Location Routing Problem (LRP)

Given, a graph $G = (D, C, A, V)$ with set of depots D with depot fixed cost π_d, depot capacity q_d, and depot fleet V_d for every depot $d \in D$, set of customer nodes C with customer demand q_c for every customer $c \in C$, set of arcs $A = \{(i,j); i,j \in N = \{D \cup C\}\}$ with arc length l_{ij} for every arc $(i,j) \in A$, and set of vehicles V with vehicle speed s_v, vehicle capacity q_v, vehicle fixed cost π_v, vehicle maintenance cost π_v^m per unit distance, vehicle's driver wage π_v^w per unit time, vehicle fuel cost π_v^f per unit fuel use, and vehicle fuel-to-energy efficiency η_v for every vehicle $v \in V$, the objective is to develop least cost routes from select depot nodes using select vehicles such that every customer node is visited exactly once while also accounting for depot and vehicle capacities.

$$\min z = \sum_{d \in D} \pi_d y_d + \sum_{v \in V} \pi_v y_v + \sum_{\substack{(i,j) \in A; \\ v \in V}} \left(\pi_v^m + \pi_v^w / s_v + \pi_v^f \eta_v q_{ij}\right) l_{ij} x_{ij}^v$$

Subject to,

Service constraint,

$$\sum_{d \in D} z_{cd} = 1 \quad \forall\, c \in C$$

Flow conservation,

$$\sum_{i \in T_j} x_{ij}^v = \sum_{k \in H_j} x_{jk}^v \qquad \forall\, j \in N$$

$$q_{ic} + M\left(1 - \sum_{v \in V} x_{ic}^v\right) \geq q_{cj} + q_c \quad \forall i \in T_c; j \in H_c; c \in C$$

Allocation constraints,

$$z_{cv} = \sum_{j \in H_c} x_{cj}^v \quad \forall\, c \in C; v \in V$$

$$z_{cd} = \sum_{v \in V_d} z_{cv} \quad \forall\, c \in C; d \in D$$

Resource use constraints,

$$y_v \leq \sum_{j \in H_d} x_{dj}^v \quad \forall\, v \in V_d$$

$$y_d \leq \sum_{v \in V_d} y_v \quad \forall\, d \in D$$

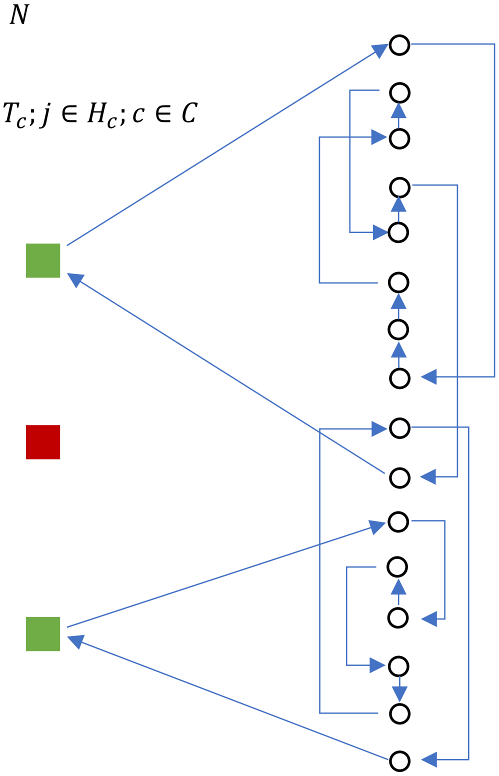

Fig. A typical LRP solution

Capacity constraints,

$$\sum\nolimits_{c \in C} q_c z_{cv} \leq q_v y_v \quad \forall\, v \in V$$

$$\sum\nolimits_{c \in C} q_c z_{cd} \leq q_d y_d \quad \forall\, d \in D$$

Other constraints,

$$x_{ij} \ \in \{0,1\} \ \ \forall\, (i,j) \in A$$

$$y_v \ \in \{0,1\} \ \ \forall\, v \in V$$

$$y_d \ \in \{0,1\} \ \ \forall\, d \in D$$

$$z_{cd} \in \{0,1\} \ \ \forall\, c \in C; d \in D$$

$$z_{cv} \ \in \{0,1\} \ \ \forall\, c \in C; d \in D$$

Where,

$$T_j = \{i; (i,j) \in A\}$$

$$H_j = \{k; (j,k) \in A\}$$