NPRE 321: Introduction to Plasmas and their Applications

Homework 7.

Constants

$g = 9.8 \quad ms^{-2}$

$m_e = 9.11\times10^{-31} \quad kg$

u = 1.66×10-27 kg

$m_H = 1.673\times10^{-27} \quad kg$

$c = 2.998\times10^{8} \quad m\ s^{-1}$

$e = 1.602\times10^{-19} \quad C \quad or \quad J/eV$

$k_B = 1.38\times10^{-23} \quad m^2 kg\ s^{-2} K^{-1}$

$N_{AV} = 6.022\times10^{23} \quad mole^{-1}$

$\mu_0 = 4\pi\times10^{-7} \quad m\ kg\ s^{-2} A^{-2}$

$\varepsilon_0 = 8.854\times10^{-12} \quad m^{-3} kg^{-1} s^4 A^2$

H = 1.007834u

D = 2.014102u

Cu = 63.546u

n = 1.008665u

T = 3.016049u

Ar = 39.948u

^{3}***He*** = 3.016029u

^{4}***He*** = 4.002602u

$N_{tor} = 40$

$N_{hel} = 4$

D = –4.25 MeV

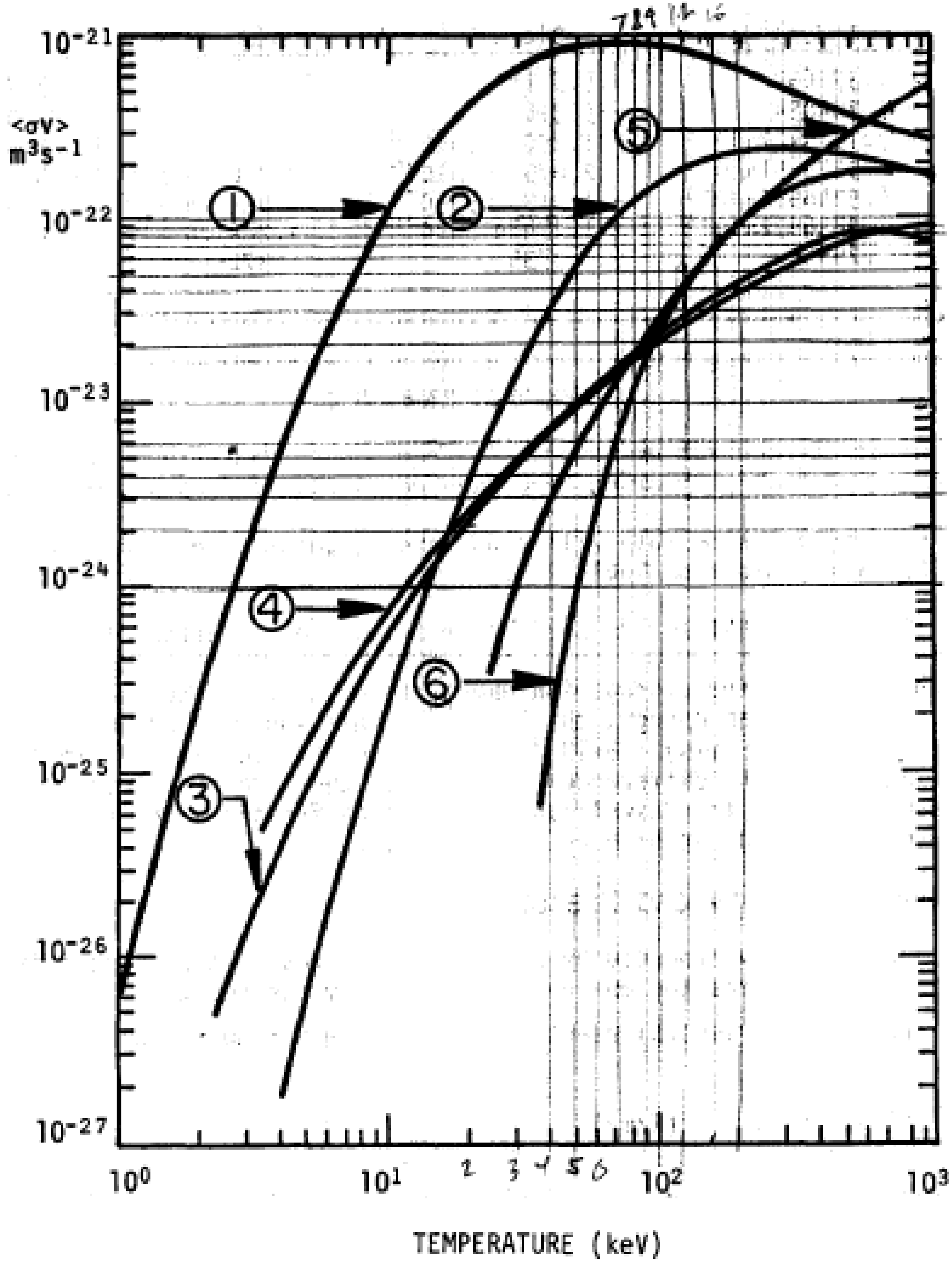
T = –0.73 MeV

^{3}***He*** = -2.57 MeV

^{4}***He*** = –7.08 MeV

1 eV = 11604 K = 1.602×10^{-19} J

