FIT5097- ASSIGNMENT 1

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30535808 || BIM

1. 1(a)

   Linear Programming Model

   (i) Set Decision Variables:

Decision Variables	For?
X_1	Production Quantity for Product 1
X_2	Production Quantity for Product 2
X_3	Production Quantity for Product 3
X_4	Production Quantity for Product 4
X_5	Production Quantity for Product 5

   (ii) Set Objective function:

   So, our problem is a profit maximization problem. Hence, the objective function is a maximization function expressed in terms of decision variables.

   Maximize: $510X_1 + 300X_2 + 510X_3 + 270X_4 + 810X_5$

   (iii) Constraints: **Resource Constraint:**

   - $2x_1 + 10x_2 + 2x_3 + 3x_4 + 6x_5 <= 2487$ (**Resource 1**): The coefficients of the decision variables are the quantities of resource 1 used for that product.

   Similarly,

   - $6x_1 + 3x_2 + 6x_3 + 3x_4 + 10x_5 <= 3030$ (**Resource 2**)
   - $2x_1 + 3x_2 + 10x_3 + 6x_4 + 2x_5 <= 5217$ (**Resource 3**)
   - $7x_1 + 6x_2 + 5x_3 + 4x_4 + 3x_5 <= 4000$ (**Resource 4**)
   - $5x_1 + 6x_2 + 3x_3 + 10x_4 + 2x_5 <= 4999$ (**Resource 5**)
   - $10x_1 + 3x_2 + 5x_3 + 3x_4 + 4x_5 <= 2769$ (**Resource 6**)

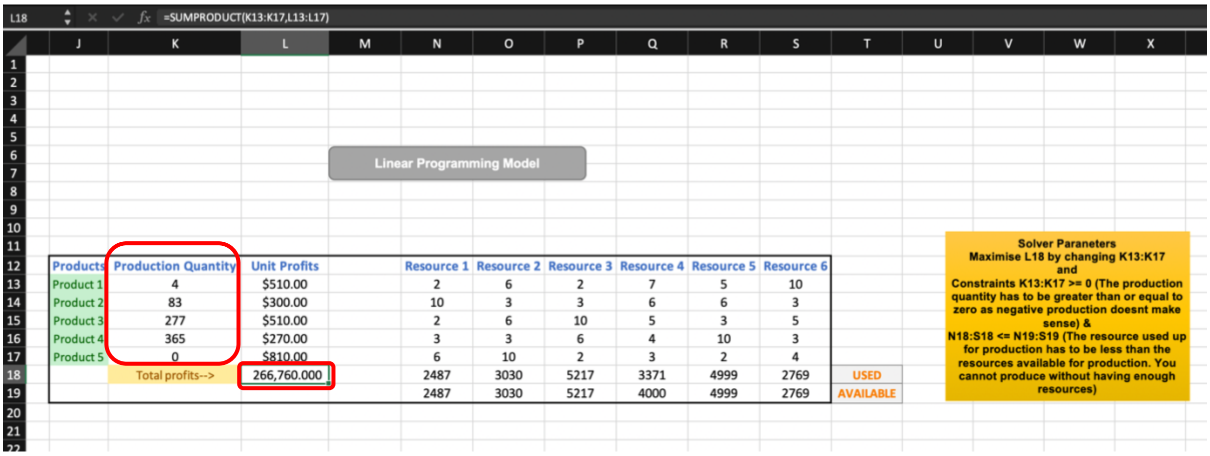
   Constraints: **Non-Negativity Constraints:** The values of production of products cannot be a negative number.

   - $X_1>0$
   - $X_2>0$
   - $X_3>0$
   - $X_4>0$
   - $X_5>0$

1(b)

Done on worksheet '1(b)'

The solver runs the solution by maximizing cell L18(Profit). The value in L18 is the sum product of the unit profits and the production numbers (Column K and L). The solver changes cells K13:K17(Production numbers) to come to a solution which maximizes the profits keeping the constraints in mind. The problem has 2 constraints which are fed in the solver. Both their reasoning are mentioned in the excel sheet.



L18 =SUMPRODUCT(K13:K17,L13:L17)

Linear Programming Model

Products	Production Quantity	Unit Profits	Resource 1	Resource 2	Resource 3	Resource 4	Resource 5	Resource 6	
Product 1	4	$510.00	2	6	2	7	5	10	
Product 2	83	$300.00	10	3	3	6	6	3	
Product 3	277	$510.00	2	6	10	5	3	5	
Product 4	365	$270.00	3	3	6	4	10	3	
Product 5	0	$810.00	6	10	2	3	2	4	
	Total profits-->	266,760.000	2487	3030	5217	3371	4999	2769	USED
			2487	3030	5217	4000	4999	2769	AVAILABLE

Solver Paraneters
Maximise L18 by changing K13:K17
and
Constraints K13:K17 >= 0 (The production quantity has to be greater than or equal to zero as negative production doesnt make sense) &
N18:S18 <= N19:S19 (The resource used up for production has to be less than the resources available for production. You cannot produce without having enough resources)

1(c)

Done on worksheet '1(c)'

Explanation: The sensitivity report gives us some additional information about our model and the data it gives in the result.
Some of the column explanations:
Shadow Price: That is the value of marginal increase in profit with increase in production by exactly 1.
For instance, in cell E18, the shadow price is 4.717. This means that product 1 is produced 1 more than usual, then it will add 4.717 dollars to the profit value.

I24

Microsoft Excel 16.0 Sensitivity Report
Worksheet: [Book1.xlsx]1(b)
Report Created: 10/18/2020 6:52:20 PM

Variable Cells

Cell	Name	Final Value	Reduced Cost	Objective Coefficient	Allowable Increase	Allowable Decrease
K13	Product 1 Production Numbers	4	0	510	1.333333333	27.69230769
K14	Product 2 Production Numbers	83	0	300	22.5	38.14285714
K15	Product 3 Production Numbers	277	0	510	8.780487805	1.487603306
K16	Product 4 Production Numbers	365	0	270	48.39622642	5
K17	Product 5 Production Numbers	0	-48.33922261	810	48.33922261	1E+30

Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
N18	Total profits--> Resource 1	2487	4.717314488	2487	105.3023256	3.39647E-12
O18	Total profits--> Resource 2	3030	82.84452297	3030	2.98944E-13	20.00589102
P18	Total profits--> Resource 3	5217	0.159010601	5217	33.54074074	9.09495E-13
Q18	Total profits--> Resource 4	3371	0	4000	1E+30	629
R18	Total profits--> Resource 5	4999	0.636042403	4999	3.23213E-12	174.1538462
S18	Total profits--> Resource 6	2769	0	2769	1E+30	5.31074E-13

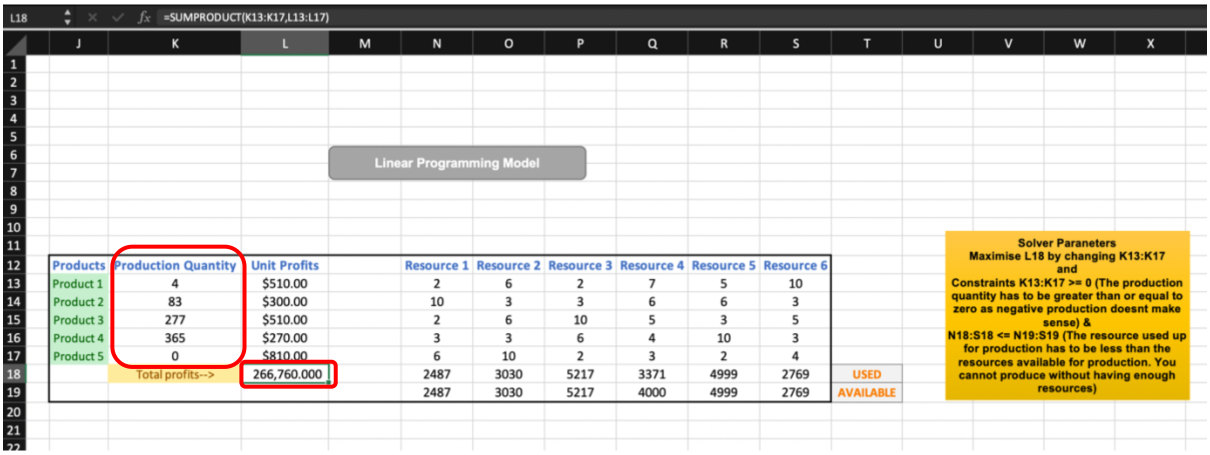
Allowable increase or decrease: The value is the maximum increase in production without shifting the optimal solution from its place.
For example, in the same example from E18, the allowable increase is mentioned in cell G18. This means that till the point 105 more product 1 are produced, it will add to the profit in a positive way, but if exceeded then it disturbs the optimal solution.

