

Solving Ordinary Differential Equations (ODE) with Simulink

Faculty of Technology and Bionics

Solving ODEs in Simulink – Example 1

Problem Statement

Ordinary Differential Equation (ODE)

$$\ddot{x}(t) + \omega_0^2 \cdot x(t) = 0$$

Initial Conditions (IC)

$$\dot{x}(t = 0) = 0$$

$$x(t = 0) = 1$$

Initial Value Problem

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The IC models an undamped, free oscillation with eigenfrequency $\omega_0=1$. Initially the system is in rest $dx/dt=0$ for $t=0$, the initial position is $x=1$.

Solving ODEs in Simulink – Example 1

Step 1

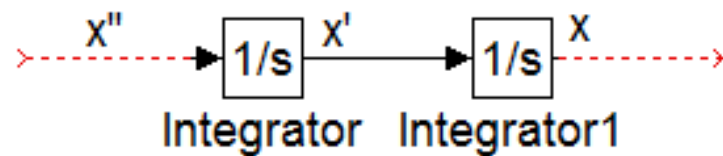
Transform the ODE into explicit form

$$\ddot{x}(t) = -\omega_0^2 \cdot x(t)$$

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Step 2

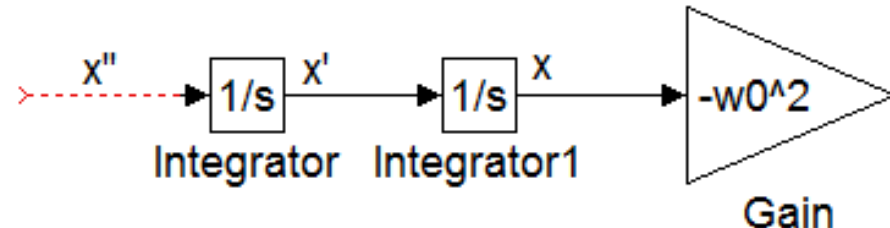
Integrate the acceleration twice



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Step 3

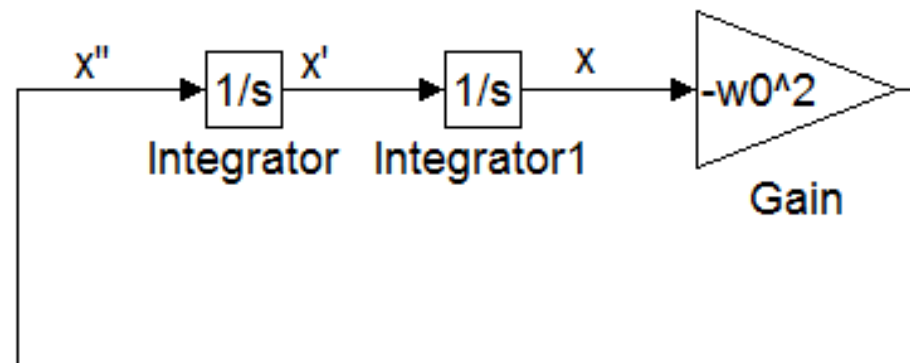
Set up the Right-hand-Side (RHS) by use of gains, summation points etc.