T, keV	$<\sigma v>_{DT}$, m^3/s		$<\sigma v>_{DD}$, m^3/s		$<\sigma_{v>D\text{-}3He}$ m^3/s
1	5.48×10^{-27}		1.52×10^{-28}		3.02×10^{-32}
1.5	5.89×10^{-26}		1.38×10^{-27}		1.32×10^{-30}
2	2.63×10^{-25}		5.42×10^{-27}		1.42×10^{-29}
3	1.71×10^{-24}		2.95×10^{-26}		2.75×10^{-28}
4	5.58×10^{-24}		8.47×10^{-26}		1.77×10^{-27}
5	1.29×10^{-23}		1.77×10^{-25}		6.66×10^{-27}
6	2.42×10^{-23}		3.09×10^{-25}		1.83×10^{-26}
8	5.94×10^{-23}		6.90×10^{-25}		7.96×10^{-26}
10	1.09×10^{-22}		1.21×10^{-24}		2.27×10^{-25}
15	2.65×10^{-22}		2.97×10^{-24}		1.27×10^{-24}
20	4.24×10^{-22}		5.16×10^{-24}		3.79×10^{-24}
25	5.59×10^{-22}		7.60×10^{-24}		8.18×10^{-24}
30	6.65×10^{-22}		1.02×10^{-23}		1.45×10^{-23}
40	8.03×10^{-22}		1.55×10^{-23}		3.23×10^{-23}
50	8.71×10^{-22}		2.08×10^{-23}		5.44×10^{-23}



<σV>
m³s⁻¹
10⁻²¹
10⁻²²
10⁻²³
10⁻²⁴
10⁻²⁵
10⁻²⁶
10⁻²⁷
10⁰
10¹
10²
10³
TEMPERATURE (keV)
1
2
3
4
5
6

