

## Location Routing Problem (LRP)

Given, a graph  $G = (D, C, A, V)$  with set of depots  $D$  with depot fixed cost  $\pi_d$ , depot capacity  $q_d$ , and depot fleet  $V_d$  for every depot  $d \in D$ , set of customer nodes  $C$  with customer demand  $q_c$  for every customer  $c \in C$ , set of arcs  $A = \{(i, j); i, j \in N = \{D \cup C\}\}$  with arc length  $l_{ij}$  for every arc  $(i, j) \in A$ , and set of vehicles  $V$  with vehicle speed  $s_v$ , vehicle capacity  $q_v$ , vehicle fixed cost  $\pi_v$ , vehicle maintenance cost  $\pi_v^m$  per unit distance, vehicle's driver wage  $\pi_v^w$  per unit time, vehicle fuel cost  $\pi_v^f$  per unit fuel use, and vehicle fuel-to-energy efficiency  $\eta_v$  for every vehicle  $v \in V$ , the objective is to develop least cost routes from select depot nodes using select vehicles such that every customer node is visited exactly once while also accounting for depot and vehicle capacities.

$$\min z = \sum_{d \in D} \pi_d y_d + \sum_{v \in V} \pi_v y_v + \sum_{(i,j) \in A, v \in V} (\pi_v^m + \pi_v^w / s_v + \pi_v^f \eta_v q_{ij}) l_{ij} x_{ij}^v$$

Subject to,

Service constraint,

$$\sum_{d \in D} z_{cd} = 1 \quad \forall c \in C$$

Flow conservation,

$$\sum_{i \in T_j} x_{ij}^v = \sum_{k \in H_j} x_{jk}^v \quad \forall j \in N$$

$$q_{ic} + M \left( 1 - \sum_{v \in V} x_{ic}^v \right) \geq q_{cj} + q_c \quad \forall i \in T_c; j \in H_c; c \in C$$

Allocation constraints,

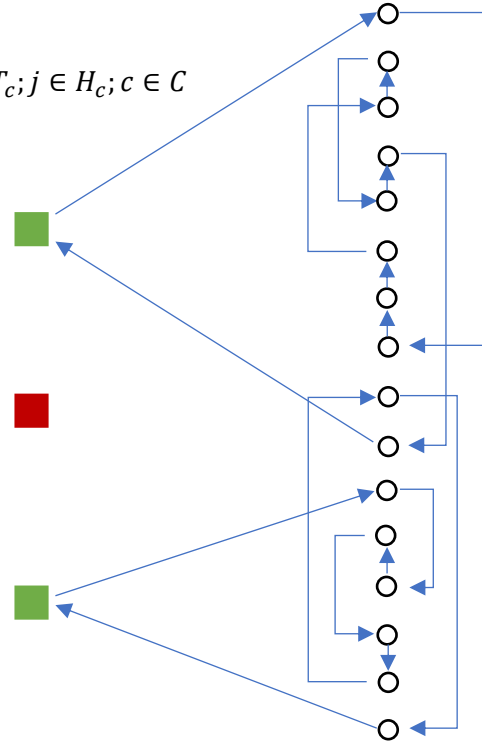
$$z_{cv} = \sum_{j \in H_c} x_{cj}^v \quad \forall c \in C; v \in V$$

$$z_{cd} = \sum_{v \in V_d} z_{cv} \quad \forall c \in C; d \in D$$

Resource use constraints,

$$y_v \leq \sum_{j \in H_d} x_{dj}^v \quad \forall v \in V_d$$

$$y_d \leq \sum_{v \in V_d} y_v \quad \forall d \in D$$



**Fig. A typical LRP solution**

Capacity constraints,

$$\sum_{c \in C} q_c z_{cv} \leq q_v y_v \quad \forall v \in V$$

$$\sum_{c \in C} q_c z_{cd} \leq q_d y_d \quad \forall d \in D$$

Other constraints,

$$x_{ij} \in \{0,1\} \quad \forall (i,j) \in A$$

$$y_v \in \{0,1\} \quad \forall v \in V$$

$$y_d \in \{0,1\} \quad \forall d \in D$$

$$z_{cd} \in \{0,1\} \quad \forall c \in C; d \in D$$

$$z_{cv} \in \{0,1\} \quad \forall c \in C; d \in D$$

Where,

$$T_j = \{i; (i,j) \in A\}$$

$$H_j = \{k; (j,k) \in A\}$$