A2 — A2-1 and 2-2 combined

local structure tensor

A^TA

$\downarrow$

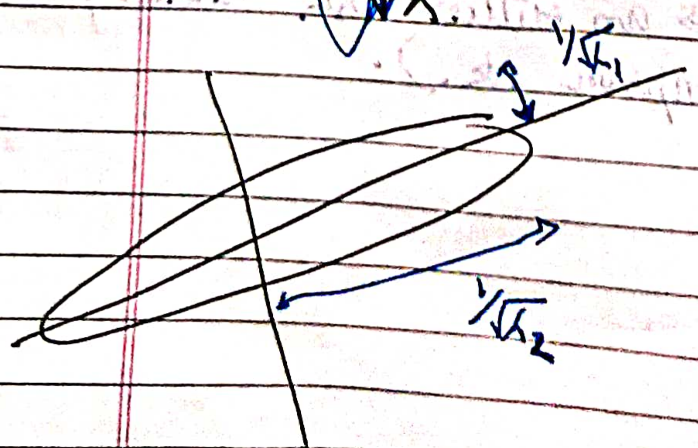
$$\begin{bmatrix} \sum_W I_x I_x & \sum_W I_x I_y \\ \sum_W I_x I_y & \sum_W I_y I_y \end{bmatrix}$$

$\downarrow$

We know that rank of any square matrix is equal to it's number of non-zero eigenvalues.

Let the eigenvalues for A^TA be (λ_1, λ_2). $[\lambda_1 > \lambda_2]$

I had proved in my midterm submission that the gradient steepness in the directions of the semi-major and semi-minor axis for the Harris matrix is $\propto \sqrt{\lambda}$.



Case 1: Rank = 0 (Smooth region)

In this case, both eigenvalues are zero.

This can happen only if $\sum I_{xx} = 0$, $\sum I_{yy} = 0$

and $\sum I_{xy} = 0$

⇒ gradients in both 'x' and 'y' direction at each point in the window is zero

⇒ surface is planar and ~~~~ has constant intensity across the pixels

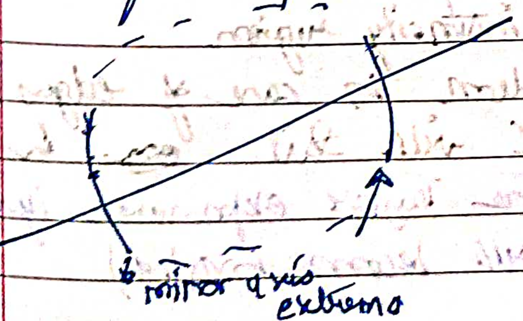
⇒ it will be very difficult to track motion in any direction.

Case 2: Rank = 1 (edge-like)

This means $\lambda_1 \neq 0$, $\lambda_2 = 0$.

Mathematically, this can happen only when image gradients for all pixels in the image are parallel ie, $I_y = m I_x \ \forall (x,y) \in W$.

Geometrically, this means that the length of ~~~~ the major axis of ellipse is infinite



⇒ however large steps we take along the direction of semi-major axis, intensity will remain the same.

This is synonymous to the pixel being in an edge like region.

~~~~ For a small window, this leads to the aperture problem.
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Case 3: Rank = 2

Even though the rank of a matrix is 2, it is possible that the eigenvalues are small i.e., length of semi-minor and semi-major axis is huge i.e., movement along either direction will hardly lead to any ~~gradient~~ intensity drop and the problems encountered are similar to the case of a flat / edge region.

As a result, to bound the length of one of the ellipse axis, ~~one of~~ the eigenvalues need to be sufficiently large for $\frac{1}{\sqrt{\lambda}}$ to be sufficiently small. This is enforced by keeping 'τ' as a threshold. (corner like behaviour).

Answering Q4 →

as I described above, in a smooth image with constant intensity across pixels, using different window sizes and sigma won't change the rank of the matrix and hence, problems which I described earlier will still persist.

∴ ① Fails on a smooth constant intensity region

② Fails due to the aperture problem in case of edges

③ Non-iterative version of LK will fail ~~as~~ in case of large motions (as the Taylor expansion ignoring of higher order terms will become invalid.)

④ As we saw in the case of Lambertian ball (answered in sec 1.1), the optical flow != actual motion due to the symmetrical nature of the ball and the changing brightness due to the moving light.