1(d)

From the excel worksheet named '1(b)', the optimal plan for production is as follows:

- $X_1 = 4$
- $X_2 = 83$
- $X_3 = 277$
- $X_4 = 365$
- $X_5 = 0$

Associated profits with the production plan are given from the result of the solver. It can be seen in cell L18 of worksheet '1(b)'. The solver gives an optimum output considering the constraints. The solver is set to maximize cell L18, which is the sumproduct of the profit column and the production quantity.



=SUMPRODUCT(K13:K17,L13:L17)

Linear Programming Model

Products	Production Quantity	Unit Profits	Resource 1	Resource 2	Resource 3	Resource 4	Resource 5	Resource 6	
Product 1	4	$510.00	2	6	2	7	5	10	
Product 2	83	$300.00	10	3	3	6	6	3	
Product 3	277	$510.00	2	6	10	5	3	5	
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	Total profits-->	266,760.000	2487	3030	5217	3371	4999	2769	USED
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Solver Paraneters
Maximise L18 by changing K13:K17
and
Constraints K13:K17 >= 0 (The production quantity has to be greater than or equal to zero as negative production doesnt make sense) &
N18:S18 <= N19:S19 (The resource used up for production has to be less than the resources available for production. You cannot produce without having enough resources)

On the other hand, it can also be calculated by manually doing the calculations as shown below.

Now, take the objective function

- $510X_1 + 300X_2 + 510X_3 + 270X_4 + 810X_5$

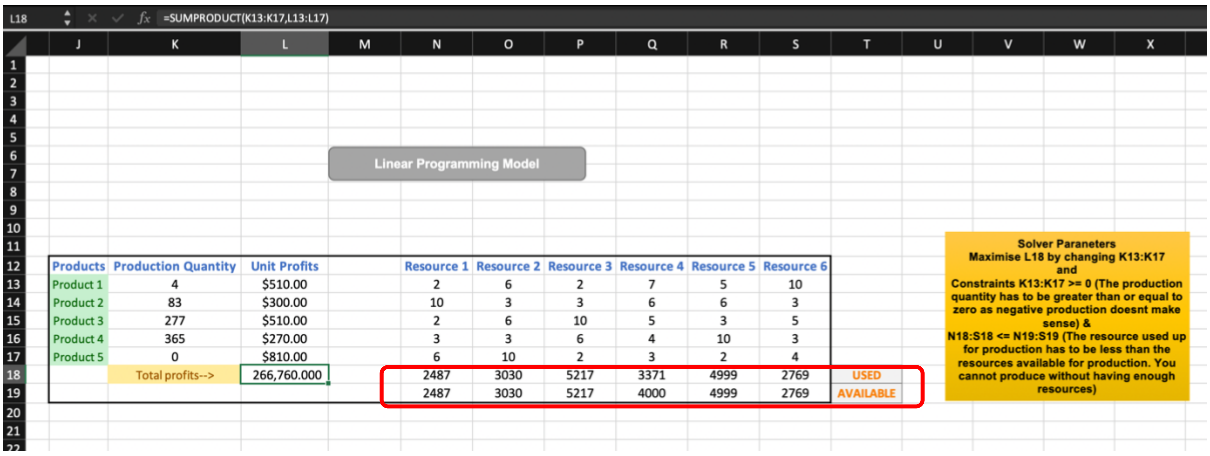
Substitute values of X_1, X_2, X_3, X_4, X_5 as 4, 83, 277, 365 and 0 respectively,

We get,

$510*4 + 300*84 + 510*277 + 270*365 + 810*0$
= $2,66,760/-

1(e)

Binding Constraints: In worksheet '1(b)', Binding constraints are constraints in which the 'Used' values(Cells N18:S18) of the resources are equal to the 'Available' values (Cells N19:S19).



=SUMPRODUCT(K13:K17,L13:L17)

Linear Programming Model

Products	Production Quantity	Unit Profits	Resource 1	Resource 2	Resource 3	Resource 4	Resource 5	Resource 6	
Product 1	4	$510.00	2	6	2	7	5	10	
Product 2	83	$300.00	10	3	3	6	6	3	
Product 3	277	$510.00	2	6	10	5	3	5	
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Solver Paraneters
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and
Constraints K13:K17 >= 0 (The production quantity has to be greater than or equal to zero as negative production doesnt make sense) &
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Also, in worksheet '1(c)', it can be seen in the 'final values' in cells D18:D23 are either equal or not equal to the 'Constraint R.H Side' in cells F18:F23. The ones that are equal are binding and others aren't.

Microsoft Excel 16.0 Sensitivity Report
Worksheet: [Book1.xlsx]1(b)
Report Created: 10/18/2020 6:52:20 PM

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S18	Total profits--> Resource 6	2769	0	2769	1E+30	5.31074E-13

Hence, the binding constraints are as follows:

1. Resource 1 Constraint
2. Resource 2 Constraint
3. Resource 3 Constraint
4. Resource 5 Constraint
5. Resource 6 Constraint

1(f)

After making the necessary changes in the resource constraint table, we get the following available resources.

RESOURCE	AVAILABLE QUANTITY
Resource 1	2477
Resource 2	3031
Resource 3	5212
Resource 4	4010
Resource 5	5099
Resource 6	2766

The changes to the resources are outside the allowable range for resources 1,2,3, and 6. For instance, in worksheet '1(c)' in cell G19, the allowable increase is 2.98944E-13 which is a very small number between 0 and 1. For the same resource 2, the new scenario adds 1 of resource 2 which is greater than the allowable range.

Microsoft Excel 16.0 Sensitivity Report
Worksheet: [Book1.xlsx]1(b)
Report Created: 10/18/2020 6:52:20 PM

Variable Cells

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K15	Product 3 Production Numbers	277	0	510	8.780487805	1.487603306
K16	Product 4 Production Numbers	365	0	270	48.39622642	5
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Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
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S18	Total profits--> Resource 6	2769	0	2769	1E+30	5.31074E-13

Hence a new worksheet named '1(f)' is created which notes these changes to the constraint and re-runs the solver.

The answer given for profit by the solver is $266,759.032/-, Which is mentioned in cell L18 of worksheet '1(f)'.
The profit in the previous scenario was = $2,66,760/-.
Therefore, taking the difference of the two value gives us

266759.032 – 266760 = -0.9679

Linear Programming Model

Products	Production Quantity	Unit Profits	Resource 1	Resource 2	Resource 3	Resource 4	Resource 5	Resource 6	
Product 1	3.88172043	$510.00	2	6	2	7	5	10	
Product 2	83.37634409	$300.00	10	3	3	6	6	3	
Product 3	280.5268817	$510.00	2	6	10	5	3	5	
Product 4	358.1397849	$270.00	3	3	6	4	10	3	
Product 5	0	$810.00	6	10	2	3	2	4	
	Total profits-->	266,759.032	2477	3031	5212	3362.6237	4942.6452	2766	USED
			2477	3031	5212	4010	5099	2766	AVAILABLE

Solver Paraneters
Maximise L18 by changing K13:K17
and
Constraints K13:K17 >= 0 (The production quantity has to be greater than or equal to zero as negative production doesnt make sense) &
N18:S18 <= N19:S19 (The resource used up for production has to be less than the resources available for production. You cannot produce without having enough resources)

Hence, in the new scenario, the company incurs a loss of 0.9679. Therefore, the offer will result in a loss if it were to be accepted. And hence should be rejected.

1(g)

For production of product 6, the resources needed are as follows:

RESOURCES	REQUIRED QUANTITY
Resource 2	2
Resource 4	4
Resource 5	5

Given: Profits associated with product 6 = $155
To come to a decision for this problem, we need to calculate the reduced cost.
The formula to be used: Profit – Σ(Shadow Price * Units of resources required)

For the above formula, the shadow prices for required resources are stated below, from the worksheet '1(c)'

RESOURCES	SHADOW PRICE
Resource 2	82.844
Resource 4	0
Resource 5	0.636

Putting these values in the Formula for reduced cost,
We get,

Reduced cost = 155 – (82.844*2 + 0*4 + 0.636*5)
= 155 – (165.688 + 0 + 3.18)
= 155 – 168.868
Reduced Cost = -13.868

The reduced cost is negative hence, we do not recommend the company to produce product 6. This is because a Negative reduced cost means it adds on to the cost and in turn, decreases the profit.

