MATH 416H HW 11

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1. Consider the base: $\left\{\frac{1}{0!}x^0, \frac{1}{1!}x^1, \frac{1}{2!}x^2 \cdots, \frac{1}{n!}x^n\right\}$,

   Then, $A = [T]_{\mathcal{B}\mathcal{B}} = \begin{bmatrix} 0 & 1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & 1 & \cdots & 0 & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 0 & 1 \\ 0 & 0 & 0 & \cdots & 0 & 0 \end{bmatrix}$

   Which is in jordan normal form

2. Notice that $\begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix}$ is diagonalizable. Therefore, $USDS^{-1}U^{-1}$ is a diagonal matrix. Then, diagonalize $\begin{pmatrix} 1 & 3 \\ 3 & 1 \end{pmatrix}$. There is $\lambda_1 = 4, \lambda_2 = -2$ and $v_1 = (s, -s)^T, v_2 = (s, s)^T$ Then the equation becomes:

   $$U \begin{bmatrix} s & s \\ s & -s \end{bmatrix} \begin{bmatrix} 1 & 3 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} \frac{1}{2s} & \frac{1}{2s} \\ \frac{1}{2s} & -\frac{1}{2s} \end{bmatrix} U^*$$

   Consider the specific case where $US = I, U^*S^{-1} = I$ while setting the imaginary part to zero, we have $UU = I, US = I, US^{-1} = I$, then $S = S^{-1}$, we have $\begin{bmatrix} \frac{1}{2s} & \frac{1}{2s} \\ \frac{1}{2s} & -\frac{1}{2s} \end{bmatrix} = \begin{bmatrix} s & s \\ s & -s \end{bmatrix}$, then $s = \frac{\sqrt{2}}{2}$, then $U = S^{-1} = \begin{bmatrix} \frac{\sqrt{2}}{2} & \frac{\sqrt{2}}{2} \\ \frac{\sqrt{2}}{2} & -\frac{\sqrt{2}}{2} \end{bmatrix} = U^*$.

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