Problem Set 9

MAT237: Advanced Calculus
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Problem 0.1. *Find the* 3^{rd} *order Taylor polynomial of the following functions:*

(a) $f(x, y) = \sin x \cos y$ *based on the point* $(0, 0)$

(b) $f(x, y) = \frac{1}{1+x-y}$ *based on the point* $(0, 0)$

(c) $f(x, y) = \log(1 + x - y)$ *based on the point* $(0, 0)$

(d) $f(x, y) = x + \cos(\pi y) + x \log y$ *based on the point* $(3, 1)$

(e) $f(x, y, z) = x^2 y + z$ *based on the point* $(1, 2, 1)$*. Why should the remainder be zero?*

(f) $f(x, y) = \cos x^2 + xy^2 - \frac{1}{1-xy}$ *based on the point* $(0, 0)$*.*

Problem 0.2. *Let* $f, g : \mathbb{R}^n \to \mathbb{R}$ *be* C^∞ *functions, and let* $T(f), T(g)$ *be the Taylor series of* f *and* g *respectively. Show that* $T(f + g) = T(f) + T(g)$*.*

Problem 0.3. *Let* $p : \mathbb{R}^n \to \mathbb{R}$ *be a multi-variate polynomial. If* $T(p, a)$ *is the Taylor series of* p *about the point* $a \in \mathbb{R}$*, show that* $T(p, a) = p$*.*

Problem 0.4. *Derive the following version of the following "product rule" for partial derivatives; if* α *is any multi-index, then:*

$$\partial^\alpha (fg) = \sum_{\beta+\gamma=\alpha} \frac{\alpha!}{\beta!\gamma!} \partial^\beta f \partial^\gamma g$$