MATH 461 Lecture 2 Note

James Liu

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1 Probability

1.1 Definitions

Sample Space: A sample space is a set containing all possible out come of a experiment.

Event: A event is a subspace of the Sample space

1.2 Axioms of probabilities

E: Event set, S: Sample Space, $P(x)$: a map between a set to probability

1) $E \subseteq S$, $0 \leqslant P(E) \leqslant 1$
2) $P(S) = 1$
3) For any sequence of $E_1, E_2, E_3, \cdots, E_n$ of events that are disjoint $P(\bigcup_{i=1}^{\infty} E_i) = \sum_{i=1}^{\infty} P(E_i)$. Or, the probability of combined disjoint event equals to the sum of the probabilities when they are seperate.

Exaples

Say we measure the liftime of a light builb. Assume that $P(A) = \int_A e^{-t}\mathrm{d}t$, where the A is a time interval $[0, \infty)$ as $\int_0^{\infty} e^{-t}\mathrm{d}t = 1$, also other function P can be chosed.

1.3 properties

1. $P(\varnothing) = 0$
2. If $E_1, E_2, E_3, \cdots, E_n$ are disjoint then $P(\bigcup_{i=1}^{n} E_i) = \sum_{i=1}^{\infty} P(E_i)$
3. If $E \subseteq F$, then $P(E) \leqslant P(F)$
4. $P(E^c) = 1 - P(E)$
5. $P(\bigcup_{i=1}^{n} E_i^c) = 1 - P(\bigcup_{i=1}^{n} E_i)$