Reduced cost also gives us insight into how much profit needs to be increased for it to be considered under the optimal solution

Hence, the profit required = Old profit + |Reduced Cost|
= 155 + 13.868
Required Profit = 168.868

Therefore, this means that if the profit for product 6 is marginally greater than 168.868, it would be included in a changed optimal solution setting.

1(h)

PRODUCTS	OLD PROFIT	NEW PROFIT	CHANGE IN PROFIT(ΔC_j)
Product 1	510	512	2
Product 2	300	301	1
Product 3	510	511	1
Product 4	270	269	-1
Product 5	810	811	1

***ΔC_j = New Profit – Old Profit**

To verify if there is a change in the optimal solution, we will apply 100% Rule

In 100% rule, we calculate a value r_j by the formula mentioned below

$$\Sigma r_j = \begin{cases} \Delta C_j / I_j, \text{ If } \Delta C_j >= 0 \\ \Delta C_j / D_j, \text{ If } \Delta C_j <= 0 \end{cases}$$

Where I_j = Allowable Increase
& D_j = Allowable Decrease
*When ΔC_j is negative, we take the D_j value from the sensitivity report and when its positive, we take the I_j value.

In worksheet '1(c)', in the sensitivity report, the associated I_J and D_j values taken from cell G9:G13.

Microsoft Excel 16.0 Sensitivity Report
Worksheet: [Book1.xlsx]1(b)
Report Created: 10/18/2020 6:52:20 PM

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Constraints

Cell	Name	Final Value	Shadow Price	Constraint R.H. Side	Allowable Increase	Allowable Decrease
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R18	Total profits--> Resource 5	4999	0.636042403	4999	3.23213E-12	174.1538462
S18	Total profits--> Resource 6	2769	0	2769	1E+30	5.31074E-13

Therefore, $\Sigma r_j = 2/1.333 + 1/22.5 + 1/8.78 -(-1)/5 + 1/48.33$
$= 1.5 + 0.0444 + 0.11 + 0.2 + 0.0206$
$\Sigma r_j = 1.875$

Now, since $\Sigma r_j > 1$, we can conclude that the current optimal solution might still remain optimal. But since it cannot be guaranteed, we need to re-run the solver which is in worksheet '1(h)'.

Linear Programming Model

Products	Production Quantity	Unit Profits	Resource 1	Resource 2	Resource 3	Resource 4	Resource 5	Resource 6	
Product 1	4	$512.00	2	6	2	7	5	10	
Product 2	83	$301.00	10	3	3	6	6	3	
Product 3	277	$511.00	2	6	10	5	3	5	
Product 4	365	$269.00	3	3	6	4	10	3	
Product 5	0	$811.00	6	10	2	3	2	4	
	Total profits-->	266,763.000	2487	3030	5217	3371	4999	2769	USED
			2487	3030	5217	4000	4999	2769	AVAILABLE

Solver Paraneters
Maximise L18 by changing K13:K17
and
Constraints K13:K17 >= 0 (The production quantity has to be greater than or equal to zero as negative production doesnt make sense) &
N18:S18 <= N19:S19 (The resource used up for production has to be less than the resources available for production. You cannot produce without having enough resources)

From the worksheet it is clear that the production quantity is the same (4, 83, 277, 365, and 0) as mentioned in worksheet '1(b)' and hence we conclude that the optimal solution hasn't changed.

1(i)

PRODUCTS	OLD PROFIT	NEW PROFIT	CHANGE IN PROFIT(ΔC_j)
Product 1	510	1020	510
Product 2	300	600	300
Product 3	510	1020	510
Product 4	270	540	270
Product 5	810	1620	810

***ΔC_j = New Profit – Old Profit**

To verify if there is a change in the optimal solution, we will apply 100% Rule

In 100% rule, we calculate a value r_j by the formula mentioned below

$$\Sigma r_j = \begin{cases} \Delta C_j / I_j, \text{ If } \Delta C_j >= 0 \\ \Delta C_j / D_j, \text{ If } \Delta C_j <= 0 \end{cases}$$

Where I_j = Allowable Increase
& D_j = Allowable Decrease
*When $\mathbf{\Delta C_j}$ is negative, we take the D_j value from the sensitivity report and when its positive, we take the I_j value.

In worksheet '1(c)', in the sensitivity report, the associated I_J and D_j values taken from cell G9:G13.

I24

Microsoft Excel 16.0 Sensitivity Report
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Variable Cells

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Therefore, $\Sigma r_j = 510/1.333 + 300/22.5 + 510/8.78 + 270/48.39 + 810/48.339$
$= 383.45 + 13.333 + 58.086 + 5.579 + 16.79$

$\Sigma r_j = 477.195$

Now, since $\Sigma r_j > 1$, we can again conclude that the current optimal solution might still remain optimal. But since it cannot be guaranteed, we need to re-run the solver which is in worksheet '1(i)'.

Linear Programming Model

Products	Production Quantity	Unit Profits	Resource 1	Resource 2	Resource 3	Resource 4	Resource 5	Resource 6	
Product 1	4	$1,020.00	2	6	2	7	5	10	
Product 2	83	$600.00	10	3	3	6	6	3	
Product 3	277	$1,020.00	2	6	10	5	3	5	
Product 4	365	$540.00	3	3	6	4	10	3	
Product 5	0	$1,620.00	6	10	2	3	2	4	
	Total profits-->	533,520.000	2487	3030	5217	3371	4999	2769	USED
			2487	3030	5217	4000	4999	2769	AVAILABLE

Solver Paraneters
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N18:S18 <= N19:S19 (The resource used up for production has to be less than the resources available for production. You cannot produce without having enough resources)

From the worksheet it is clear that the production quantity is the same (4, 83, 277, 365, and 0) as mentioned in worksheet '1(b)' and hence we conclude that the optimal solution hasn't changed.

1(j)

PRODUCTS	OLD PROFIT	NEW PROFIT	CHANGE IN PROFIT(ΔC_j)
Product 1	510	255	-255
Product 2	300	150	-150
Product 3	510	255	-255
Product 4	270	135	-135
Product 5	810	405	-405

***ΔC_j = New Profit – Old Profit**

To verify if there is a change in the optimal solution, we will apply 100% Rule

In 100% rule, we calculate a value r_j by the formula mentioned below

$$\Sigma r_j = \begin{cases} \Delta C_j / I_j, & \text{If } \Delta C_j >= 0 \\ \Delta C_j / D_j, & \text{If } \Delta C_j <= 0 \end{cases}$$

Where I_j = Allowable Increase
& D_j = Allowable Decrease
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Therefore, $\Sigma r_j = -(-255)/27.69 + -(-150)/38.14 + -(-255)/1.487 + -(-135)/5 + -(-405)/\infty$
$= 9.209 + 3.93 + 171.48 + 27 + 0$
$\Sigma r_j = 211.619$

Now, since $\Sigma r_j > 1$, we can again conclude that the current optimal solution might still remain optimal. But since it cannot be guaranteed, we need to re-run the solver which is in worksheet '1(j)'.

Linear Programming Model

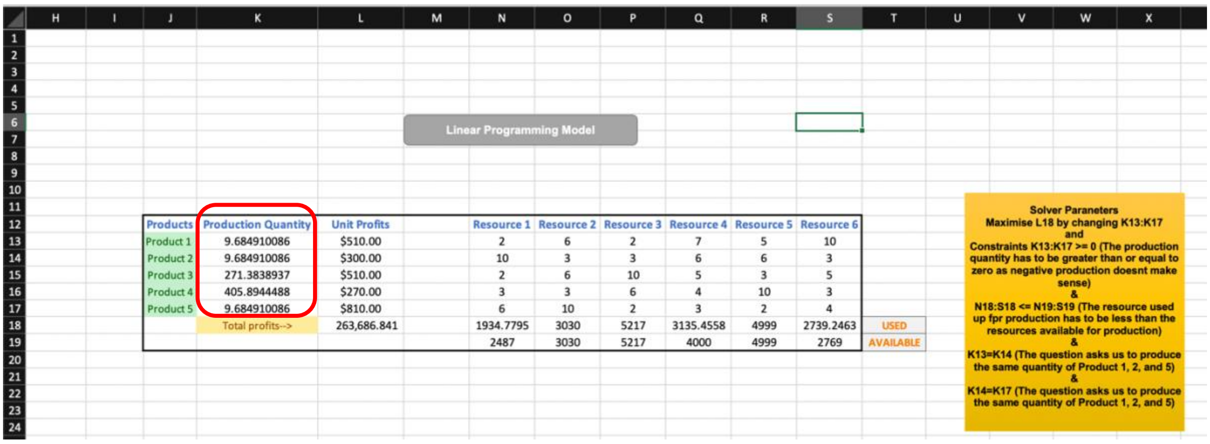
Products	Production Quantity	Unit Profits	Resource 1	Resource 2	Resource 3	Resource 4	Resource 5	Resource 6	
Product 1	4	$255.00	2	6	2	7	5	10	
Product 2	83	$150.00	10	3	3	6	6	3	
Product 3	277	$255.00	2	6	10	5	3	5	
Product 4	365	$135.00	3	3	6	4	10	3	
Product 5	0	$405.00	6	10	2	3	2	4	
	Total profits-->	133,380.000	2487	3030	5217	3371	4999	2769	USED
			2487	3030	5217	4000	4999	2769	AVAILABLE

Solver Paraneters
Maximise L18 by changing K13:K17
and
Constraints K13:K17 >= 0 (The production quantity has to be greater than or equal to zero as negative production doesnt make sense) &
N18:S18 <= N19:S19 (The resource used up for production has to be less than the resources available for production. You cannot produce without having enough resources)

From the worksheet it is clear that the production quantity is the same (4, 83, 277, 365, and 0) as mentioned in worksheet '1(b)' and hence we conclude that the optimal solution hasn't changed.

1(k)

A new worksheet '1(k)' is made in which the same specifications from 1(d) are set.
Since the question asks us to produce the same amounts of product 1, 2, and 5, this has to be incorporated in the constraints of the solver. 2 more constraints are added to the solver, which equate the cells K13 and K14 as one constraint and K14 and K17 as another. The solver is re-run again.
This time, the solver gives a changed values.


Linear Programming Model

Products	Production Quantity	Unit Profits	Resource 1	Resource 2	Resource 3	Resource 4	Resource 5	Resource 6	
Product 1	9.684910086	$510.00	2	6	2	7	5	10	
Product 2	9.684910086	$300.00	10	3	3	6	6	3	
Product 3	271.3838937	$510.00	2	6	10	5	3	5	
Product 4	405.8944488	$270.00	3	3	6	4	10	3	
Product 5	9.684910086	$810.00	6	10	2	3	2	4	
	Total profits-->	263,686.841	1934.7795	3030	5217	3135.4558	4999	2739.2463	USED
			2487	3030	5217	4000	4999	2769	AVAILABLE

Solver Paraneters
Maximise L18 by changing K13:K17
and
Constraints K13:K17 >= 0 (The production quantity has to be greater than or equal to zero as negative production doesnt make sense)
&
N18:S18 <= N19:S19 (The resource used up fpr production has to be less than the resources available for production)
&
K13=K14 (The question asks us to produce the same quantity of Product 1, 2, and 5)
&
K14=K17 (The question asks us to produce the same quantity of Product 1, 2, and 5)

In cells K13, K14, and K17 it can be seen that the production quantity is equal in the 3 products and sits at 9.6849.

Comparing the original optimal answer to the current optimal answer:

Product	ORIGINAL OPTIMAL PRODUCTION QUANTITY	CURRENT OPTIMAL PRODUCTION QUANTITY	DIFFERENCE(CURRENT – ORIGINAL)
Product 1	4	9.684910086	-5.684910086
Product 2	83	9.684910086	-73.315089914
Product 3	277	271.3838937	0.3838937
Product 4	365	405.8944488	40.8944488
Product 5	0	9.684910086	9.684910086

Now, if you add up the difference column we get (-28.036747414). This negative value effects the feasible region in a negative way and decreases it.

The feasible region is nothing but the area under the graph. The area under the graph in this case is basically the Total Profit. In terms of the objective function, area under the graph is the value we want to maximise.
Therefore, When the net change in production quantity is negative, it decreases the profit because in the graph the corner points move closer to the origin and hence decrease the feasibility region.

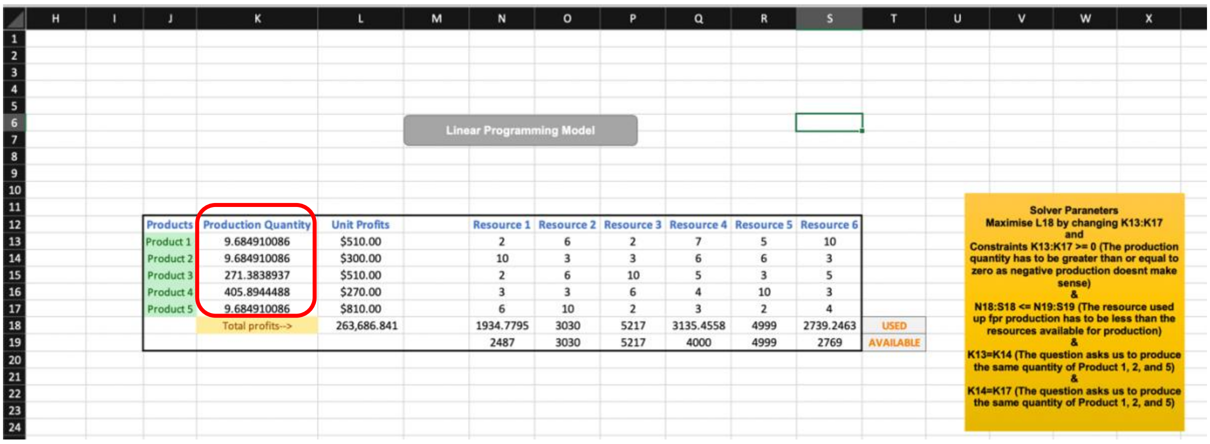
Also, if we directly look at the Total profit, there is negative change in that as well.

Current - Original
= 263686.841 - 266760
= (- 3073.159)

The negative difference also means that the feasible region will decrease.

1(l)

In the same worksheet '1(k)' in which the same specifications from 1(d) are set.
Since the question asks us to produce the same amounts of product 1, 2, and 5, this has to be incorporated in the constraints of the solver. 2 more constraints are added to the solver, which equate the cells K13 and K14 as one constraint and K14 and K17 as another. The solver is re-run again.
This time, the solver gives a changed values.



Linear Programming Model

Products	Production Quantity	Unit Profits	Resource 1	Resource 2	Resource 3	Resource 4	Resource 5	Resource 6	
Product 1	9.684910086	$510.00	2	6	2	7	5	10	
Product 2	9.684910086	$300.00	10	3	3	6	6	3	
Product 3	271.3838937	$510.00	2	6	10	5	3	5	
Product 4	405.8944488	$270.00	3	3	6	4	10	3	
Product 5	9.684910086	$810.00	6	10	2	3	2	4	
	Total profits-->	263,686.841	1934.7795	3030	5217	3135.4558	4999	2739.2463	USED
			2487	3030	5217	4000	4999	2769	AVAILABLE

Solver Paraneters
Maximise L18 by changing K13:K17
and
Constraints K13:K17 >= 0 (The production quantity has to be greater than or equal to zero as negative production doesnt make sense)
&
N18:S18 <= N19:S19 (The resource used up fpr production has to be less than the resources available for production)
&
K13=K14 (The question asks us to produce the same quantity of Product 1, 2, and 5)
&
K14=K17 (The question asks us to produce the same quantity of Product 1, 2, and 5)

In cells K13, K14, and K17 it can be seen that the production quantity is equal in the 3 products and sits at 9.6849.

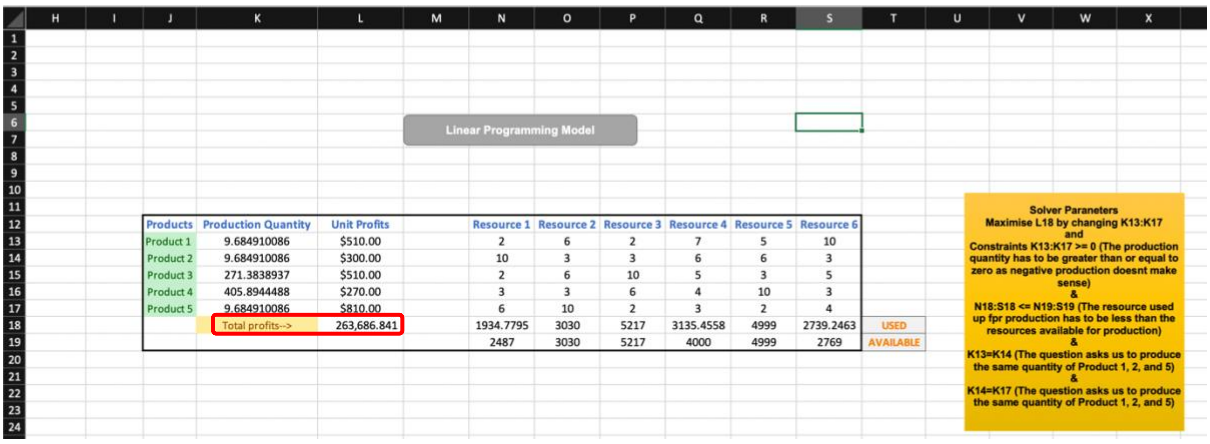
PART 1

Product	OPTIMAL PRODUCTION QUANTITY
Product 1	9.684910086
Product 2	9.684910086
Product 3	271.3838937
Product 4	405.8944488
Product 5	9.684910086

The reason for the changed values is the addition of two additional constraints which forces the solver to change its value of production quantities and hence in turn the Total Profit.

