Problem Set 15

MAT237: Advanced Calculus
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Problem 0.1. *Let $Z_i \subseteq \mathbb{R}^n, i = 1, \ldots, n$ be a collection of zero Jordan measure sets. Show that $\cup Z_i$ has also Jordan measure zero.*

Solution. Let $\mathcal{Z} = \cup_{0 \leq i \leq k} \mathcal{Z}_i$. We are trying to show there exists a function $M : \mathcal{P}(\mathbb{R}) \to \mathbb{R}$ such that:

$$M(\mathcal{Z}) = inf\{ \sum_{k=1}^{n} l(I_k) : I_k \text{ is an interval such that } \mathcal{Z} \subseteq \cup_{k=1}^{n} I_k \}$$

Consider $I_1, \ldots, I_n$ where $I_1 \subseteq Z_1, I_2 \subseteq Z_2, \ldots, I_n \subseteq Z_n$. Since Z_k for $1 \leq k \leq n$ is a Jordan Measure Zero set, there exists a subinterval such that $I_k \subseteq \cup I_{k_i}$ where $inf\{\sum_{i=1}^{h} I_{k_i}\} = 0$. Thus, it follows by the linear property of $infimum$ that $inf\{\sum_{k=1}^{n} l(I_k)\} = 0$. This implies, $M(\mathcal{Z})$ has Jordan Measure Zero. ■