MATH 461 Lecture 2 Note

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Edit: August 28, 2024, Class: Aug 28

Combination

notation

$\binom{n}{r} = \frac{1}{r!} \times \frac{n!}{(n-r)!}$ As $\frac{n!}{(n-r)!}$ shows howmany ways of selecting r items from n items considering sequence. Divied by $r!$ gives the total ways not considering order.

Examples

1. From a group of 5 women, 7 men, how many different committes of 5 people (2 women, 3 men) can be formed?

   $\binom{5}{2} \cdot \binom{7}{3}$

   What if 2 men do not want to stay together?

   a.
   $$\binom{5}{2} \cdot \left(\binom{5}{3} + \binom{2}{1} \cdot \binom{5}{2} \right)$$

   b.
   $$\binom{5}{2} \cdot \left(\binom{7}{3} - \binom{2}{2} \cdot \binom{5}{1} \right)$$

   a. is correct because of is choose the approch to add the special case , as $\binom{5}{3}$ is the combination number without the 2 special people. and the rest accounted for having only one of the 2 special ones chosed.
   b. is as it removes the unsatisfied ones by removing the amount similar with chossing only one from the normal men group.

2. If there are n antenna with m of them defacted, which is $m \leq n-m+1$, how many ways to arrange it that no 2 defacted antenna are sitting next to each other?

   $\binom{n-m+1}{m}$ as there are $n-m$ working antenna, there are $n-m+1$ places we can put defacting ones in between. and wee need to put in total m defacting ones in these positions.

Useful Identity

$$\binom{n}{r} = \binom{n-1}{r-1} + \binom{n-1}{r}$$

$$(x+y)^n = \sum_{k=0}^{n} \binom{n}{k} x^k y^{n-k}$$

Intuitive Prof

$$(x_1+y_1)(x_2+y_2)(x_3+y_3) \times \cdots \times (x_n+y_n)$$

$$= \underbrace{\overbrace{x_1x_2x_3\cdots x_n}^{n \text{ terms}} + y_1x_2x_3x_4\cdots x_n + \cdots + y_1y_2y_3\cdots y_n}_{2^n \text{terms}}$$

The terms that have k numbers of x_i is $\binom{n}{k}$.

Multinomial Coefficient

$$(x_1+x_2+x_3+x_4)^n = \sum \binom{n}{p_1, p_2, p_3, p_4} x_1^{p_1} x_2^{p_2} x_3^{p_3} x_4^{p_4}$$

where $\binom{n}{p_1, p_2, p_3, p_4}$ means $\binom{n}{p_1}\binom{n-p_1}{p_2}\binom{n-p_1-p_2}{p_1}\binom{n-p_1-p_2-p_3}{p_4} = \frac{n!}{p_1!p_2!p_3!p_4!}$

Examples

suppose there are n balls to be put into r boxes, and no boxes are empty, how many ways?
consider there are $n - 1$ possible divider place that gives no empty boxes:

$$\square_\wedge\square_\wedge\square_\wedge\square_\wedge\square_\wedge\square_\wedge\square_\wedge\square$$

and we need to put $r - 1$ dividers inside to divide the 8 balls into r groups. Therefore it is straight forward that th answer is $\binom{n-1}{r-1}$

what if allows empty?

we add extra r balls to be the one ball that fills every empty boxes, thus, the answer goes $\binom{n+r-1}{r-1}$