PART 2

The resultant profit is 263686.841. It is a decrease from the original value because of the drop in overall production values as shown in question 1(k)



Linear Programming Model

Products	Production Quantity	Unit Profits	Resource 1	Resource 2	Resource 3	Resource 4	Resource 5	Resource 6	
Product 1	9.684910086	$510.00	2	6	2	7	5	10	
Product 2	9.684910086	$300.00	10	3	3	6	6	3	
Product 3	271.3838937	$510.00	2	6	10	5	3	5	
Product 4	405.8944488	$270.00	3	3	6	4	10	3	
Product 5	9.684910086	$810.00	6	10	2	3	2	4	
	Total profits-->	263,686.841	1934.7795	3030	5217	3135.4558	4999	2739.2463	USED
			2487	3030	5217	4000	4999	2769	AVAILABLE

Solver Paraneters
Maximise L18 by changing K13:K17
and
Constraints K13:K17 >= 0 (The production quantity has to be greater than or equal to zero as negative production doesnt make sense)
&
N18:S18 <= N19:S19 (The resource used up fpr production has to be less than the resources available for production)
&
K13=K14 (The question asks us to produce the same quantity of Product 1, 2, and 5)
&
K14=K17 (The question asks us to produce the same quantity of Product 1, 2, and 5)

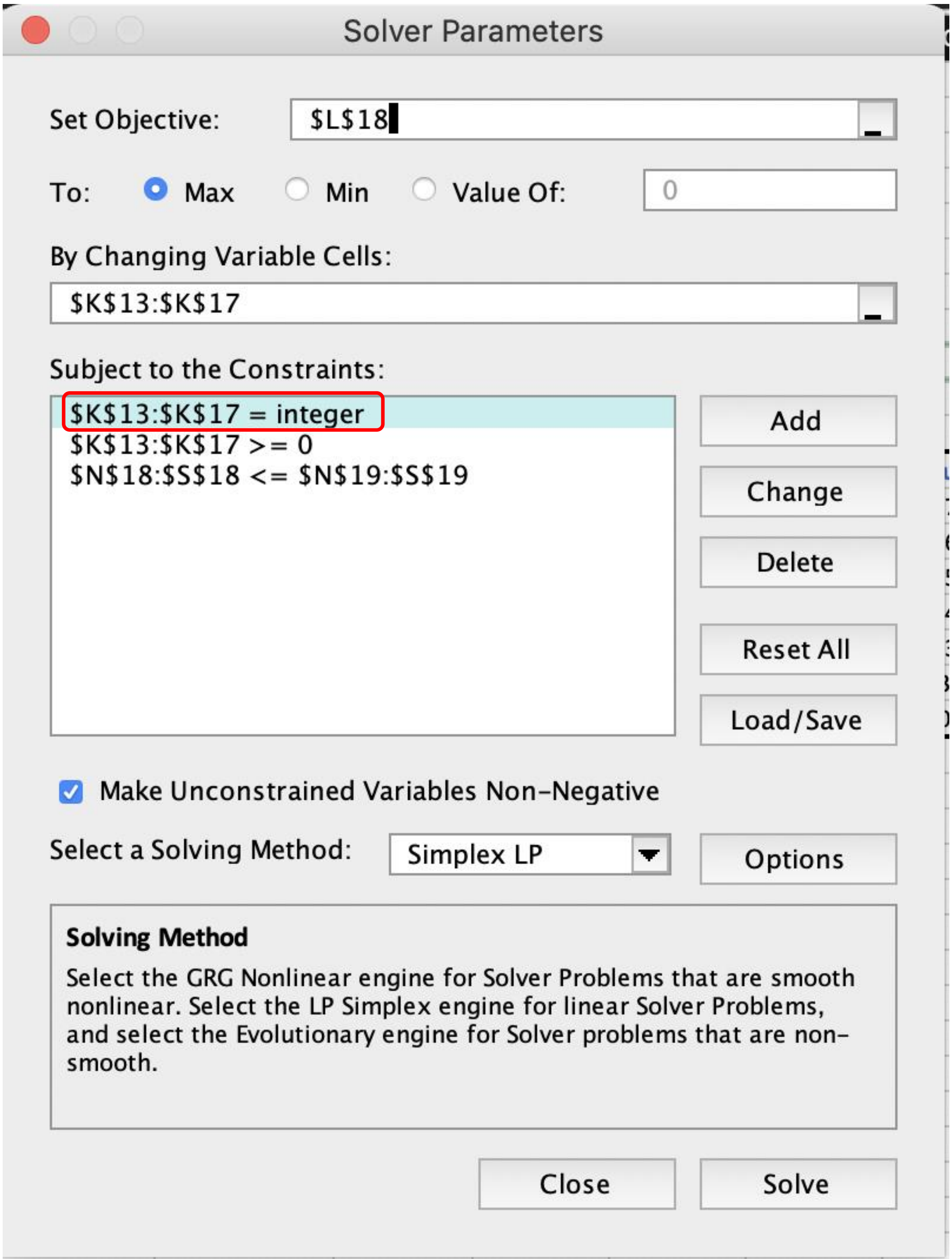
Current - Original
= 263686.841 - 266760
= (- 3073.159)

Since the Total Profit is sumproduct of production quantity and unit profit, a decrease in overall production can plummet the Total Profit as shown below:

510* 9.684910086 + 300* 9.684910086 + 510* 271.3838937 + 270* 405.8944488 + 810*9.684910086 = 263686.841

1(m)

In the worksheet '1(m)', in order to comply with the question, we added a constraint to the solver which makes all the production quantities as integer values. After this addition to the list of constraints, we re-run the solver.



PART 1

Product	OPTIMAL PRODUCTION QUANTITY
Product 1	4
Product 2	83
Product 3	277
Product 4	365
Product 5	0

The solver goes back to the original production values of 4, 83, 277, 365, and 0. Since the production quantities from part 1(d) weren't integers anyway, adding this new constraint doesn't change the answer at all.

PART 2

From the excel worksheet '1(m)', the optimal plan for production is as follows:

- $X_1 = 4$
- $X_2 = 83$
- $X_3 = 277$
- $X_4 = 365$
- $X_5 = 0$

Linear Programming Model

Products	Production Quantity	Unit Profits	Resource 1	Resource 2	Resource 3	Resource 4	Resource 5	Resource 6	
Product 1	4	$510.00	2	6	2	7	5	10	
Product 2	83	$300.00	10	3	3	6	6	3	
Product 3	277	$510.00	2	6	10	5	3	5	
Product 4	365	$270.00	3	3	6	4	10	3	
Product 5	0	$810.00	6	10	2	3	2	4	
	Total profits-->	266,760.000	2487	3030	5217	3371	4999	2769	USED
			2487	3030	5217	4000	4999	2769	AVAILABLE

Solver Paraneters
Maximise L18 by changing K13:K17
and
Constraints K13:K17 >= 0 (The production quantity has to be greater than or equal to zero as negative production doesnt make sense) &
N18:S18 <= N19:S19 (The resource used up fpr production has to be less than the resources available for production)
&
K13:K17 = Integer (As required by the question)

Associated Total profits with the production plan is given from the result of the solver. It can be seen in cell L18 of worksheet '1(m)'. The solver gives an optimum output considering the constraints. The solver is set to maximize cell L18, which is the sumproduct of the profit column and the production quantity.

On the other hand, it can also be calculated by manually doing the calculations as shown below.

