PHYS 225 HW 9

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1. a) $f(x,y,z) = x^2 + y^2 + z^2$

Cartesian:

$$\begin{aligned} f(x,y,z) &= x^2 + y^2 + z^2 \\ \nabla f &= \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right) \\ &= (2x, 2y, 2z) \end{aligned}$$

Cylindrical:

$$\begin{aligned} f(x,y,z) &= x^2 + y^2 + z^2 \\ f(r,\theta,z) &= r^2 + z^2 \\ \nabla f &= (2r, 0, 2z) \end{aligned}$$

Spherical:

$$\begin{aligned} f(x,y,z) &= x^2 + y^2 + z^2 \\ f(\rho,\theta,\phi) &= \rho^2 \\ \nabla f &= (2\rho, 0, 0) \end{aligned}$$

b) $f(x,y,z) = \sin(z)$

Cartesian:

$$\begin{aligned} f(x,y,z) &= \sin(z) \\ \nabla f &= (0, 0, \cos(z)) \end{aligned}$$

Cylindrical:

$$\begin{aligned} f(x,y,z) &= \sin(z) \\ f(r,\theta,z) &= \sin(z) \\ \nabla f &= (0, 0, \cos(z)) \end{aligned}$$

Spherical:

$$\begin{aligned} f(x,y,z) &= \sin(z) \\ f(\rho,\theta,\phi) &= \sin(\rho \times \cos(\theta)) \\ \nabla f &= (\cos(\theta)\cos(\rho\cos(\theta)), -\cos(\rho \times \cos(\theta))\rho \times \sin(\theta), 0) \end{aligned}$$

c) $f(x, y, z) = x + y + z$

Cartesian:

$$\begin{aligned} f(x, y, z) &= x + y + z \\ \nabla f &= (1, 1, 1) \end{aligned}$$

Cylindrical:

$$\begin{aligned} f(x, y, z) &= x + y + z \\ f(r, \theta, z) &= r\cos(\theta) + r\sin(\theta) + z \\ \nabla f &= (\cos(\theta) + \sin(\theta), r\cos(\theta) - r\sin(\theta), 1) \end{aligned}$$

Spherical:

$$\begin{aligned} f(x, y, z) &= x + y + z \\ f(\rho, \theta, \phi) &= \rho\sin(\theta)\cos(\phi) + \rho\sin(\theta)\sin(\phi) + \rho\cos(\theta) \\ \nabla f &= \begin{bmatrix} \sin(\theta)\cos(\phi) + \sin(\theta)\sin(\phi) + \cos(\theta) \\ \rho\cos(\theta)\cos(\phi) + \rho\cos(\theta)\sin(\phi) - \rho\sin(\theta) \\ -\rho\sin(\theta)\sin(\phi) + \rho\sin(\theta)\cos(\phi) \end{bmatrix} \end{aligned}$$

2. a) $v = x\hat{x} + y\hat{y} + z\hat{z}$

Cartesian:

$$\nabla \cdot v = 1 + 1 + 1 = 3$$

Cylindrical:

$$\begin{aligned} v &= x\hat{x} + y\hat{y} + z\hat{z} \\ v &= x\hat{\rho} + y\hat{\theta} + z\hat{z} \end{aligned}$$

Spherical:

b) $v = \hat{\rho}$

Cartesian:

Cylindrical:

c) $v = \hat{\theta}$

Cartesian:

Spherical: