PHYS 225 HW 12

James Liu

Due:Dec 5 Edit: December 4, 2024

1. a)

$$\begin{aligned}\nabla \cdot (\nabla \times A) &= \partial_i \cdot (\epsilon^{ijk} \partial_j A_k) \\ &= \epsilon^{ijk} \partial_i \cdot (\partial_j A_k) \\ &= \partial_i \partial_j A_k - \partial_i \partial_k A_j - \partial_j \partial_i A_k + \partial_j \partial_k A_i + \partial_k \partial_j A_i - \partial_k \partial_i A_j\end{aligned}$$

b) If $\epsilon_{abc}\epsilon^{ijk} = -1$ then the total number of transpositions is also odd as if the total number is even, then it is either 2 odd which gives $-1 \times (-1) = 1$ or 2 even thich is $1 \times 1 = 1$ and none of then equals -1. similar for $\epsilon_{abc}\epsilon^{ijk} = 1$, the total transpositions will be even. Therefore, we can add the same number of transpositions to ijk and abc which will not change the oddity of the sum and permute ijk back to ijk then the set of transpositions affacting the sign would be abc therefore, we just need to permute it and set it to right sign and look up whether it is with ijk. which is what the right side of the euqality did.

c)

$$\begin{aligned}&\delta^i_a\delta^j_b\delta^k_c + \delta^i_b\delta^j_c\delta^k_a + \delta^i_c\delta^j_a\delta^k_b - \delta^i_b\delta^j_a\delta^k_c - \delta^i_a\delta^j_c\delta^k_b - \delta^i_c\delta^j_b\delta^k_a \\ =&\delta^i_i\delta^j_b\delta^k_c + \delta^i_b\delta^j_c\delta^k_i + \delta^i_c\delta^j_i\delta^k_b - \delta^i_b\delta^j_i\delta^k_c - \delta^i_i\delta^j_c\delta^k_b - \delta^i_c\delta^j_b\delta^k_i \\ =&\end{aligned}$$

as $\delta^i_i = 1$ and anything contains i and is not δ^i_i euqlas zero and takes the term with it.

d)

$$
\begin{aligned}
&\nabla \times (\nabla \times F)_i \\
=&\epsilon^{ijk}\partial_j(\epsilon_{abc}\partial^b A_c)^k \\
=&\epsilon^{ijk}\partial_j(\epsilon_{kbc}\partial^b A_c)^k \\
=&\epsilon^{kij}\partial_j(\epsilon_{kbc}\partial^b A_c)^k \\
=&(\delta^i_b\delta^j_c - \delta^i_c\delta^j_b)(\partial_j\partial^b A_c)
\end{aligned}
$$

Thus, after considering all the transpositions, there will be:

$$\nabla \times (\nabla \times F)_i = \nabla(\nabla \cdot F) - \nabla^2 F$$

2. a)

$$
\begin{aligned}
&\int_1^3 \langle t, 3t+1, 2t\rangle \cdot \langle 1, 3, 2\rangle dt \\
=&\int_1^3 14t + 3dt \\
=&62
\end{aligned}
$$

b) Parametrize x, y, z, we get $\begin{cases} x = t \\ y = 2t \\ z = 5t^2 \end{cases}$

$$
\begin{aligned}
&\int_0^2 \langle 4t^2, 25t^4, 3\rangle \cdot \langle 1, 2, 10t\rangle dt \\
=&\int_0^2 50t^4 + 4t^2 + 30tdt \\
=&16010.7
\end{aligned}
$$

c) Parametrize x, y, we get $\begin{cases} x = t^2 \\ y = \sqrt{4-t} \end{cases}$

$$
\begin{aligned}
&\int_{\sqrt{2}}^{-\sqrt{2}} \langle -\sqrt{4-t}, t^2, 0\rangle \cdot \langle 2t, -(4-t)^{-\frac{1}{2}}, 0\rangle dt \\
=&\int_{\sqrt{2}}^{-\sqrt{2}} -2t\sqrt{4-t} - \frac{t^2}{\sqrt{4-t}} dt \\
=& -1.92322
\end{aligned}
$$