NPRE200 HW 4

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1. Using $\Sigma_f(E) = \frac{E_0}{E}\Sigma_f(E),\ \phi(E) = \frac{1}{(kT)^2}E\exp(-E/kT)$

$$
\begin{aligned}
\bar{\Sigma}_f &= \frac{R_f}{\phi} \\
&= \frac{\int_0^\infty \Sigma(E)\varphi(E)\ \mathrm{dE}}{\int_0^\infty \varphi(E)\ \mathrm{dE}} \\
&= \frac{\int_0^\infty \frac{E_0}{E}\Sigma_f(E_0)\frac{1}{(kT)^2}E\exp(-E/kT)}{\int_0^\infty \varphi(E)\ \mathrm{dE}} \\
&= \frac{\int_0^\infty E_0\Sigma_f(E_0)\frac{1}{(kT)^2}\exp(-E/kT)}{\int_0^\infty E\frac{1}{(kT)^2}\exp(-E/kT)} \\
&= E_0\Sigma_f E_0\frac{\int_0^\infty \exp(-E/kT)\ \mathrm{d}E}{\int_0^\infty E\exp(-E/kT)\ \mathrm{d}E}
\end{aligned}
$$

$$
\begin{aligned}
\because \int_0^\infty x\exp(-x)\ \mathrm{dx} &= 1 \\
\int_0^\infty E/kT\exp(-E/kT)\ \mathrm{d}E &= 1 \\
\int_0^\infty E\exp(-E/kT)\ \mathrm{d}E &= (kT)^2 \\
\int_0^\infty \exp(-E/kT)\ \mathrm{d}E &= kT
\end{aligned}
$$

$$
\begin{aligned}
\bar{\Sigma}_f &= \frac{E_0\Sigma_f(E_0)}{kT} \\
\sigma_f &= \frac{\bar{\Sigma}_f}{N} = \frac{R_f}{\phi N}
\end{aligned}
$$

2.

$$
\begin{aligned}
\bar{\Sigma}_f &= \frac{R_f}{\phi} \\
&= \frac{\int_0^\infty \Sigma(E)\varphi(E)\ \mathrm{dE}}{\int_0^\infty \varphi(E)\ \mathrm{dE}} \\
&= \frac{\int_0^\infty \frac{E_0}{E}\Sigma_f(E_0)\frac{1}{(kT)^2}E\exp(-E/kT)}{\int_0^\infty \varphi(E)\ \mathrm{dE}} \\
&= \frac{\int_0^\infty E_0\Sigma_f(E_0)\frac{1}{(kT)^2}\exp(-E/kT)}{\int_0^\infty E\frac{1}{(kT)^2}\exp(-E/kT)} \\
&= E_0\Sigma_f E_0\frac{\int_0^\infty \exp(-E/kT)\ \mathrm{d}E}{\int_0^\infty E\exp(-E/kT)\ \mathrm{d}E}
\end{aligned}
$$

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\because \int_0^\infty x\exp(-x)\ \mathrm{dx} &= 1 \\
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\int_0^\infty E\exp(-E/kT)\ \mathrm{d}E &= (kT)^2 \\
\int_0^\infty \exp(-E/kT)\ \mathrm{d}E &= kT \\
\bar{\Sigma}_f &= \frac{E_0\Sigma_f(E_0)}{kT}
\end{aligned}
$$

3.

$$
\Sigma_t(E)\phi(E) = \int \Sigma_s(E' \to E)\phi(E')\mathrm{d}E' + \chi(E)s_f'''
$$

4. a)

$$
q(E) = -\int_E^\infty \Sigma_a E'\varphi(E')\ \mathrm{d}E' + \int_E^\infty \chi(E')dE's_f''',\ E > 1.0\ \mathrm{eV}
$$

b) Assume that in the intermidiate range, $\int_0^\infty \chi(E)\ \mathrm{d}E = 1$ as the production of neutron by fission is in significant,

$$
q(E) = -\int_E^\infty \Sigma_a(E')\varphi(E')\ \mathrm{d}E' + s_f'''
$$

5. Assuming that we are at energy below where all the neutrons are formed by fission.

$$\begin{aligned}
\Sigma_s(E)\varphi(E) &= \int p(E' \to E)\Sigma_s(E')\varphi(E')\ \mathrm{d}E' + \chi(E)s_f''' \\
&= \int p(E' \to E)\Sigma_s(E')\varphi(E')\ \mathrm{d}E' \\
&= \int_E^{E/\alpha} \frac{1}{(1-\alpha)E'}\Sigma_s(E')\varphi(E')\ \mathrm{d}E'
\end{aligned}$$

6. Suppose it holds:

$$\begin{aligned}
C/E &= \int_E^{E/\alpha} \frac{1}{(1-\alpha)E'} \cdot \frac{C}{E'}\ \mathrm{d}E' \\
&= \frac{C}{(1-\alpha)} \int_E^{E/\alpha} \frac{1}{E'^2}\ \mathrm{d}E' \\
&= \frac{C}{(1-\alpha)}\ -\frac{1}{E'}\bigg|_E^{E/\alpha} \\
&= \frac{C}{E}
\end{aligned}$$

7. a)

$$\begin{aligned}
\varphi_M(E) &= \frac{1}{(kT)^2}E\exp(-E/kT) \\
\bar{\sigma}_{aT} &= \int \sigma_a(E)\varphi(E)\ \mathrm{d}E \\
\bar{\sigma}_{aT} &= \int \sqrt{E-0/E}\ \sigma_a(E_0)\frac{1}{(kT)^2}E\exp(-E/kT)\ \mathrm{d}E \\
&= \frac{\sqrt{\pi}}{2}\sqrt{\frac{E_0}{kT}}
\end{aligned}$$

$$\because E_0 = kT_0$$

$$\begin{aligned}
&= 0.8862(T_0/T)^{1/2}\sigma_a(E_0) \\
\therefore \bar{\sigma}_a(T) &= (T_0/T)^{1/2}\sigma_a(T_0)
\end{aligned}$$

b)

$$\begin{aligned}
\bar{\sigma}_{aT}(T) &= (T_0/T)^{1/2}\sigma_a(T_0) \\
&= \sqrt{293.6/573} \times 0.5896 \\
&= 0.422\ \text{barn}
\end{aligned}$$