

MATH 461 Homework 3

James Liu

Due: Sep 20 Edit: September 19, 2024

Part 1

3.57 a) $P(2S) = 2 \times p(1-p)$

b) $P(3I) = 3 \times p^2(1-p)$

c) $P = \frac{2p^2(1-p)}{3p^2(1-p)} = \frac{2}{3}$

3.59 a) $P = p^4$

b) $P = p^3(1-p)$

c) $1 - p^4$ as something else appear in first 4 toses which means at least a T is produced

3.64 a) $P_1 = P(C) = p$

b) $P_2 = P(C) = p^2 + 0.5 \times 2p(1-p)$

$P_2 - P_1 = p^2 + p(1-p) - p = 0$, thus both strategy shall have same probability of wining.

3.66 a) $P = P(C_1C_2C \cup C_3C_4C_5) = (p_1p_2 + p_3p_3 - p_1p_2p_3p_4)p_5$

b)

$$\begin{aligned} P(E) &= P(C_1C_4 \cup C_2C_5 \cup C_3C_1C_5 \cup C_3C_2C_4) \\ &= P[C_3^c(C_1C_4 \cup C_2C_5) \cup C_3(C_1C_4 \cup C_2C_5 \cup C_1C_5 \cup C_2C_4)] \\ &= P(C_3^c)P(C_1C_4 \cup C_2C_5) + P(C_3)P(C_1C_4 \cup C_2C_5 \cup C_1C_5 \cup C_2C_4) \\ &= p_1p_4 + p_2p_5 + p_3(p_1p_5 + p_2p_4) - (p_1p_2p_3p_4 + p_1p_2p_3p_5 \\ &\quad + p_1p_3p_4p_5 + p_2p_3p_4p_5) + 2p_1p_2p_3p_4p_5 \end{aligned}$$

3.78 a) $P = 2 \times p^3(1-p) + 2 \times p(1-p)^3$

b) $P = \frac{p^2}{p^2 + (1-p)^2}$

3.81 $P = \frac{0.55^{15}}{0.45^{15} + 0.55^{15}} = 95.30\%$

3.83 a) $P = 0.5 \times \frac{4}{6} + 0.5 \times \frac{2}{6} = \frac{3}{6} = \frac{1}{2}$

$$\text{b) } P = \frac{3}{5}$$

$$\text{c) } P = \frac{4}{5}$$

$$3.84 \quad \text{a) } P(A) = \frac{1}{3} \sum_{i=1}^{\infty} \left(\frac{2}{3}\right)^{3(i-1)} = \frac{1}{3} \times \frac{-1}{\frac{8}{27} - 1} = \frac{9}{19}$$

$$P(B) = \frac{2}{9} \sum_{i=1}^{\infty} \left(\frac{2}{3}\right)^{3(i-1)} = \frac{2}{9} \times \frac{-1}{\frac{8}{27} - 1} = \frac{6}{19}$$

$$P(C) = \frac{19 - 9 - 6}{19} = \frac{4}{19}$$

$$\text{b) } P(A) = P(A1) + P(A2) + P(A3) = \frac{7}{15}$$

$$P(B) = P(B1) + P(B2) + P(A3) = \frac{68}{165}$$

$$P(C) = 1 - P(A) - P(B) = \frac{4}{33}$$

Part 2

4.1 $\binom{3}{2} = 6$ types of X.

$$\text{X=4: } P(X = 4) = \frac{4}{14} \frac{3}{13} = \frac{6}{91} \approx 6.59\%$$

$$\text{X=2: } P(X = 2) = \frac{4}{14} \frac{2}{13} + \frac{2}{14} \frac{4}{13} = \frac{\binom{4}{1}\binom{2}{1}}{\binom{14}{2}} = \frac{8}{91} \approx 8.79\%$$

$$\text{X=1: } P(X = 1) = \frac{\binom{4}{1}\binom{8}{1}}{\binom{14}{2}} = \frac{32}{91} \approx 35.16\%$$

$$\text{X=0: } P(X = 0) = \frac{\binom{2}{2}}{\binom{14}{2}} = \frac{1}{91} \approx 1.10\%$$

$$\text{X=-1: } P(X = -1) = \frac{\binom{8}{1}\binom{2}{1}}{\binom{14}{2}} = \frac{16}{91} \approx 17.58\%$$

$$\text{X=-2: } P(X = -2) = \frac{\binom{8}{2}}{\binom{14}{2}} = \frac{28}{91} \approx 30.77\%$$

4.4

$$X = 1: P(X = 1) = \frac{\binom{5}{1}9!}{10!} = \frac{1}{2} = 50\%$$

$$X = 2: P(X = 2) = \frac{\binom{5}{1}\binom{5}{1}8!}{10!} = \frac{5}{18} \approx 27.78\%$$

$$X = 3: P(X = 3) = \frac{2! \times \binom{5}{2}\binom{5}{1} \times 7!}{10!} = \frac{5}{36} \approx 13.89\%$$

$$X = 4: P(X = 4) = \frac{3! \times \binom{5}{3}\binom{5}{1} \times 6!}{10!} = \frac{5}{84} \approx 5.95\%$$

$$X = 5: P(X = 5) = \frac{4! \times \binom{5}{4}\binom{5}{1} \times 5!}{10!} = \frac{5}{252} \approx 1.98\%$$

$$X = 6: P(X = 6) = \frac{5! \times \binom{5}{5}\binom{5}{1} \times 4!}{10!} = \frac{1}{252} \approx 0.40\%$$

$$X > 6: P(X > 6) = 0$$

4.5 $X \in \{y|y = (n - 2t), t \in \mathbb{Z}, (n - 2t) \geq 0\}$

Part 3

4.13 There are total of 5 possible variants of X 's value.

$$P(X = 0) = 0.7 \times 0.4 = 0.28$$

$$P(X = 500) = 0.5 \times (0.3 \times 0.4 + 0.7 \times 0.6) = 0.27$$

$$P(X = 1000) = 0.5 \times (0.3 \times 0.4 + 0.7 \times 0.6) + 0.5^2 \times 0.3 \times 0.6 = 0.315$$

$$P(X = 1500) = 0.3 \times 0.6(0.5^2) \times 2! = 0.09$$

$$P(X = 2000) = 0.3 \times 0.6(0.5^2) = 0.045$$

4.14

$$P(X = 4) = \frac{1}{5} = 20\%$$

$$P(X = 3) = \frac{3!}{5!} = \frac{1}{20} = 5\%$$

$$P(X = 2) = \frac{1 \times \binom{3}{2} \times 2!}{5!} + \frac{1 \times \binom{2}{2} \times 2! \times 2!}{5!} = \frac{1}{12} \approx 8.33\%$$

$$P(X = 1) = \frac{1 \times \binom{3}{1} \times 1 \times 2!}{5!} + \frac{1 \times \binom{2}{1} \binom{2}{1} \times 2!}{5!} + \frac{1 \times 1 \times \binom{3}{1} \times 2!}{5!} = \frac{1}{6} \approx 16.67\%$$

$$P(X = 0) = \frac{\binom{4}{1} \times 3!}{5!} + \frac{\binom{3}{1} \times 3!}{5!} + \frac{\binom{2}{1} \times 3!}{5!} + \frac{3!}{5!} = 0.5 = 50\%$$

Part 4

4.17

a)

$$P(X = 1) = \frac{1}{2} + \frac{0}{4} - \frac{1}{4} = \frac{1}{4}$$

$$P(X = 2) = \frac{11}{12} - \frac{1}{2} - \frac{1}{4} = \frac{1}{6}$$

$$P(X = 3) = 1 - \frac{11}{12} = \frac{1}{12}$$

b)

$$P(E) = P\left(\frac{3}{2}\right) - P\left(\frac{1}{2}\right) = \left(\frac{1}{2} + \frac{\frac{1}{2}}{4}\right) - \frac{\frac{1}{2}}{4} = \frac{1}{2}$$

4.19

$$\left\{ \begin{array}{ll} \frac{1}{2} & x = 0 \\ \frac{1}{10} & x = 1 \\ \frac{1}{5} & x = 2 \\ \frac{1}{10} & x = 3 \\ \frac{1}{10} & x = 3.5 \end{array} \right.$$