Make sure to show all working including any diagrams

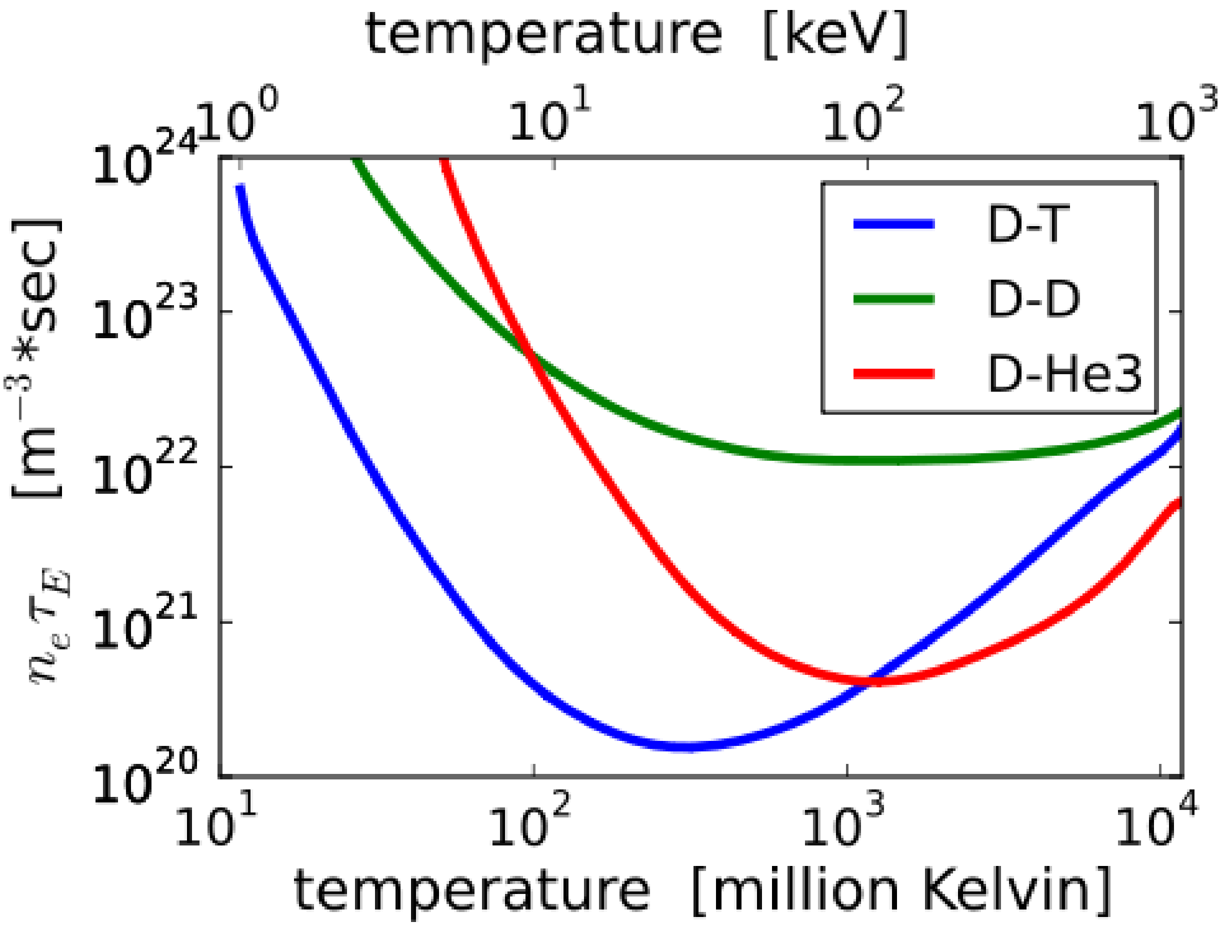
Question 1a:

Using a similar process for the DT reaction, what is the Power density (Wm^{-3}) generated by the D-D reaction and thus the formula for the power that can be generated (W)? You will need to start with the formula

$$r = n_D n_D < \sigma v >_{DT}$$

Hint: Remember that there are two just as equally likely paths.

(10 marks)



Question 1b:

Lawsons criteria is a measure of merit used in fusion to compare the rate of energy that is being produced by a system to the rate of energy lost to the surrounding environment. Using the confinement time, τ_E, formula

$$\tau_E = \frac{W}{P_{loss}}$$

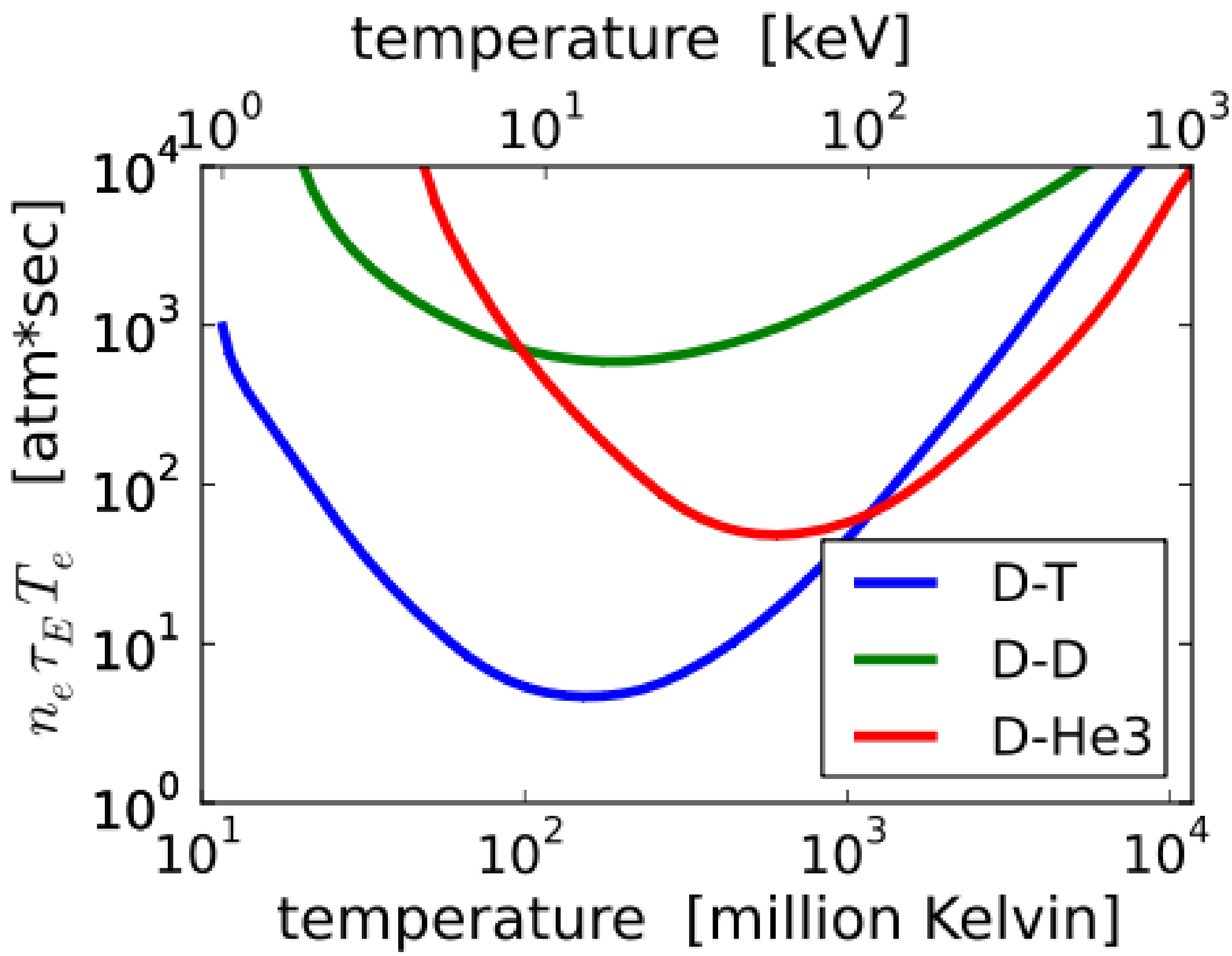
And the stored energy in the system

$$W = \frac{3}{2}(n_i T_i + n_e T_e)V$$

What is the Lawson criteria formula for the D-D reaction?

Hint: Remember that only the charged particles will heat the plasma and that the plasma is in thermal equilibrium.

(10 marks)



Question 1c:

In fact, it turns out that an even better measure of merit is the triple product, $nT\tau_E$. The minimum in the curve occurs at about 25 keV for the DD reaction. Using the formula, you derived in part b and the reaction rate parameter for DD at this temperature, what is the value of the triple product? What does this tell you about the reaction about how easy it is to achieve comparing it to the DT reaction looked at in class?

Hint: Remember that the triple product has an extra factor or T applied to it. Also remember to have the final answer in terms of $m^{-3} \cdot keV \cdot s$

(15 marks)

Question 1d:

Let's assume that a machine the size of HIDRA was able to achieve the conditions for DD fusion. The plasma dimensions and parameters are

$$R_o = 0.72 \text{ m}$$

$$a = 0.15 \text{ m}$$

$$n = 5\times10^{22} \text{ m}^{-3}$$

$$T = 25 \text{ keV}$$

What is the total fusion power output from the plasma? **What is the required confinement time** using the triple product?

Hint: You can approximate the volume of the plasma to be a straight cylinder.

(10 marks)

Question 2:

A microwave interferometer is used to measure the density in a plasma. The path length through the plasma is 20 cm and the frequency used for the interferometer is 100 GHz. A phase difference of $\Delta\varphi = 17.45$ rad is measured. ***What is the plasma density***?

(10 marks)

Question 3:

A Rogowski coil which measured the current in the plasma can be used to measure the overall ohmic heating power in a plasma. This is done by measuring the current of the plasma and then using the ohmic power law

$$P = I^2 R$$

To get the heating power of the plasma the resistivity needs to be found first and then the resistance. For a hydrogen plasma the resistivity can be simply be given by the ***Spitzer Resistivity***,

$$\eta = 5.25 \times 10^{-5} \frac{Z \ln \Lambda}{T_e^{3/2}}$$

Where Z is the atomic number for hydrogen, Te is given in eV and Λ is the ***Coulomb logarithm***,