Now, take the objective function

- $510X_1 + 300X_2 + 510X_3 + 270X_4 + 810X_5$

Substitute values of X_1, X_2, X_3, X_4, X_5 as 4, 83, 277, 365 and 0 respectively,

We get,

510*4 + 300*84 + 510*277 + 270*365 + 810*0
= $2,66,760/-

1(n)

PRODUCTS	FIXED COSTS
Product 1	2000
Product 2	4000
Product 3	8000
Product 4	16000
Product 5	1000

Note: It is worthy to note that the fixed costs associated to the products only affect the profit when that particular product is produced. Hence a method to avoid those costs whenever necessary are to be devised. A binary constraint method.

$$Y_i = \begin{cases} 1, \text{ If } x_i > 0 & ; \quad i = 1,2,3,4,5 \\ 0, \text{ If } x_i = 0 & \end{cases}$$

In this scenario, if X_i (Production quantity) is +ve, then $Y_i = 1$, hence that particular fixed cost will be included in the objective function

The new objective function = $510x_1 + 300x_2 + 510x_3 + 270x_4 + 810x_5 – 2000Y_1 - 4000Y_2 – 8000Y_3 – 16000Y_4 – 1000Y_5$

Constraint definition:

CONSTRAINTS TYPE	CONSTRAINT SPECIFICATION
Resource Constraints	$2x_1 + 10x_2 + 2x_3 + 3x_4 + 6x_5 <= 2487$ (**Resource 1**) $6x_1 + 3x_2 + 6x_3 + 3x_4 + 10x_5 <= 3030$ (**Resource 2**) $2x_1 + 3x_2 + 10x_3 + 6x_4 + 2x_5 <= 5217$ (**Resource 3**) $7x_1 + 6x_2 + 5x_3 + 4x_4 + 3x_5 <= 4000$ (**Resource 4**) $5x_1 + 6x_2 + 3x_3 + 10x_4 + 2x_5 <= 4999$ (**Resource 5**) $10x_1 + 3x_2 + 5x_3 + 3x_4 + 4x_5 <= 2769$ (**Resource 6**)
Non-Negativity Constraints	$X_1>0$ $X_2>0$ $X_3>0$ $X_4>0$ $X_5>0$
Integer Constraint	For I = 1 to 5, X_i = Integer

<table>
<tr><td>Binary Constraints</td><td>All the Binary column cells (T13:T17) should be binary with formula as follows (also explained above):

$$Y_i = \begin{cases} 1, \text{ If } x_i > 0 \quad ; \quad i = 1,2,3,4,5 \\ 0, \text{ If } x_i = 0 \end{cases}$$</td></tr>
<tr><td>Linking Constraints</td><td>Linking constraints are used to set the upper bound of the object function values of X_i in the graph. This is done to increase the area under the graph (Total Profits). The constraints use X_i in the equation to push the upper bound and also includes Y_i to check for inclusion of the product in production.

The push for upper bound is done through the ‘Big M’ value (M_i) which is the minimum value of X_i in the constraint equations.

In order to get the maximum value of X_i in the resource constraints equations we take the instance of X_1.

$$2x_1 + 10x_2 + 2x_3 + 3x_4 + 6x_5 <= 2487$$
To get the maximum capacity of production of product $X_{1,}$ we put X_2, $X_{3,}$ $X_{4,}$ $X_5 = 0$.
We get,
$$X_1 <= 2487/2$$
Similarly we calculate for each resource constraint equation and get 5 values of X_1 which are as follows,

2487/2, 3030/6, 5217/2, 4000/7, 4999/5, 2769/10

It is important to note* that to select value for M_i we choose the minimum from the 5. The reason is that to produce a single unit of product 1, we require 2 units of resource 1, 6 units of resource 2, 2 units of resource 3, 7 units of resource 4, 5 units of resource 5, and 10 units of resource 6. It is not possible to produce the product even if 1 resource is not available partially. Therefore, if we take the minimum value from all the values, we ensure that it satisfies the equation i.e.: it ensures that all the other resource requirements will be met.

Therefore in this case M = 2769/10 = 276.9.

Similarly, for all X_i ‘s, we calculate the value of M_i

- $M_1 = 276.9$
- $M_2 = 248.7$
- $M_3 = 505$
- $M_4 = 499.5$
- $M_5 = 303$</td></tr>
</table>

Now, The constraint equation is $X_i <= M_iY_i$
This equation sets the maximum value range for X_i to maximise the objective function as both are positively correlated, i.e.: Increase in X_i increases the objective function.

This equation can also be expressed as $X_i - M_iY_i <= 0$

Hence, our linking constraints are as follows:

$X_1 – 276.9Y_1 <=0$

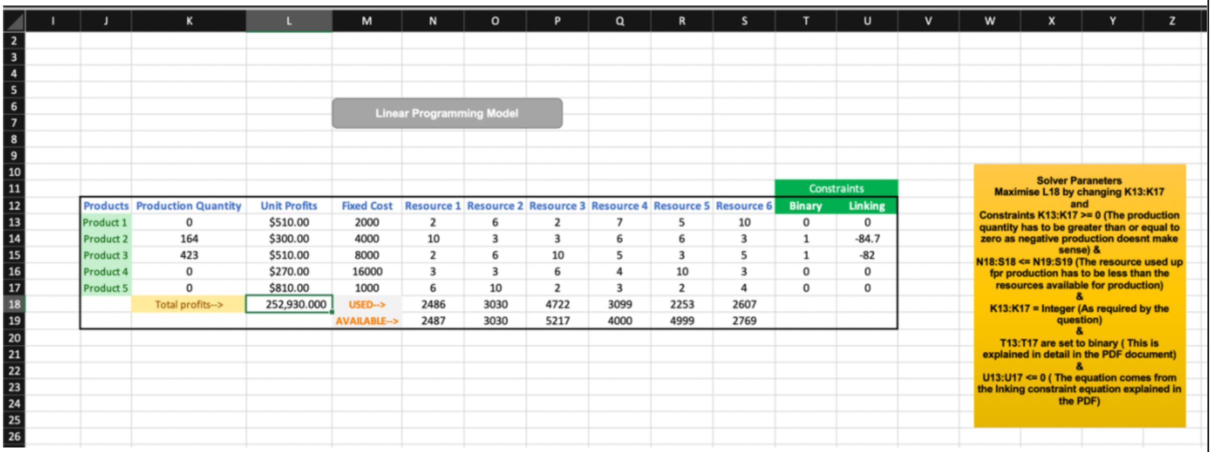
$X_2 – 248.7Y_2 <=0$

$X_3 – 505Y_3 <=0$

$X_4 – 499.5Y_4 <=0$

$X_5 – 303Y_5 <=0$

This can be seen in the worksheet '1(n)' in the constraints of the solver. The LHS of the equation is also fed in the cells N19:S19.



Linear Programming Model

Products	Production Quantity	Unit Profits	Fixed Cost	Resource 1	Resource 2	Resource 3	Resource 4	Resource 5	Resource 6	Constraints Binary	Constraints Linking
Product 1	0	$510.00	2000	2	6	2	7	5	10	0	0
Product 2	164	$300.00	4000	10	3	3	6	6	3	1	-84.7
Product 3	423	$510.00	8000	2	6	10	5	3	5	1	-82
Product 4	0	$270.00	16000	3	3	6	4	10	3	0	0
Product 5	0	$810.00	1000	6	10	2	3	2	4	0	0
	Total profits-->	252,930.000	USED-->	2486	3030	4722	3099	2253	2607		
			AVAILABLE-->	2487	3030	5217	4000	4999	2769		

