NPRE200 HW 2

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1. $N_{H_2O} = \frac{\rho A}{m} = 0.036 \times 6.023 \times 10^{23} \div 18 = 1.2046 \times 10^{21}$ atom/cm^3
   $\sigma = 2 \times 38 + 4.2 \times 10^{-5} = 76.000042$ b$= 7.600042 \times 10^{-23}$ cm^2
   $\Sigma = N_{H_2O} \times \sigma = 0.09155$ cm^{-1}

2. i: Deuterium

   Average: $\xi = \frac{2}{A+2/3} = 0.75$
   $n = \frac{1}{\xi}\ln\left(\frac{E_0}{E_n}\right) = \frac{4}{3}\ln\left(\frac{1\times10^6}{1}\right) = 18.42$

   Minimum: $\xi = \ln\left(\frac{E_1}{E_2}\right) = \ln\left(\left[\frac{A+1}{A-1}\right]^2\right) = 2\ln\left(\frac{A+1}{A-1}\right) = 2.197$
   $n = \frac{1}{\xi}\ln\left(\frac{E_0}{E_n}\right) = \frac{1}{2.197}\ln\left(\frac{10^6}{1}\right) = 6.28771$
   The average collision is significantly larger than the minimum number required.

   ii: Carbon-12

   Average: $\xi = \frac{2}{A+2/3} = \frac{3}{19}$
   $n = \frac{1}{\xi}\ln\left(\frac{E_0}{E_n}\right) = \frac{19}{3}\ln\left(\frac{1\times10^6}{1}\right) = 87.4982$

   Minimum: $\xi = \ln\left(\frac{E_1}{E_2}\right) = \ln\left(\left[\frac{A+1}{A-1}\right]^2\right) = 2\ln\left(\frac{A+1}{A-1}\right) = 0.334108$
   $n = \frac{1}{\xi}\ln\left(\frac{E_0}{E_n}\right) = \frac{1}{0.334108}\ln\left(\frac{10^6}{1}\right) = 41.3504$
   The average collision is significantly larger than the minimum number required.

3. $\Sigma = N\sigma = \frac{\rho A}{m}\sigma = \frac{7.86\times6.023\times10^{23}}{55.847} \times 2.56 \times 10^{-24} = 0.217$ cm $^{-1}$

4. $$n = \frac{1}{\xi}\ln\left(\frac{E_0}{E_n}\right)$$

5. Suppose there is a equal probability for a neutron to be scattered from E to αE, then for it to land on any E' is:

   $$P(E \to E') = \frac{1}{E - \alpha E}$$

6. Atomic percent: $\dfrac{e\times 235}{(1-e)\times 238+e\times 235}=0.0072$, $e=0.007291=\frac{N_{235}}{N_{238}}$.

$$N_{235}=\int_t^{t_0}{}^0N_{235}\cdot e^{-\lambda t}dt={}^0N_{235}\times e^{-\lambda_{235}(t_0-t)}$$
$$N_{238}=\int_t^{t_0}{}^0N_{238}\cdot e^{-\lambda t}dt={}^0N_{238}\times e^{-\lambda_{238}(t_0-t)}$$

Then $k=\dfrac{{}^0N_{235}e^{-\lambda_{235}\Delta t}}{{}^0N_{238}e^{-\lambda_{238}\Delta t}+{}^0N_{235}e^{-\lambda_{235}\Delta t}}=0.007291$

$$\begin{cases}\lambda_{235}=\ln(2)/7.1\times 10^8\times 356\times 24\times 60\times 60=3.17397\times 10^{-17}\\ \lambda_{238}=\ln(2)/4.5\times 10^9\times 356\times 24\times 60\times 60=5.00783\times 10^{-18}\\ \Delta t=2\times 10^9\times 356\times 24\times 60\times 60=6.15168\times 10^{16}\text{ s}\end{cases}$$

Thus:

$$\begin{aligned}k=\frac{{}^0N_{235}\times 0.141915}{{}^0N_{235}\times 0.141915+{}^0N_{238}\times 0.734867}&=0.007291\\ 0.007291({}^0N_{235}\times 0.141915+{}^0N_{238}\times 0.734867)&={}^0N_{235}\times 0.141915\\ 0.14088{}^0N_{235}&=0.005358{}^0N_{238}\\ {}^0N_{235}&=0.038032{}^0N_{238}\\ \frac{0.038032{}^0N_{238}}{0.038032{}^0N_{238}+{}^0N_{238}}&=0.036638\end{aligned}$$