MATH 416H Lecture 3 Note

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$F = \mathbb{C}$ or $F = \mathbb{R}$ distinguish by context.

1 Linear maps

$$A = \begin{bmatrix} a_{11} & a_{12} & a_{13} & \cdots & a_{1n} \\ a_{21} & a_{22} & a_{23} & \cdots & a_{2n} \\ a_{31} & a_{32} & a_{33} & \cdots & a_{3n} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ a_{m1} & a_{m2} & a_{m3} & \cdots & a_{mn} \end{bmatrix}$$

Claim that for a linear map $L_A : F^n \to F^m$ $\exists A$ that $L_A(\overrightarrow{x}) = A\overrightarrow{x}$ prof is similar with the ones in L1 note.

2 Vector Space

2.1 Linear Combinations

Let V be a Vector space, $\{\overrightarrow{v_1}, \overrightarrow{v_2}, \cdots, \overrightarrow{v_k}\} \subseteq V$ A linear combination of $v_1, \cdots, v_k$ is a vector of the form $\overrightarrow{u} = \lambda_1\overrightarrow{v_1} + \lambda_2\overrightarrow{v_2} + \lambda_3\overrightarrow{v_3} + \cdots + \lambda_k\overrightarrow{v_k}$ for some $\lambda_i \in F$

2.1.1 Example

In $\mathbb{R}^2$, $\overrightarrow{u} = \binom{1}{3}$ is a linear combination from $\overrightarrow{v_1} = \binom{1}{1}$ and $\overrightarrow{v_2} = \binom{0}{1}$, as $\overrightarrow{u} = 1 \times \overrightarrow{v_1} + 2 \times \overrightarrow{v_2}$

2.2 Span

A set of vectors $\{\overrightarrow{v_1}, \overrightarrow{v_2}, \cdots, \overrightarrow{v_k}\} \subseteq V$ in a vector space V span V if $\forall\overrightarrow{u} \in V$, $\exists\lambda_1 \cdots \lambda_k \in F$ that $\overrightarrow{u} = \lambda_1\overrightarrow{v_1} + \lambda_2\overrightarrow{v_2} + \lambda_3\overrightarrow{v_3} + \cdots + \lambda_k\overrightarrow{v_k}$

2.2.1 Example

in $\mathbb{R}^2$, $\binom{1}{0}, \binom{1}{1}$ spans the vector space of $\mathbb{R}^2$ while $\binom{1}{0}, \binom{2}{0}$ does not.

2.3 Linear dependency

A set of vectors $\{\overrightarrow{v_1}, \overrightarrow{v_2}, \cdots, \overrightarrow{v_k}\} \subseteq V$ in vector space V is **linearly dependent** if $\exists \lambda_1, \lambda_2, \lambda_3, \cdots, \lambda_k \in F$ not all being zeros while $\lambda_1 \overrightarrow{v_1} + \lambda_2 \overrightarrow{v_2} + \cdots + \lambda_k \overrightarrow{v_k} = \overrightarrow{0}$ And if a set of vectors are not linearly dependent then it is linear independent.

2.4 Basis

Let V be a vector space over F, A set of vectors $\{\overrightarrow{v_1}, \overrightarrow{v_2}, \cdots, \overrightarrow{v_k}\} \subseteq V$ is a basis if it is linearly independent and spans V.

2.4.1 Definition

Suppose V is a vector space, $\{\overrightarrow{v_1}, \overrightarrow{v_2}, \cdots, \overrightarrow{v_k}\} \subseteq V$. $\forall v \in V$, $\exists \{\lambda_1, \lambda_2, \lambda_2, \cdots, \lambda_k\}$, $\lambda_i \in F$ that $v = \lambda_1 \overrightarrow{v_1} + \lambda_2 \overrightarrow{v_2} + \lambda_3 \overrightarrow{v_3} + \cdots + \lambda_k \overrightarrow{v_k}$ then the set of vectors is a set of basis of V

2.5 Examples

1. NO-EXAMPLE
   $\left\{ \begin{pmatrix} 1 \\ 0 \end{pmatrix}, \begin{pmatrix} 0 \\ 1 \end{pmatrix}, \begin{pmatrix} 1 \\ 1 \end{pmatrix} \right\}$ is not a set of basis for $\mathbb{R}^2$
2.