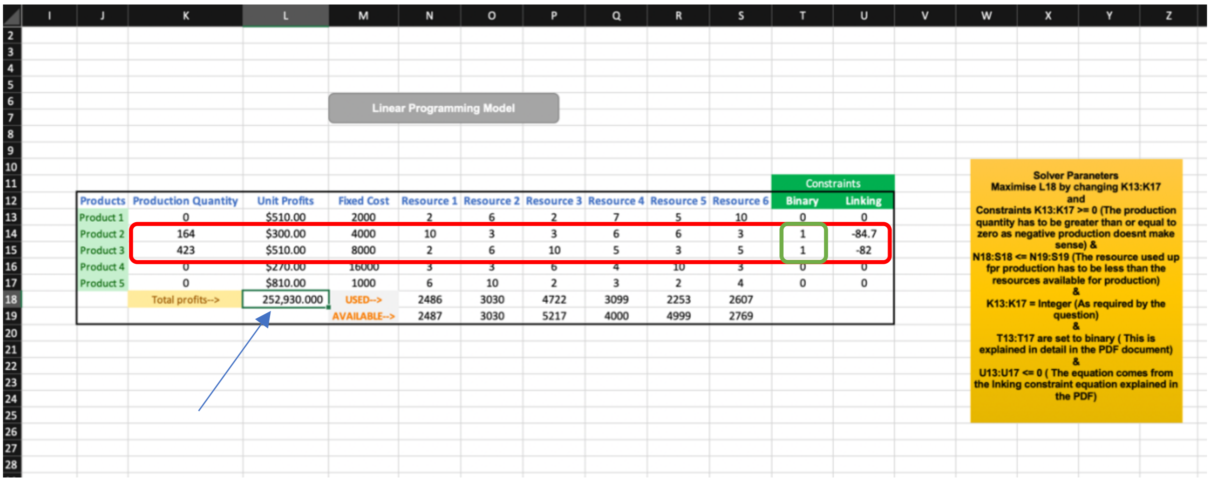
Solver Paraneters
Maximise L18 by changing K13:K17
and
Constraints K13:K17 >= 0 (The production quantity has to be greater than or equal to zero as negative production doesnt make sense) &
N18:S18 <= N19:S19 (The resource used up fpr production has to be less than the resources available for production)
&
K13:K17 = Integer (As required by the question)
&
T13:T17 are set to binary (This is explained in detail in the PDF document)
&
U13:U17 <= 0 (The equation comes from the lnking constraint equation explained in the PDF)

PART 1

Optimal solution(Production Quantity) produced in this scenario is as follows

PRODUCTS	OPTIMAL PRODUCTION QUANTITY
Product 1	0
Product 2	164
Product 3	423
Product 4	0
Product 5	0

This means we produce Product 2 and Product 3 only. In worksheet '1(n)' we can see in cells T13:T17 that the corresponding binary value for product 2 and 3 are one hence the production quantities are positive whereas, others have 0 binary value and hence have a production quantity value of 0.



Linear Programming Model

Products	Production Quantity	Unit Profits	Fixed Cost	Resource 1	Resource 2	Resource 3	Resource 4	Resource 5	Resource 6	Constraints Binary	Constraints Linking
Product 1	0	$510.00	2000	2	6	2	7	5	10	0	0
Product 2	164	$300.00	4000	10	3	3	6	6	3	1	-84.7
Product 3	423	$510.00	8000	2	6	10	5	3	5	1	-82
Product 4	0	$270.00	16000	3	3	6	4	10	3	0	0
Product 5	0	$810.00	1000	6	10	2	3	2	4	0	0
	Total profits-->	252,930.000	USED-->	2486	3030	4722	3099	2253	2607		
			AVAILABLE-->	2487	3030	5217	4000	4999	2769		

Solver Paraneters
Maximise L18 by changing K13:K17
and
Constraints K13:K17 >= 0 (The production quantity has to be greater than or equal to zero as negative production doesnt make sense) &
N18:S18 <= N19:S19 (The resource used up fpr production has to be less than the resources available for production)
&
K13:K17 = Integer (As required by the question)
&
T13:T17 are set to binary (This is explained in detail in the PDF document)
&
U13:U17 <= 0 (The equation comes from the lnking constraint equation explained in the PDF)

PART 2

In the worksheet '1(n)' we can see in cell L18 the objective function which the solver has maximized.
The optimum value of the objective function is $ 252930.000.

In other ways we can calculate it though the objective function as well by substituting values of X_i from the cells K13:K17 and the value of Y_i from the cells T13:T17.

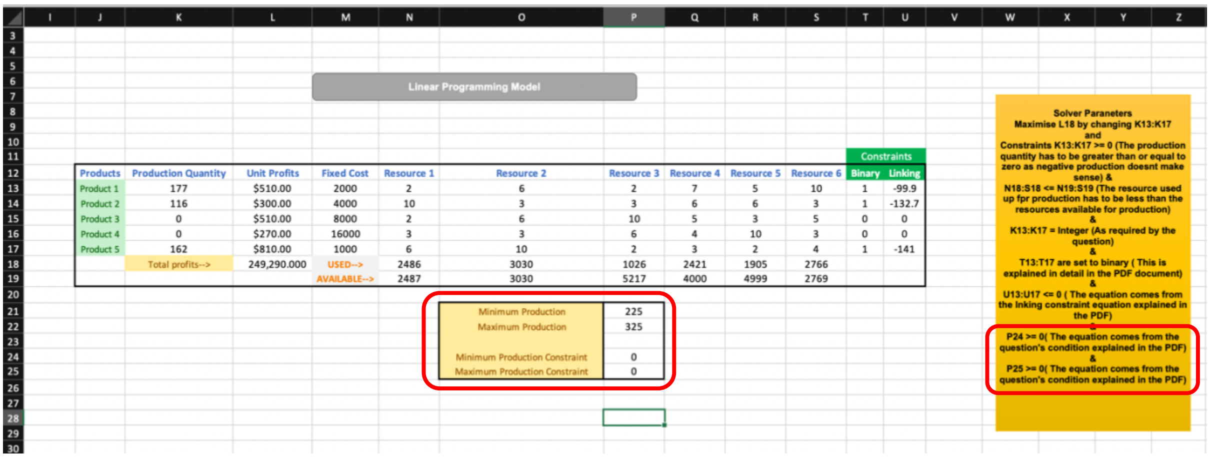
Objective function = $510x_1 + 300x_2 + 510x_3 + 270x_4 + 810x_5 – 2000Y_1 - 4000Y_2 – 8000Y_3 – 16000Y_4 – 1000Y_5$

= 510*0 + 300*164 + 510*423 + 270*0 + 810*0 – 2000*0 -4000*1 – 8000*1 – 16000*0 – 1000*0

= 252930.000

1(o)

In worksheet '1(o)', we add two constraints to incorporate the condition from the question about the minimum(225) and maximum(325) quantity of product 3 to be produced, IF at all it is produced.



Linear Programming Model

Products	Production Quantity	Unit Profits	Fixed Cost	Resource 1	Resource 2	Resource 3	Resource 4	Resource 5	Resource 6	Binary	Linking
Product 1	177	$510.00	2000	2	6	2	7	5	10	1	-99.9
Product 2	116	$300.00	4000	10	3	3	6	6	3	1	-132.7
Product 3	0	$510.00	8000	2	6	10	5	3	5	0	0
Product 4	0	$270.00	16000	3	3	6	4	10	3	0	0
Product 5	162	$810.00	1000	6	10	2	3	2	4	1	-141
	Total profits-->	249,290.000	USED-->	2486	3030	1026	2421	1905	2766		
			AVAILABLE-->	2487	3030	5217	4000	4999	2769		

Minimum Production	225
Maximum Production	325
Minimum Production Constraint	0
Maximum Production Constraint	0

Hence, we add two more equations in cell P24 and P25.
So, to set a maximum limit we can put the LHS of $X_3 <= 325Y_3$ in cell P25.

Or

$$X_3 - 325Y_3 <= 0$$

Similarly,
To set minimum limit we can put the LHS of $X_3 >= 225Y_3$ in cell P24.

Or

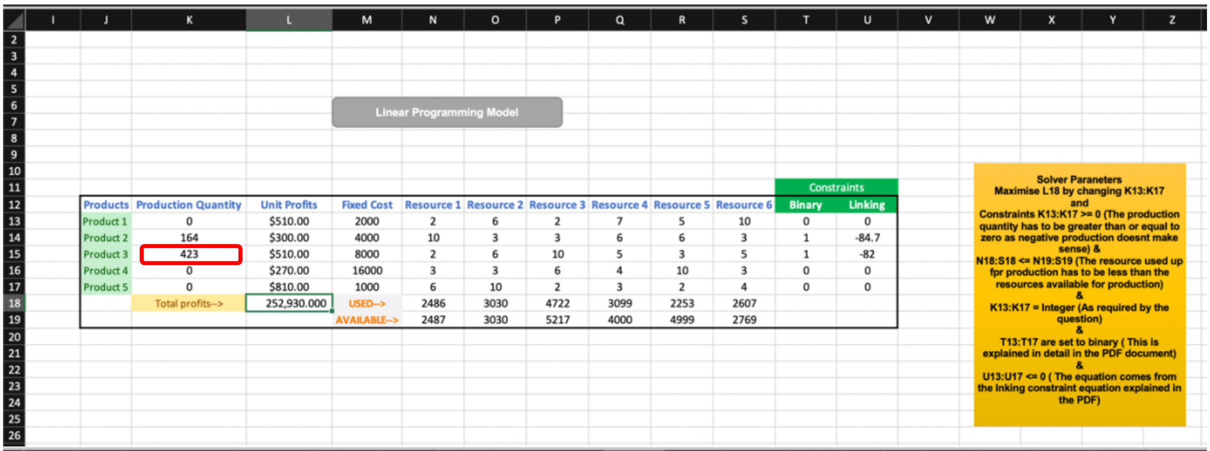
$$X_3 - 225Y_3 >= 0$$

PART 1

PRODUCTS	OPTIMAL PRODUCTION QUANTITY
Product 1	177
Product 2	116
Product 3	0
Product 4	0
Product 5	162

This means we do not produce Product 3 and Product 4. In worksheet '1(o)' we can see in cells T13:T17 that the corresponding binary value for product 3 and 4 is 0 hence the production quantities are 0 whereas, others have binary value 1 and hence have a positive production quantity value.

