MATH 461 Homework 10

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7.50

$$\int_0^\infty x^2 \cdot \frac{e^{-\frac{x}{y}} e^{-y}}{y e^{-y}} \, \mathrm{d}x = \frac{1}{y} 2y^3$$
$$= 2y^2$$

7.51

$$f(x|y) = \frac{f(x,y)}{f_Y(y)} = \frac{e^{-y}}{y} \frac{1}{e^{-y}} = \frac{1}{y} E(X^3|Y=y) \qquad = \int_0^y x^3 \frac{1}{y} = \frac{y^3}{4}$$

7.56

$$E(X) = \sum_{n=0}^\infty E(X|N=n)P(N=n)$$
$$= \sum_{n=0}^\infty N E(I_1) P(N=n)$$
$$= \sum_{n=0}^\infty N\left(1 - \left(\frac{N_1}{N}\right)^n\right) \frac{10^n}{n!} e^{-10}$$
$$= N - N e^{-\frac{10}{N}}$$

7.57

$$\mathrm{Cov}(N,X) = E(NX) - E(N)E(X) = 0$$
$$E(NX) = E(N)E(X) = 5 \times 2.5 = 12.5$$

7.75 a)

$$M_X(t) = e^{2e^t - 2}$$
$$M_Y(t) = \left(\frac{3}{4}e^t + \frac{1}{4}\right)^{10}$$
$$P(X+Y=2) = P(X=0, Y=2) + P(X=1, Y=1) + P(X=2, Y=0)$$
$$= e^{-2}(0.000386) + 2e^{-2}(0.0000286) + \frac{2^2 e^{-2}}{2!}(9.537 \times 10^{-7})$$
$$= 6.024 \times 10^{-5}$$

b)

$$P(XY = 0) = P(X = 0) + P(Y = 0) - P(X = Y = 0)$$
$$= e^{-2} + 9.5367 \times 10^{-7} - \left(e^{-2}\left(9.5367 \times 10^{-7}\right)\right)$$
$$= 0.1353$$

c)

$$E(XY) = E(X)E(Y)$$
$$= \lambda \cdot np$$
$$= 15$$

8.1   a)

$$P(X > 18) \leq \frac{E(X)}{18} = \frac{16}{18} = \frac{8}{9}$$

b)

$$P(X > 254) \leq \frac{E(X)}{25} = \frac{16}{25}$$

8.2   a)

$$\sigma^2 = \mathrm{Var}(X) = 9$$
$$P(|X_1 6| \geq 6) \leq \frac{\sigma^2}{6^2} = \frac{1}{4}$$
$$P(10 \leq X \leq 22) = 1 - P(|X_1 6| \geq 6) \geq \frac{3}{4}$$

b)

$$P(X > 16 + 2) \leq \frac{\sigma^2}{\sigma^2 + 2^2} = \frac{9}{9 + 4} = \frac{9}{13}$$