$$\Lambda = 12\pi n_e \lambda_D^3$$

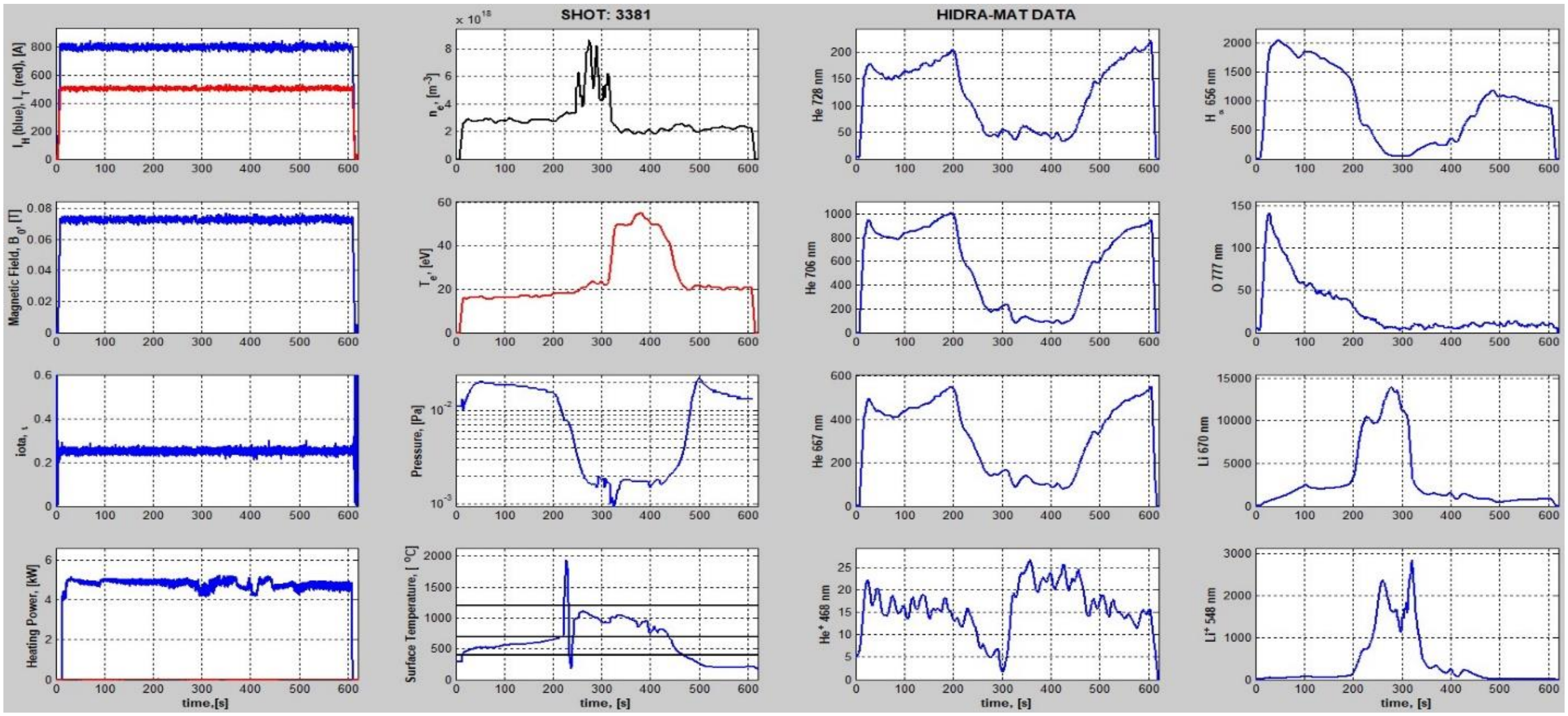
What is the plasma current power for a machine like HIDRA if a Rogowski coil measures a voltage of 3.9 V with a loop radius of 1 mm, 10,000 loops and integration time of 1 ms. The plasma density is $n_e = 1\times10^{19}$ m^{-3} and the plasma temperature is T_e = 1000 eV.

Hint: Assume that the HIDRA plasma has a minor radius of a = 0.15 m and major radius of R_0 = 0.72 m. the resistance for the plasma can be simply found via $R = \eta \frac{L_{plasma}}{A_{plasma}}$

(20 marks)

Question 4:

In lectures, I showed some data from HIDRA, with a helium plasma interacting with liquid lithium on a plasma facing surface. I did discuss this in class, but I am interested to see what you take out of this discussion and what insights you may get from the plasma material interactions that are happening here. Here is the data again.



Hint: think back about the PMI question in homework 6 and some of the things that we have looked at. There will be a further complication that not just from sputtering there will be evaporation as well since we are dealing with a liquid. Look up the specific heat of lithium and see what it tells you.

(20 marks)

Question 5:

I am curious to know where you think the future of plasma is. I have discussed some things in class, however I want you to do some research and talk about what you think plasmas can be used for.

(20 marks)