Location Routing Problem (LRP)

Given, a graph $G = (D, C, A)$ with set of depot nodes D with capacity q_d, operational cost π_d^o, fixed cost π_d^f, and fleet of delivery vehicles V_d with capacity q_v, speed s_v, refueling time τ_v^f, service time at the depot node τ_v^d (per unit demand), service time at the customer node τ_v^c, operational cost π_v^o (per unit distance traveled), fixed cost π_v^f, working hours w_v for every vehicle $v \in V_d$ for every depot $d \in D$; set of customer nodes C with demand q_c, delivery time-window $[t_c^e, t_c^l]$ for every customer $c \in C$; set of arcs A with length l_{ij} for every arc $(i,j) \in A$, the objective is to develop least cost routes from select depot nodes using select vehicles such that every customer node is visited exactly once while also accounting for vehicle capacities.

$$\min z = \sum_{d\in D}\left(\pi_d^f + \pi_d^o \sum_{v\in V_d}\sum_{r\in R_v} z_{cr} q_c\right) y_d + \sum_{v\in V}\left(\pi_v^f + \sum_{(i,j)\in A} \pi_v^o l_{ij} x_{ij}^v\right) y_v$$

Subject to,

Service constraint,

$$\sum_{d\in D}\sum_{v\in V_d}\sum_{r\in R_v} z_{cr} = 1 \qquad \forall\, c \in C$$

Flow conservation,

$$\sum_{i\in T_j} x_{ij}^r = \sum_{k\in H_j} x_{jk}^r \qquad \forall\, j \in \{D \cup C\}; r \in R_v; v \in V_d; d \in D$$

Allocation constraints,

$$z_{cr} = \sum_{v\in V_d}\sum_{j\in H_c} x_{cj}^r \qquad \forall\, c \in C; r \in R_v; v \in V_d; d \in D$$

Resource use constraints,

$$y_v \le \sum_{r\in R_v}\sum_{j\in H_d} x_{dj}^r \qquad \forall\, v \in V_d; d \in D$$

$$y_d \le \sum_{v\in V_d} y_v \qquad \forall\, d \in D$$

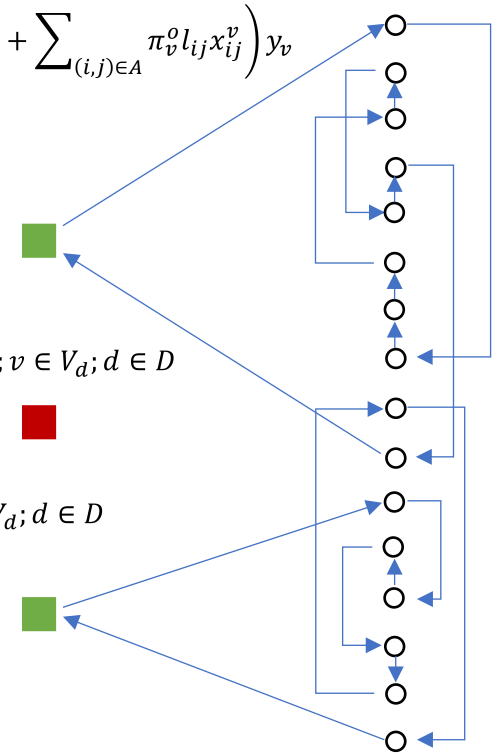

Fig. A typical LRP solution

Route constraints,

$$l_r = \sum_{i\in T_j} x_{ij}^r\, l_{ij} \qquad \forall\, j \in \{D \cup C\}; r \in R_v; v \in V_d; d \in D$$

$$q_r = \sum_{c\in C} z_{cr}\, q_c \qquad \forall\, c \in C; r \in R_v; v \in V_d; d \in D$$

Capacity constraints,

$$\sum\nolimits_{c \in C} q_c z_{cr} \leq q_v y_v \qquad \forall\, v \in V_d; d \in D$$

$$\sum\nolimits_{c \in C} q_c \sum\nolimits_{v \in V_d} \sum\nolimits_{r \in R_v} z_{cr} \leq q_d y_d \; \forall\, d \in D$$

Vehicle range constraint,

$$\sum\nolimits_{d \in D} \sum\nolimits_{v \in V_d} \sum\nolimits_{r \in R_v} l_r \leq l_v \qquad \forall\, v \in V_d; d \in D$$

Route start and end time constraint,

$$t_{r_1}^{s} = 0 \qquad \forall\, v \in V_d; d \in D$$

$$t_{r_k}^{s} = t_{r_{k-1}}^{e} + \tau_v^{f} l_r / l_v + \tau_v^{d} q_r \qquad \forall\, r \in R_v; \in V_d; d \in D$$

Driver working hours constraint,

$$t_{r_K}^{e} \leq w_v \qquad \forall\, v \in V_d; d \in D$$

Customer node arrival and departure time constraint,

$$t_c^{a} + M(1 - x_{ic}^{r}) \geq t_i^{d} + x_{ic}^{r}\, l_{ic}/s_v \qquad \forall\, i \in T_c; c \in C; r \in R_v; v \in V_d; d \in D$$

$$t_c^{d} \geq t_c^{a} + \max(0, t_c^{e} - t_c^{a}) + \tau_c^{v} \qquad \forall\, c \in C$$

Time-window constraint,

$$t_c^{a} \leq t_c^{l} \qquad \forall\, c \in C$$

Binary constraints,

Arc use,

$$x_{ij} \in \{0,1\} \qquad \forall\, (i,j) \in A$$

Vehicle use,

$$y_v \in \{0,1\} \qquad \forall\, v \in V_d; d \in D$$

Depot use,

$$y_d \in \{0,1\} \qquad \forall\, d \in D$$

Customer node allocation,

$$z_{cr} \in \{0,1\} \qquad \forall\, c \in C; r \in R_v; v \in V_d; d \in D$$

Where,

$$T_j = \{i; (i,j) \in A\}$$

$$H_j = \{k; (j,k) \in A\}$$

$$R_v = \{r_1, r_2, \dots, r_k, \dots, r_K\} \qquad \forall\, v \in V_d; d \in D$$