MATH 416H Lecture 1 Note

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Matrix

Definition

A $m \times n$ Matrix A is an array of numbers (integers, rationals, real, complex)

Linearity

We say a function (map) is linear when:

$$F(\lambda \overrightarrow{x} + \mu \overrightarrow{y}) = \lambda F(\overrightarrow{x}) + \mu F(\overrightarrow{y})$$

Note that linear in calculus is not nessesarily linear in linear algebra!
e.g.

$$\begin{aligned} f(x) &= 2x + 3 \\ f(x+y) &= 2(x+y) + 3 \\ f(x) + f(y) &= 2x + 3 + 2y + 3 \neq 2(x+y) + 3 \end{aligned}$$

examples

Try prof that if $\exists f(\overrightarrow{x}) : \mathbb{R}^n \to \mathbb{R}^m$ is linear, then $\exists A$ that A is a matrix and $f(\overrightarrow{x}) = A \times \overrightarrow{x}$.

Say $\overrightarrow{x}$ is a n dimensional vector, then $\overrightarrow{x} = \sum_1^n \overrightarrow{e_i} \times x_i$ where e_i are base vectors like:
$\begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \cdots \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ \vdots \\ 1 \end{bmatrix}$ and x_i are respect value at the direction of base vectors

therefore, as $f(\overrightarrow{x})$ is linear, there is:

$$\begin{aligned} f(\overrightarrow{x}) &= f\left(\sum_{1}^{n} \overrightarrow{e_i} x_i\right) \\ &= \sum_{1}^{n} x_i f(\overrightarrow{e_i}) \end{aligned}$$

thus,

$$A = \begin{bmatrix} f(\overrightarrow{e_1}) & f(\overrightarrow{e_2}) & \cdots & f(\overrightarrow{e_n}) \end{bmatrix}$$

therefore such matrix exist.