C T_H \\
T &= \sqrt{T_C T_H}
\end{aligned}$$

2. It the system is reversible then:

$$\eta_{\max} = 1 - \frac{T_C}{T_H} = 1 - \frac{600}{1800} = \frac{2}{3}$$

   a. Impossible as $\eta > \eta_{\max}$:

   $$\eta = 1 - \frac{100}{500} = \frac{4}{5} > \frac{2}{3}$$

   b. Is irreversiable as:

   $$\eta = \frac{W_c}{Q_H} = \frac{250}{500} = \frac{1}{2} < \frac{2}{3}$$

   c. Impossible as $\eta > \eta_{\max}$:

   $$\begin{aligned}
   Q_H &= W_c + Q_C = 500\text{kW} \\
   \eta &= \frac{W_c}{Q_H} = \frac{350}{500} = \frac{7}{10} > \frac{2}{3}
   \end{aligned}$$

   d. Is irreversiable, as:

   $$\eta = 1 - \frac{Q_C}{Q_H} = \frac{3}{5} < \frac{2}{3}$$

3.

$$\eta_{\max} = 1 - \frac{T_C}{T_H} = 1 - \frac{15 + 273}{1100 + 273} = 79.024\%$$

Thus, such device is possible and operates a nearlly reversible processes as 79% is so close to the maximum efficiency

4. a)

$$\gamma_{\max} = \frac{T_H}{T_H - T_C} = \frac{20+273}{5} = 58.6$$
$$W_{\min} = \frac{Q_H}{\gamma_{\max}} = \frac{3\times 10^6}{58.6} = 5.12\times 10^4 \text{ kJ}$$

b)

$$\gamma_{\max} = \frac{T_H}{T_H - T_C} = \frac{20+273}{10} = 29.3$$
$$W_{\min} = \frac{Q_H}{\gamma_{\max}} = \frac{3\times 10^6}{29.3} = 1.02\times 10^5 \text{ kJ}$$

c)

$$\gamma_{\max} = \frac{T_H}{T_H - T_C} = \frac{20+273}{15} = 19.533$$
$$W_{\min} = \frac{Q_H}{\gamma_{\max}} = \frac{3\times 10^6}{19.533} = 1.535\times 10^5 \text{ kJ}$$

5. a)

$$\gamma_{\max} = \frac{T_H}{T_H - T_C} = \frac{20+273}{27} = 10.851$$
$$W_{\min} = \frac{Q_H}{\gamma_{\max}} = 24000/10.851 = 2211.6\text{kJ/h} = 0.6143\text{kJ/s} = 0.6143\text{kW}$$
$$0.6143 \times 1 \times 0.085 \times 24 = 1.25317 \text{ \$/day}$$

b)

$$\gamma_{\max} = \frac{T_H}{T_H - T_C} = \frac{20+273}{15} = 19.533$$
$$W_{\min} = \frac{Q_H}{\gamma_{\max}} = 24000/19.533 = 1228.67\text{kJ/h} = 0.3413\text{kJ/s} = 0.3413\text{kW}$$
$$0.3413 \times 1 \times 0.085 \times 24 = 0.6924 \text{ \$/day}$$

6.

$$
\begin{aligned}
W &= \frac{Q_2}{\gamma} = \frac{Q_2}{\frac{T_d}{T_d - T_0}} \\
\eta &= \frac{W}{Q_s} = 1 - \frac{T_d}{T_s} \\
Q_s &= \frac{W T_s}{T_s - T_d} \\
Q_1 &= Q_s - W = \frac{W T_s}{T_s - T_d} - W = \frac{W T_d}{T_s - T_d} \\
Q_2 &= \frac{T_d W}{T_d - T_0} \\
\frac{Q_1 + Q_2}{Q_s} &= \frac{\frac{W T_d}{T_s - T_d} + \frac{T_d W}{T_d - T_0}}{\frac{W T_s}{T_s - T_d}} \\
&= \frac{\frac{T_d}{T_s - T_d} + \frac{T_d}{T_d - T_0}}{\frac{T_s}{T_s - T_d}} \\
&= \frac{T_d + \frac{T_d(T_s - T_d)}{T_d - T_0}}{T_s} \\
&= \frac{T_d(T_s - T_0)}{T_s(T_d - T_0)}
\end{aligned}
$$