troop of why a Productes based SVD alignment gives
the best aligning transform between two point clouds with
Known correspondences.

And $P = \{\vec{r}_1, \vec{r}_2, \dots, \vec{r}_n\} \rightarrow \text{Reint cloud.} \}$ $Q = \{\vec{r}_1, \vec{r}_2, \dots, \vec{r}_n\} \rightarrow \text{Reint cloud.} \}$ $\text{cach point belongs to } R^d.$

Ain: To find a rigid transfouration that optimally aligns The 2 nets in least squares source is e, we neek a rotation R and a translation 't' such that:-

(Rot) = resident augmin SwillRp: +t) - 9:112

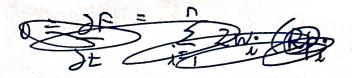
tend

where wi >0 for each point point point.

Assume R is Brid.

At F(t) = 2 will Rp: +t-9:112.

we can find the applicable translation by taking the derivative of F with respect to "t" and searching for voots as:



Nong 12 d And ofte

$$||R|_{i}^{2}+t-q_{i}^{2}||^{2} = (p_{i}^{T}R^{T}+t^{T}-q_{i}^{T})(Rp_{i}+t-q_{i}^{T})$$

$$= p_{i}^{T}R^{T}Rp_{i} + (p_{i}^{T}R^{T}t) - p_{i}^{T}R^{T}q$$

$$+ (t^{T}Rp_{i}) + (t^{T}t) - (t^{T}q_{i})$$

$$-q_{i}^{T}Rp_{i} - (q_{i}^{T}t) + q_{i}^{T}q_{i}$$

only the circled toms are defendent on 't'

$$\frac{\partial F}{\partial t} = 2R \left(\sum_{i=1}^{n} w_{i} p_{i} \right) + 2t \left(\sum_{i=1}^{n} w_{i} \right) - 2 \stackrel{?}{\leq} w_{i} p_{i} = 0$$

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The second second second second

For a fixed irotation mention R',

The most optimal eTo is $\overline{q} - R\overline{p}$

Without loss of generality, we can arrive both posit clouds around neive combroids is $\chi_{i} := p_{i} - \bar{p} \qquad | \qquad y_{i} := q_{i} - \bar{q} \qquad | \qquad |$ i. The problem can now be stold as:- $R = \underset{R \in (SOGL)}{\operatorname{asymin}} = \underset{i=1}{\overset{\sim}{\sum}} |W_{i}|^{2} |R(\bar{p}+z_{i}) + \bar{q}-R\bar{p}| - (\bar{q}+y_{i})|^{2}$ Now = RiTRT-yiT) (Rxi-yi) [RRT=I] = xitzi + yiyi - xiktyi = -yikzi ->/Rxi-jil2= xiz: +yiji)- 2yiRzi Indyendent of R Noting B and B, R= originin (- \(\frac{2}{2}\) w.y. TR \(\frac{1}{2}\). RESULA) WY RX:

Lit W= diag(w,, ..., wn) - D man diagramal multiere with Wi as do singular sonly i 7 = den nature with you as its columno X = den matrix with x; us its columns Then, Now, tr(WYTRX) = { wigiTRX; Using 5,6,9 R= angmax tr(WYTRX)

RESOLD R= organize tr (RXWYT) Let us denote died covariance nature Thing SVD of (S= USVT ormugoral

wing of ma (9) VTRUE ormogenal as R= arymat (RUEVT) => R = ENGMAX (EVTRU) Let VTRU= m be ormogenal nature All entire of the first Since @ M = ormogenal = columns of M we ormonormal rector > Each /Mig/ </ using OOO R= angmax (tr(Em)) \Rightarrow $R = \operatorname{argmax} \left(\begin{array}{c} \sigma_1 \\ \sigma_2 \\ \vdots \\ \sigma_d \end{array} \right) \left(\begin{array}{c} \sigma_1 \\ \sigma_2 \\ \vdots \\ \sigma_d \end{array} \right)$ $=)_{R=mymax}\left(\sum_{i=1}^{d}\sigma_{i}m_{ii}\right)\leq\sum_{i=1}^{d}\sigma_{i}^{2}\sum_{i=1}^{d}\omega_{i}m_{ii}$ -Jm = I => VTRV = I => | R=VUT orphogonal maling

Nong ormogonal matrices include both rotation + explactions.

If det (NUT) = +1, men There is no reflection.

IF LUT(VUT) = -1

=) we want det(17) = -1

The want om such that det (m) = -1

and (ou with the to a wind) is workinged.

The net of all diagonal of rotation mattuces of only on is equal to:

Conver rule of the points (II, II, ..., II) with an even

number of coordinates which one c-1?

Now, for det(m) = -1, we want odd number of -13.

Since we event to noveinize and of is smallest singular value, we want old to be -1.

 $\frac{1}{2} = \left(\frac{1}{2} \right)$

Hone, This EVD bard soln via the products argument leads to least squared over.