```
1 def RecPowerTwo(n):
    # Pre:  n is a natural number
3   # Post: Returns 2^n.
    if n == 0:
5       result = 1
    else:
7       result =  2 * RecPowerTwo(n-1)
    return result
```

p2–RecPowerTwo.py

Theorem 1. *Assuming the precondition holds before the execution of the program, RecPowerTwo will terminate and the postcondition holds.*

Proof:

First, we will prove the following predicate: For all $i \in \mathbb{N}$, let $P(i)$ be the predicate:

$$P(i): \qquad \text{The function call RecPowerTwo(i) returns } 2^i$$

Base Case: i = 0. We fulfill the condition on line 4 since i = 0 so, by line 5, RecPowerTwo puts the value of result at 1 and then exits. Since $2^0 = 1$, we are done. Assume that P(i) holds. We want to show P(i + 1) holds. Assume RecPowerTwo(i + 1) is called. Since $i \in N, i + 1 > 0$ holds so the condition on line 4 is not fulfilled. Thus the program skips to line 6 where the function call RecPowerTwo(i) is called. By our induction hypothesis, P(i) holds and so RecPowerTwo(i) returns 2^i. Thus, we have:

$$\text{result } = 2 * 2^i$$

and so result= 2^{i+1}. Thus, the program terminates and returns 2^{i+1} as wanted. Therefore, P(i+1) holds as wanted.

The predicate shows the program terminates and the value which it returns satisfies the postcondition so we are done.