Traveling Salesman Problem (TSP)

Given a graph $G = (N, A)$ with set of nodes N and set of arcs $A = \{(i,j); i, j \in N\}$ with arc traversal cost $c_{ij}; (i,j) \in A$, the objective is to develop a least cost route visiting every node exactly once.

$$\min_{x_{ij} \in \{0,1\}} z = \sum_{(i,j) \in A} c_{ij} x_{ij}$$

Subject to,

Service constraint,

$$\sum_{i \in T_j} x_{ij} = 1 \quad \forall j \in N$$

Flow constraint,

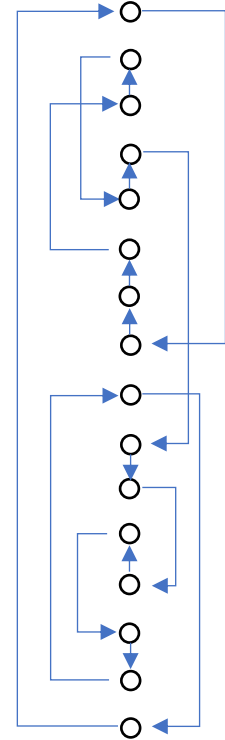$$\sum_{i \in T_j} x_{ij} = \sum_{k \in H_j} x_{jk} \quad \forall j \in N$$

Sub-tour elimination constraint,

$$\sum_{(i,j) \in S} x_{ij} \leq |S| - 1 \quad \forall S \subset N, |S| \geq 2$$

Where,

$$T_j = \{i; (i,j) \in A\}$$

$$H_j = \{k; (j,k) \in A\}$$

Fig. A typical TSP solution