

Point-to-Point routing formulation

N :	Set of nodes
A :	Set of arcs
P :	Set of parameters
d_{ij} :	Length of arc (i, j)
v_{ij} :	Vehicle speed on arc (i, j)
t_{ij} :	Travel time on arc (i, j)
x_{ij} :	Flow on arc (i, j)
c_{ij} :	Cost of traversing arc (i, j)
r, s :	Origin, Destination
θ_p :	Cost of parameter p
η_n^p :	Coefficient of v_{ij}^n for parameter p
φ_p :	Binary variable indicating inclusion of parameter p in the generalized cost function

To begin, authors introduce the network as a directed graph - $G(N, A)$, with set of nodes N and a set of arcs $A = \{(i, j); i, j \in N\}$. A vehicle traversing through this network observes costs pertaining to travel-related parameters $p \in P$, which manifest on the arc at a rate defined by a polynomial function on arc speed, given by, $\sum_n \eta_n^p v_{ij}^n$. Thus, the arc traversal cost is,

$$c_{ij} = \sum_{p \in P} \varphi_p \theta_p t_{ij} \sum_n \eta_n^p v_{ij}^n \quad (1)$$

$$t_{ij} = d_{ij}/v_{ij} \quad (2)$$

Assuming deterministic nature of the network, authors formulate the point-to-point routing problem between origin node - r and destination node - s as a simple cost minimization problem,

$$\min_{x_{ij}; (i, j) \in A} z = \sum_{(i, j) \in A} c_{ij} x_{ij} \quad (3)$$

Subject to flow conservation,

$$\sum_{i \in T(j)} x_{ij} = \sum_{k \in H(j)} x_{jk} \quad (4)$$

$$\sum_{j \in H(r)} x_{rj} = 1 \quad (5)$$

$$\sum_{i \in T(s)} x_{is} = 1 \quad (6)$$

$$H(i) = \{k; (i, k) \in A\} \quad (7)$$

$$T(i) = \{k; (k, i) \in A\} \quad (8)$$

Equation 4 balances incoming flow with outgoing flow for node j , while **Equation 5** and **Equation 6** balance flow at the terminal nodes, i.e., origin and destination. H, T are the node successor (head) and predecessor (tail) functions, respectively (**Equation 7** and **Equation 8**). This optimization problem is solved using the Dijkstra's algorithm (25).