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Step 4

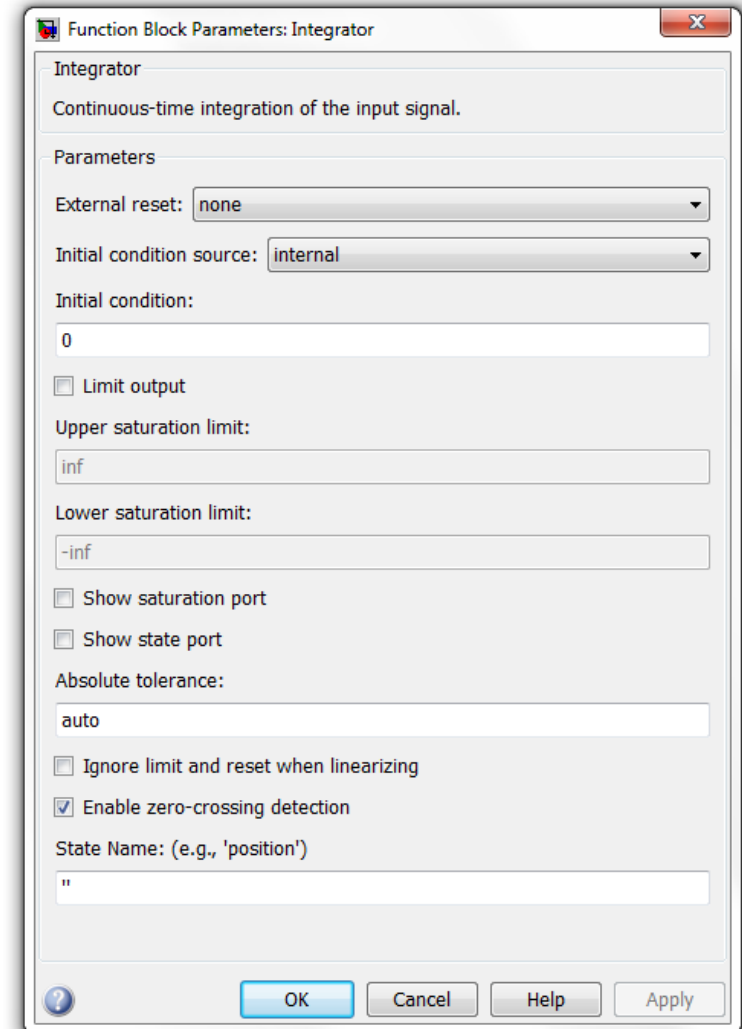
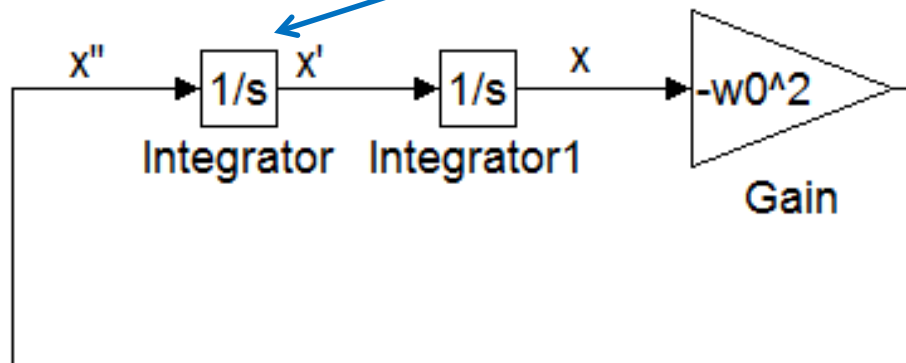
Set up the feedback loop



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Step 5

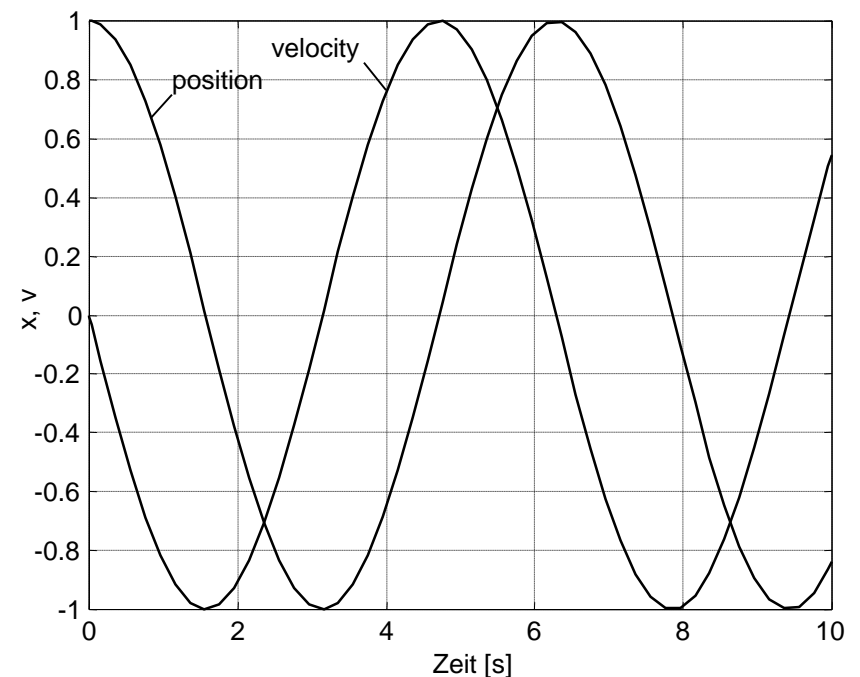
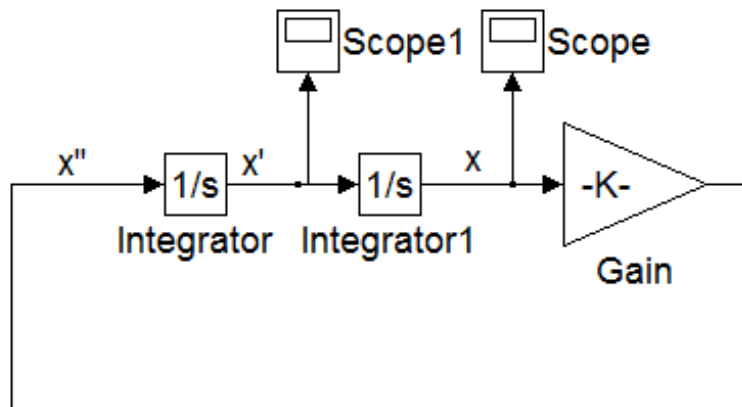
Set the Initial Condition



