Location Routing Problem (LRP)

Given, a graph $G = (D, C, A)$ with set of depots D with capacity q_d, fleet V_d, operational cost π_d^o, and fixed cost π_d^f for every depot $d \in D$; set of customer nodes C with demand q_c, delivery time-window $[t_c^e, t_c^l]$ for every customer $c \in C$; set of arcs $A = \{(i,j); i,j \in N = \{D \cup C\}\}$ with length l_{ij} for every arc $(i,j) \in A$; and set of vehicles V with capacity q_v, speed s_v, operational cost π_v^o (per unit distance), and fixed cost π_v^f for every vehicle $v \in V$, the objective is to develop least cost routes from select depot nodes using select vehicles such that every customer node is visited exactly once while also accounting for depot and vehicle capacities.

$$\min z = \sum_{d \in D} \left(\pi_d^f + \pi_d^o \sum_{c \in C} z_{cd} q_c\right) y_d + \sum_{v \in V} \left(\pi_v^f + \sum_{(i,j) \in A} \pi_v^o l_{ij} x_{ij}^v\right) y_v$$

Subject to,

Service constraint,

$$\sum_{d \in D} z_{cd} = 1 \quad \forall\, c \in C$$

Flow conservation,

$$\sum_{i \in T_j} x_{ij}^v = \sum_{k \in H_j} x_{jk}^v \qquad \forall\, j \in N$$

$$q_{ic} + M\left(1 - \sum_{v \in V} x_{ic}^v\right) \geq q_{cj} + q_c \quad \forall\, i \in T_c; j \in H_c; c \in C$$

Allocation constraints,

$$z_{cd} = \sum_{v \in V_d} \sum_{j \in H_c} x_{cj}^v \quad \forall\, c \in C; d \in D$$

Resource use constraints,

$$y_v \leq \sum_{j \in H_d} x_{dj}^v \quad \forall\, v \in V_d$$

$$y_d \leq \sum_{v \in V_d} y_v \qquad \forall\, d \in D$$

Capacity constraints,

$$\sum_{c \in C} q_c \sum_{j \in H_c} x_{cj}^v \leq q_v y_v \quad \forall\, v \in V$$

$$\sum_{c \in C} q_c z_{cd} \leq q_d y_d \qquad \forall\, d \in D$$

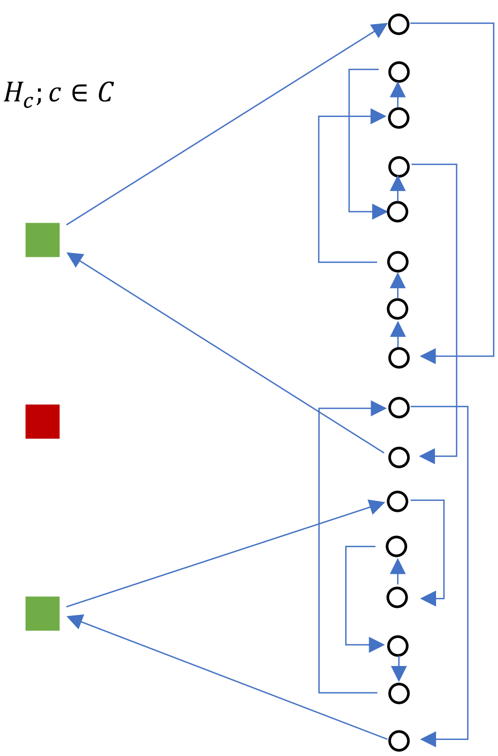

Fig. A typical LRP solution

Time-window constraints,

$$t_c^a + M(1 - x_{ic}^v) \geq t_i^d + x_{ic}^v \, l_{ic}/s_v \quad \forall\, i \in T_c; c \in C$$

$$t_c^d \geq t_c^a + \max(0, t_c^e - t_c^a) + q_c \tau_c^v \quad \forall\, c \in C$$

$$t_c^a \leq t_c^l \quad \forall\, c \in C$$

Other constraints,

$$x_{ij} \in \{0,1\} \quad \forall\, (i,j) \in A$$

$$y_v \in \{0,1\} \quad \forall\, v \in V$$

$$y_d \in \{0,1\} \quad \forall\, d \in D$$

$$z_{cd} \in \{0,1\} \quad \forall\, c \in C; d \in D$$

$$z_{cv} \in \{0,1\} \quad \forall\, c \in C; d \in D$$

Where,

$$T_j = \{i; (i,j) \in A\}$$

$$H_j = \{k; (j,k) \in A\}$$