# Location Routing Problem (LRP)

A typical LRP is defined on a directed graph $G = (D, C, A)$, where node set $D$ represents potential distribution facility nodes, node set $C$ represents customer nodes, and arc set $A$ represents the set of arcs connecting these nodes, wherein an arc connecting nodes $i$ and $j$ spans a length $l_{ij}$. Note, each distribution facility node $d \in D$ has an associated set of delivery vehicles $V_d$, capacity $q_d$, service start and end time $t_d^s$ and $t_d^e$, respectively, as well as fixed cost $\pi_d^f$, and operational cost $\pi_d^o$ per package. Further, each customer node $c \in C$ has an associated demand $q_c$, service time $\tau_c$, and must be catered within a specified time-window $[t_c^e, t_c^l]$ from a distribution facility using a delivery vehicle. These delivery vehicles have an associated set of delivery routes $R_v$ encompassing at most $r_v^{\bar{k}}$ routes, capacity $q_v$, range $l_v$, speed $s_v$, refueling time $\tau_v^f$, loading time per package $\tau_v^d$, parking time $\tau_v^c$, driver working hours $\tau_v^w$, fixed cost $\pi_v^f$, and operational costs $\pi_v^{od}$ per unit distance and $\pi_v^{ot}$ per unit time, respectively.

The objective of a LRP is to develop least cost routes from select distribution facilities using select vehicles such that every customer node is visited exactly once while also accounting for depot capacity, vehicle capacity, vehicle range, driver working-hours, and customers' time-windows.

$$min \; \Pi = \sum_{d \in D} \left( \pi_d^f + \pi_d^o \sum_{v \in V_d} \sum_{r \in R_v} \sum_{c \in C} z_{cr} q_c \right) y_d + \sum_{v \in V} \left( \pi_v^f + \sum_{r \in R_v} \sum_{(i,j) \in A} \pi_v^{od} x_{ij}^r l_{ij} + \pi_v^{ot} (t_v^e - t_v^s) \right) y_v \tag{1}$$

Subject to,

$$\sum_{r \in R} z_{cr} = 1 \qquad \forall \, c \in C \tag{2}$$

$$\sum_{j \in H_c} x_{cj}^r = z_{cr} \qquad \forall \, c \in C, r \in R \tag{3}$$

$$\sum_{i \in T_j} x_{ij}^r = \sum_{k \in H_j} x_{jk}^r \qquad \forall \, j \in N, r \in R \tag{4}$$

$$\sum_{c \in C} z_{cr} q_c \leq q_v y_v \qquad \forall \, r \in R_v, v \in V \tag{5}$$

$$\sum_{v \in V_d} \sum_{r \in R_v} \sum_{c \in C} z_{cr} q_c \leq q_d y_d \qquad \forall \, d \in D \tag{6}$$

$$t_c^a + M(1 - x_{ic}^r) \geq \begin{cases} t_r^s; & i \in D \\ t_i^d; & i \in C \end{cases} + x_{ic}^r \tau_{ic} \qquad \forall \, i \in T_c, c \in C, r \in R \tag{7}$$

$$t_c^d \geq t_c^a + \tau_v^c + max(0, t_c^e - t_c^a - \tau_v^c) + \tau_c \qquad \forall \, c \in C \tag{8}$$

$$t_c^a \leq t_c^l \qquad \forall \, c \in C \tag{9}$$

$$t_{r_v^1}^s = t_d^s \qquad \forall \, r_v \in R_v, v \in V_d \tag{10}$$

$$t_{r_v^k}^s = t_{r_v^{k-1}}^e + \tau_v^f \sum_{(i,j) \in A} \frac{x_{ij}^{r_v^k} l_{ij}}{s_v} + \tau_v^d \sum_{c \in C} z_{cr_v^k} q_c \quad \forall\, r_v^{k-1}, r_v^k \in R_v, v \in V \tag{11}$$

$$t_v^s = t_d^s \qquad\qquad\qquad\qquad\qquad \forall\, v \in V \tag{12}$$

$$t_v^e = t_{r_v^{\bar{k}_v}}^e \qquad\qquad\qquad\qquad\qquad \forall\, v \in V \tag{13}$$

$$t_v^e \leq min(t_v^s + w_v, t_d^e) \qquad\qquad\qquad \forall\, v \in V_d, d \in D \tag{14}$$

$$\sum_{r \in R_v} \sum_{(i,j) \in A} x_{ij}^r l_{ij} \leq l_v \qquad\qquad\qquad \forall\, v \in V \tag{15}$$

$$x_{ij}^r \in \{0,1\} \qquad\qquad\qquad\qquad \forall\, (i,j) \in A, r \in R \tag{16}$$

$$y_v \in \{0,1\} \qquad\qquad\qquad\qquad\qquad \forall\, v \in V \tag{17}$$

$$y_d \in \{0,1\} \qquad\qquad\qquad\qquad\qquad \forall\, d \in D \tag{18}$$

$$z_{cr} \in \{0,1\} \qquad\qquad\qquad\qquad \forall\, c \in C; r \in R \tag{19}$$