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Step 6

Start Simulation and analyse results



Solving ODEs in Simulink – Example 2

Solve the following equation in Simulink:

$$\ddot{x} + d \cdot \operatorname{sgn}(\dot{x}) \cdot \dot{x}^2 + \omega_0^2 \cdot x = \sin(\omega \cdot t)$$

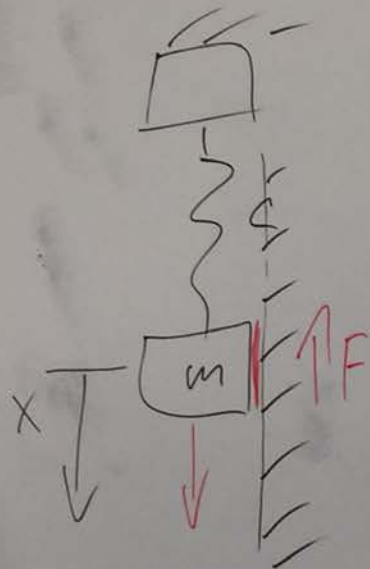
Parameter are:

$$\omega_0^2 = 5$$

$$\omega = 1$$

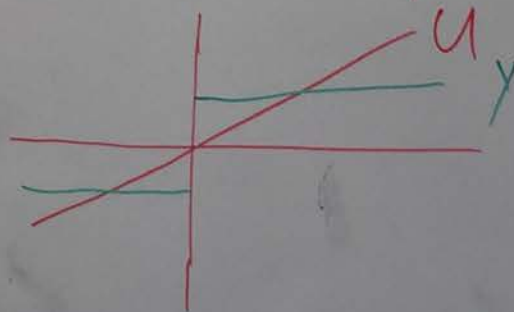
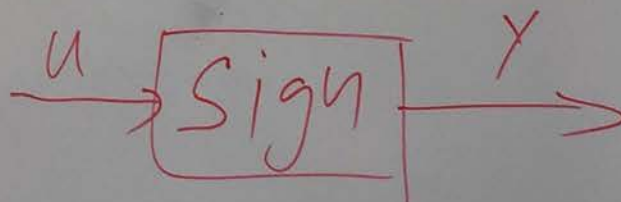
$$d = 0.4$$

Limit the maximal step-size of the integrator to $h=0.01$.



$$x(t=0) = \text{initial position}$$

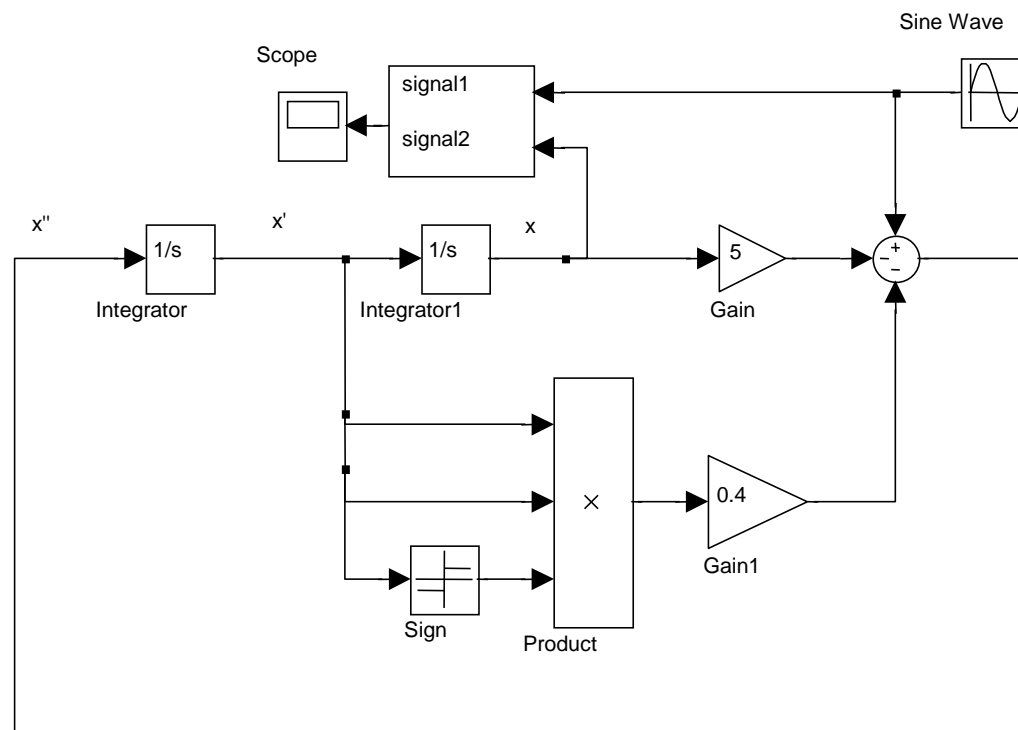
$$\dot{x}(t=0) = \text{initial velocity}$$



$$y = \begin{cases} 1, & u > 0 \\ 0, & u = 0 \\ -1, & u < 0 \end{cases}$$

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Possible block diagram



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Simulation results

