PHYS 225 HW 3

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1. a) i: $f(x) = \sum f^n(0) \times \frac{x^n}{n!}$. $f(0)' = \ln(x)\sin(x) + \frac{\cos(x)}{x}$ does not exist and limit does not exist. as $\ln(-0)$ also does not exist.

   ii: $\sin(x) = 0 + 1x + 0x^2 - \frac{1}{3!}x^3 + \cdots$
   $x = 0 + 1x + 0 + \cdots$
   $\frac{\sin(x)}{x} = 0 + 1 + 0x - \frac{1}{3!}x^2 + \cdots$

   b)

$$
\begin{aligned}
f(x) &= \frac{1}{1+x^2} \\
f(0) &= 1 \\
f'(0) &= -2x(1+x)^{-2} = 0 \\
f''(0) &= -2(1+x^2)^{-2} + 4x^2(1+x^2)^{-3} = -2 \\
f'''(0) &= 4x(1-x^2)^{-3} + 8x(1+x^2)^{-3} + 8x^3(1+x^2)^{-4} = 0
\end{aligned}
$$

   Thus, $\frac{1}{1+x^2} = 1 - \frac{2}{2!}x^2 = 1 - x^2 + \cdots$
   $\arctan(x) = \int 1 - x^2 + \cdots = x - \frac{x^3}{3} + \cdots$

   c)

$$
\begin{aligned}
e^x &= 1 + t + \frac{t^2}{2!} + \frac{t^3}{3!} + \frac{t^4}{4!} + \frac{t^5}{5!} + \cdots \\
\sin(x) &= t - \frac{t^3}{3!} \\
e^{\sin(x)} &= 1 + \left(x - \frac{x^3}{6}\right) + \frac{\left(x - \frac{x^3}{6}\right)^2}{2} + \frac{x^3}{6} + \cdots \\
&= 1 + x + \frac{x^2}{2} - \frac{x^4}{8} + \cdots
\end{aligned}
$$

2. expand $\frac{1}{\sqrt{1-k^2\sin^2(\theta)}}$ around $k=0$

$$
\begin{aligned}
f(0) &= 1 \\
f'(0) &= -\frac{1}{2}(1-k^2\sin^2(\theta))^{-\frac{3}{2}} \times 2k\sin^2(\theta) = 0 \\
f''(0) &= -2\sin^2(\theta)\frac{1}{2}(1-k^2\sin^2(\theta))^{-\frac{3}{2}} + \cdots = -\sin^2(\theta) \\
f(k) &= 1 - \frac{1}{2}k^2\sin^2(\theta) + \cdots \\
\int_0^{\pi/2} f(k)\mathrm{d}\theta &= \frac{\pi}{2} - k^2\frac{\pi}{8}
\end{aligned}
$$

Thus, it is $-\frac{\pi}{8}$

3. a)

$$
\begin{aligned}
\overrightarrow{w} &= R(\theta)\overrightarrow{v} + \overrightarrow{a} \\
\overrightarrow{u} &= R(\phi)\overrightarrow{w} + \overrightarrow{b} \\
&= R(\phi)(R(\theta)\overrightarrow{v} + \overrightarrow{a}) + \overrightarrow{b} \\
&= R(\phi)R(\theta)\overrightarrow{v} + R(\phi)\overrightarrow{a} + \overrightarrow{b}
\end{aligned}
$$

b)

$$
\begin{aligned}
\overrightarrow{w} &= R(\phi)\overrightarrow{v} + \overrightarrow{b} \\
\overrightarrow{u} &= R(\theta)\overrightarrow{w} + \overrightarrow{a} \\
&= R(\theta)(R(\phi)\overrightarrow{v} + \overrightarrow{b}) + \overrightarrow{a} \\
&= R(\theta)R(\phi)\overrightarrow{v} + R(\theta)\overrightarrow{b} + \overrightarrow{a}
\end{aligned}
$$

A different vector is rotated, it was $\overrightarrow{a}$ rotating the second time while it is now $\overrightarrow{b}$, the $\overrightarrow{v}$ is same when rotating 2 times.

c)

$$
\begin{aligned}
M_B &= \begin{bmatrix} \cos(\phi) & -\sin(\phi) & x_1 \\ \sin(\phi) & \cos(\phi) & y_1 \\ 0 & 0 & 1 \end{bmatrix} \\
M_B M_A &= \begin{bmatrix} \cos(\phi) & -\sin(\phi) & x_1 \\ \sin(\phi) & \cos(\phi) & y_1 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\theta) & -\sin(\theta) & x_0 \\ \sin(\theta) & \cos(\theta) & y_0 \\ 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} \cos\theta\cos\phi - \sin\theta\sin\phi & -\cos\theta\sin\phi - \sin\theta\cos\phi & \cos\theta x_0 - \sin\theta y_0 + x_1 \\ \sin\theta\cos\phi + \cos\theta\sin\phi & -\sin\theta\sin\phi + \cos\theta\cos\phi & \sin\theta x_0 + \cos\theta y_0 + y_1 \\ 0 & 0 & 1 \end{bmatrix} \\
M_A M_B &= \begin{bmatrix} \cos(\theta) & -\sin(\theta) & x_0 \\ \sin(\theta) & \cos(\theta) & y_0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos(\phi) & -\sin(\phi) & x_1 \\ \sin(\phi) & \cos(\phi) & y_1 \\ 0 & 0 & 1 \end{bmatrix} \\
&= \begin{bmatrix} \cos\theta\cos\phi - \sin\theta\sin\phi & -\cos\theta\sin\phi - \sin\theta\cos\phi & \cos\theta x_1 - \sin\theta y_1 + x_0 \\ \sin\theta\cos\phi + \cos\theta\sin\phi & -\sin\theta\sin\phi + \cos\theta\cos\phi & \sin\theta x_1 + \cos\theta y_1 + y_0 \\ 0 & 0 & 1 \end{bmatrix}
\end{aligned}
$$

As $M_A M_B \neq M_B M_A$ Thus the result is similar with the results above. also,it is clear that there is a different vector that is rotated.