Location Routing Problem (LRP)

A typical LRP is defined on a directed graph $G = (D, C, A)$, where node set D represents potential distribution facility nodes, node set C represents customer nodes, and arc set A represents the set of arcs connecting these nodes, wherein an arc connecting nodes i and j spans a length l_{ij}. Note, each distribution facility node $d \in D$ has an associated set of delivery vehicles V_d, capacity q_d, service start and end time t_d^s and t_d^e, respectively, as well as fixed cost π_d^f, and operational cost π_d^o per package. Further, each customer node $c \in C$ has an associated demand q_c, service time τ_c, and must be catered within a specified time-window $[t_c^e, t_c^l]$ from a distribution facility using a delivery vehicle. These delivery vehicles have an associated set of delivery routes R_v encompassing at most $r_v^{\bar{k}}$ routes, capacity q_v, range l_v, speed s_v, re-fueling time τ_v^f, loading time per package τ_v^d, parking time τ_v^c, driver working hours τ_v^w, fixed cost π_v^f, and operational costs π_v^{od} per unit distance and π_v^{ot} per unit time, respectively.

The objective of a LRP is to develop least cost routes from select distribution facilities using select vehicles such that every customer node is visited exactly once while also accounting for depot capacity, vehicle capacity, vehicle range, driver working-hours, and customers' time-windows.

$$min\ \Pi = \sum_{d \in D} \left(\pi_d^f + \pi_d^o \sum_{v \in V_d} \sum_{r \in R_v} \sum_{c \in C} z_{cr} q_c \right) y_d + \sum_{v \in V} \left(\pi_v^f + \sum_{r \in R_v} \sum_{(i,j) \in A} \pi_v^{od} x_{ij}^r l_{ij} + \pi_v^{ot} (t_v^e - t_v^s) \right) y_v \tag{1}$$

Subject to,

$$\sum_{r \in R} z_{cr} = 1 \qquad \forall\, c \in C \tag{2}$$

$$\sum_{j \in H_c} x_{cj}^r = z_{cr} \qquad \forall\, c \in C, r \in R \tag{3}$$

$$\sum_{i \in T_j} x_{ij}^r = \sum_{k \in H_j} x_{jk}^r \qquad \forall\, j \in N, r \in R \tag{4}$$

$$\sum_{c \in C} z_{cr} q_c \leq q_v y_v \qquad \forall\, r \in R_v, v \in V \tag{5}$$

$$\sum_{v \in V_d} \sum_{r \in R_v} \sum_{c \in C} z_{cr} q_c \leq q_d y_d \qquad \forall\, d \in D \tag{6}$$

$$t_c^a + M(1 - x_{dc}^r) \geq x_{dc}^r \left(t_r^s + \frac{l_{dc}}{s_v} \right) \qquad \forall\, c \in C, r \in R_v, v \in V_d, d \in D \tag{7}$$

$$t_c^a + M(1 - x_{ic}^r) \geq x_{ic}^r \left(t_i^d + \frac{l_{ic}}{s_v} \right) \qquad \forall\, i \in T_c, c \in C, r \in R_v, v \in V \tag{8}$$

$$t_c^d \geq t_c^a + \tau_v^c + max(0, t_c^e - t_c^a - \tau_v^c) + \tau_c \qquad \forall\, c \in C \tag{9}$$

$$t_{r_v^1}^s = t_d^s \qquad \forall\, r_v \in R_v, v \in V_d, d \in D \tag{10}$$

$$t^{s}_{r^{k}_{v}} = t^{e}_{r^{k-1}_{v}} + \tau^{d}_{v} \sum_{c \in C} z_{cr^{k}_{v}} \, q_c \qquad \forall \, r^{k-1}_{v}, r^{k}_{v} \in R_v, v \in V \qquad (11)$$

$$t^{e}_{r} + M(1 - x^{r}_{cd}) \geq x^{r}_{cd} \left(t^{d}_{c} + \frac{l_{cd}}{s_v} \right) \qquad \forall \, r^{k-1}_{v}, r^{k}_{v} \in R_v, v \in V \qquad (12)$$

$$t^{s}_{v} = t^{s}_{d} \qquad \forall \, v \in V \qquad (13)$$

$$t^{e}_{v} = t^{e}_{r^{\bar{k}_v}_{v}} \qquad \forall \, v \in V \qquad (14)$$

$$t^{a}_{c} \leq t^{l}_{c} \qquad \forall \, c \in C \qquad (15)$$

$$t^{e}_{v} \leq min(t^{s}_{v} + w_v, t^{e}_{d}) \qquad \forall \, v \in V_d, d \in D \qquad (16)$$

$$\sum_{r \in R_v} \sum_{(i,j) \in A} x^{r}_{ij} \, l_{ij} \leq l_v \qquad \forall \, v \in V \qquad (17)$$

$$x^{r}_{ij} \in \{0,1\} \qquad \forall \, (i,j) \in A, r \in R \qquad (18)$$

$$y_v \in \{0,1\} \qquad \forall \, v \in V \qquad (19)$$

$$y_d \in \{0,1\} \qquad \forall \, d \in D \qquad (20)$$

$$z_{cr} \in \{0,1\} \qquad \forall \, c \in C; r \in R \qquad (21)$$