PHYS 225 HW 8

James Liu

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1. a)

$$
\begin{aligned}
\Delta E &= F\Delta x \\
(\Delta E)^2 &= F^2(\Delta x)^2 \\
E^2 &= m^2c^4 + p^2c^2 \\
F^2(\Delta x)^2 &= m^2c^4 + p^2c^2 \\
p^2c^2 &= F^2(\Delta x)^2 - m^2c^4 \\
p^2 &= \frac{F^2(\Delta x)^2 - m^2c^4}{c^2} \\
t &= p/\frac{\mathrm{d}p}{\mathrm{d}t} \\
t &= p/F \\
t &= \sqrt{\frac{F^2(\Delta x)^2 - m^2c^4}{F^2c^2}}
\end{aligned}
$$

relativistic: when $x \to \infty$, $t \to \infty$, meaning that it would take infinity amount of time to go to infinitly away, meaning that

non-relativistic: when $x \ll mc^2/F$, $t = \sqrt{\frac{m^2c^4 - F^2(\Delta x)^2}{F^2c^2}}$ which is finite and reasonable as er cahn push something to a close distance in finite amount of time.

b)

$$
\begin{aligned}
F^2 x_m^2 &= m^2c^4 + p^2c^2 \\
x_m^2 &= \frac{m^2c^4}{F^2} + \frac{p^2c^2}{F^2} \\
x_m^2 &= \frac{m^2c^4}{F^2} + t^2c^2 \\
x_m &= \sqrt{\frac{m^2c^4}{F^2} + t^2c^2} \\
x_p &= tc \\
x_m &= tc \times \frac{1}{tc}\sqrt{\frac{m^2c^4}{F^2} + t^2c^2} \\
&= tc\sqrt{\frac{m^2c^4}{t^2c^2F^2} + 1} \\
&\approx tc(1 + \frac{m^2c^4}{2t^2c^2F^2}) \\
&= tc + \frac{m^2c^4}{2tcF^2} \\
\Delta x &= \frac{m^2c^4}{2tcF^2}
\end{aligned}
$$

2. a)

$$
\begin{aligned}
E_{ini} &= E_{final} \\
M^2c^4 &= M^2c^4 + p^2c^2 + dm^2c^4 + p'^2c^2 \\
p &= \gamma_u MU \\
p' &= \gamma_w dmw \\
p + p' &= 0 \\
\frac{MdU}{\sqrt{1-U^2/c^2}} + \frac{dmw}{\sqrt{1-w^2/c^2}} &= 0 \\
dU &= \frac{dm}{M}\frac{\sqrt{1-U^2/c^2}}{\sqrt{1-w^2/c^2}} \\
\frac{M}{dm} &= \frac{\sqrt{1-U^2/c^2}}{dU\sqrt{1-w^2/c^2}} \\
\frac{dm}{M} &= \frac{dU\sqrt{1-w^2/c^2}}{\sqrt{1-U^2/c^2}} \\
\frac{dm}{M} &= -\frac{dU}{w\sqrt{1-U^2/c^2}}
\end{aligned}
$$

b)

$$
\begin{aligned}
\frac{dM}{M} &= -\frac{dU}{w\left(1-\frac{U^2}{c^2}\right)} \\
\frac{dM}{M} &= -\frac{dU}{w} \cdot \frac{1}{1-\frac{U^2}{c^2}} \\
\int_{M_0}^{M(U)} \frac{dM}{M} &= -\int_0^U \frac{dU}{w} \cdot \frac{1}{1-\frac{U^2}{c^2}} \\
\ln\left(\frac{M(U)}{M_0}\right) &= -\frac{1}{w}\int_0^U \frac{dU}{1-\frac{U^2}{c^2}} \\
z &= \frac{U}{c} \\
-\frac{1}{w}\int_0^U \frac{dU}{1-\frac{U^2}{c^2}} &= -\frac{1}{w}\int_0^z \frac{c \cdot dz}{1-z^2} \\
&= -\frac{c}{w}\int_0^z \frac{dz}{1-z^2} \\
-\frac{c}{w}\left[\frac{1}{2}\ln\left|\frac{1+z}{1-z}\right|\right]_0^z &= -\frac{c}{w}\left[\frac{1}{2}\ln\left|\frac{1+\frac{U}{c}}{1-\frac{U}{c}}\right|\right]_0^U \\
\ln\left(\frac{M(U)}{M_0}\right) &= -\frac{c}{2w}\ln\left|\frac{1+\frac{U}{c}}{1-\frac{U}{c}}\right| \\
&= \ln\left[\left(\frac{1+\frac{U}{c}}{1-\frac{U}{c}}\right)^{-\frac{c}{2w}}\right] M(U) \quad = M_0\left(\frac{1+\frac{U}{c}}{1-\frac{U}{c}}\right)^{-\frac{c}{2w}}
\end{aligned}
$$