PHYS 225 HW 11

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1. a)

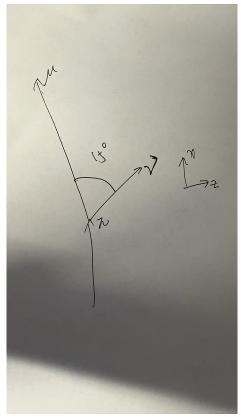
$$\begin{aligned} p_\pi &= \sqrt{E^2 - m^2} \\ &= 990.15 \times 10^6 \text{ eV} \end{aligned}$$

in the lab frame:

$$\begin{aligned} \mathbf{p}_1 &= (E_1, p_1, 0, 0) \\ \mathbf{p}_2 &= (E_2, p_2, 0, 0) \\ \mathbf{p}_3 &= (p_3, p_3, 0, 0) \\ \mathbf{p}_1 &= \mathbf{p}_2 + \mathbf{p}_3 \\ &\begin{cases} 990.15 \times 10^6 = p_2 + p_3 \\ \qquad 10^9 = E_2 + p_3 \end{cases} \\ E_2 &= \sqrt{m_\mu^2 + p_2^2} \\ &= \sqrt{(106 \times 10^6)^2 + p_2^2} \\ &\begin{cases} p_2 = 564.43 \text{ MeV} \\ p_3 = 424.72 \text{ MeV} \end{cases} \\ E_\nu &= p_3 = 424.72 \text{ MeV} \end{aligned}$$

the the direction is also colinear with the pion's momentum direction.

b)



$$
\begin{aligned}
\mathbf{p}_1 &= (E_1, p_1, 0, 0) \quad (\pi) \\
\mathbf{p}_2 &= (E_2, p_{2x}, p_{2y}, p_{2z}) \quad (\mu) \\
\mathbf{p}_3 &= (||p||, p_{3x}, p_{3y}, p_{3z}) \quad (\nu) \\
\mathbf{p}_1 &= \mathbf{p}_2 + \mathbf{p}_3 \\
||p_3|| &= E_1 - E_2 \\
&= 100 \text{ MeV} \\
||p_2|| &= \sqrt{E_2^2 - m^2} \\
&= \sqrt{900^2 - 106^2} \\
&= 893.736 \text{ MeV}
\end{aligned}
$$

Set $p_{2y} = p_{3y} = 0$, then:

$$
\begin{cases}
p_{2x} + p_{3x} = 990.15 \\
p_{2z} + p_{3z} = 0 \\
||p_2|| = 893.736 \\
||p_3|| = 100
\end{cases}
$$

Solving this will give:

$$
\begin{aligned}
\overrightarrow{p_\mu} &= (893.38, 0, -25.2117) \\
\overrightarrow{p_\nu} &= (96.7697, 0, 25.2117) \\
\cos(\theta) &= \frac{\overrightarrow{p_\nu} \cdot \overrightarrow{p_\mu}}{||\overrightarrow{p_\mu}|| \; ||\overrightarrow{p_\nu}||} \\
&= 0.962199 \\
\theta &= 15.8041^\circ
\end{aligned}
$$

2.

$$\begin{aligned}
p_1 &= \frac{E}{c} = p_{2x} + p_{3x} \\
&= p_2 \cos(\theta) + p_3 \cos(\theta) \\
p_{2y} \sin(\theta) - p_{3y} \sin(\theta) &= 0 \\
p_2 &= p_3 \\
\frac{E}{c} &= 2p_{rst} \cos(\theta) \\
p_{rst}c &= \frac{E}{\cos(\theta)} \\
E'_m &= \sqrt{p^2c^2 + m^2c^4} \\
E + E_m &= E'_m + E_{rst} \\
E + E_m &= \sqrt{E_{rst}^2 + m^2c^4} + E_{rst} \\
((E + mc^2)^2 - E_{rst})^2 &= E_{rst}^2 + m^2c^4 \\
E_{rst} &= \frac{E(E + 2mc^2)}{2(E + mc^2)} \\
\cos(\theta) &= \frac{E}{2E_{rst}} = \frac{E + mc^2}{E + 2mc^2} \\
E \to 0,\ \cos(\theta) &\to \frac{1}{2},\ \theta \to 60^\circ
\end{aligned}$$