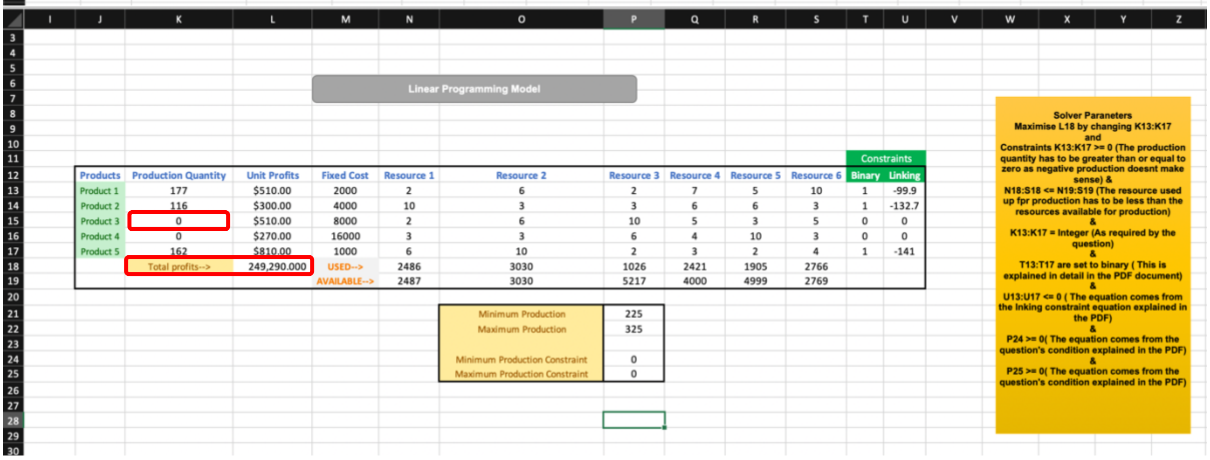
The production value of product 3 and 4 are 0 because if you compare from the question 1(n), the X_3 value there was 423. Here in question '1(o)' the value is 0 because the previous optimal solution value of X_i is outside the maximum value constraint.



Linear Programming Model

Products	Production Quantity	Unit Profits	Fixed Cost	Resource 1	Resource 2	Resource 3	Resource 4	Resource 5	Resource 6	Binary	Linking
										Constraints	
Product 1	0	$510.00	2000	2	6	2	7	5	10	0	0
Product 2	164	$300.00	4000	10	3	3	6	6	3	1	-84.7
Product 3	423	$510.00	8000	2	6	10	5	3	5	1	-82
Product 4	0	$270.00	16000	3	3	6	4	10	3	0	0
Product 5	0	$810.00	1000	6	10	2	3	2	4	0	0
	Total profits-->	252,930.000	USED-->	2486	3030	4722	3099	2253	2607		
			AVAILABLE-->	2487	3030	5217	4000	4999	2769		

Solver Paraneters
Maximise L18 by changing K13:K17
and
Constraints K13:K17 >= 0 (The production quantity has to be greater than or equal to zero as negative production doesnt make sense) &
N18:S18 <= N19:S19 (The resource used up fpr production has to be less than the resources available for production)
&
K13:K17 = Integer (As required by the question)
&
T13:T17 are set to binary (This is explained in detail in the PDF document)
&
U13:U17 <= 0 (The equation comes from the Inking constraint equation explained in the PDF)



Linear Programming Model

Products	Production Quantity	Unit Profits	Fixed Cost	Resource 1	Resource 2	Resource 3	Resource 4	Resource 5	Resource 6	Binary	Linking
										Constraints	
Product 1	177	$510.00	2000	2	6	2	7	5	10	1	-99.9
Product 2	116	$300.00	4000	10	3	3	6	6	3	1	-132.7
Product 3	0	$510.00	8000	2	6	10	5	3	5	0	0
Product 4	0	$270.00	16000	3	3	6	4	10	3	0	0
Product 5	162	$810.00	1000	6	10	2	3	2	4	1	-141
	Total profits-->	249,290.000	USED-->	2486	3030	1026	2421	1905	2766		
			AVAILABLE-->	2487	3030	5217	4000	4999	2769		

Minimum Production	225
Maximum Production	325
Minimum Production Constraint	0
Maximum Production Constraint	0

Solver Paraneters
Maximise L18 by changing K13:K17
and
Constraints K13:K17 >= 0 (The production quantity has to be greater than or equal to zero as negative production doesnt make sense) &
N18:S18 <= N19:S19 (The resource used up fpr production has to be less than the resources available for production)
&
K13:K17 = Integer (As required by the question)
&
T13:T17 are set to binary (This is explained in detail in the PDF document)
&
U13:U17 <= 0 (The equation comes from the Inking constraint equation explained in the PDF)
&
P24 >= 0(The equation comes from the question's condition explained in the PDF)
&
P25 >= 0(The equation comes from the question's condition explained in the PDF)

PART 2

In the worksheet '1(o)' we can see in cell L18 the objective function which the solver has maximized.
The optimum value of the objective function is $ 249290.000.

In other ways we can calculate it though the objective function as well by substituting values of X_i from the cells K13:K17 and the value of Y_i from the cells T13:T17.

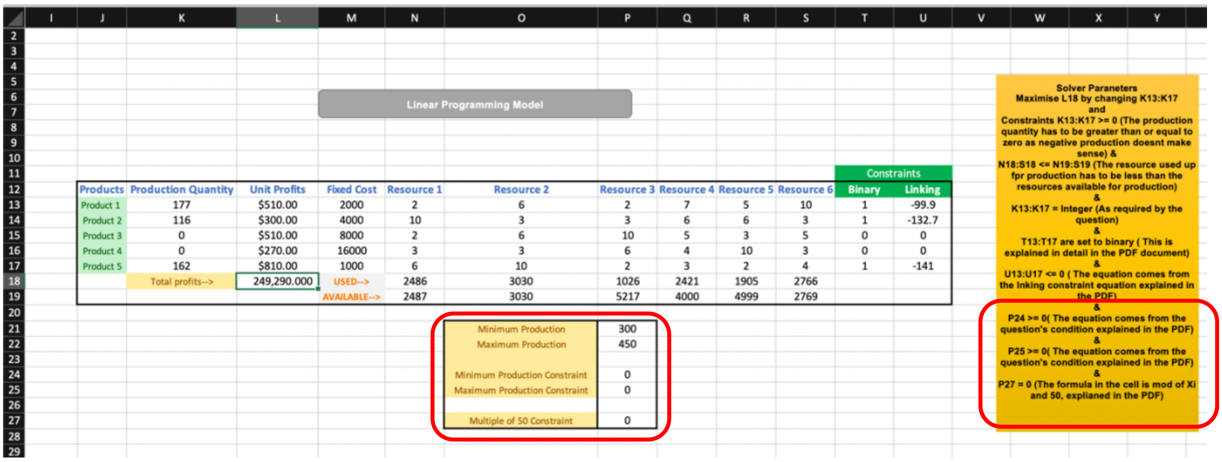
Objective function = $510x_1 + 300x_2 + 510x_3 + 270x_4 + 810x_5 – 2000Y_1 - 4000Y_2 – 8000Y_3 – 16000Y_4 – 1000Y_5$

= 510*177+ 300*116 + 510*0 + 270*0 + 810*162 – 2000*0 -4000*1 – 8000*1 – 16000*0 – 1000*1

= 249290.000

1(p)

In worksheet '1(p)' , we add three more constraints to incorporate the condition from the question about the minimum(300) and maximum(450) quantity of product 3 to be produced, IF at all it is produced.



Linear Programming Model

Products	Production Quantity	Unit Profits	Fixed Cost	Resource 1	Resource 2	Resource 3	Resource 4	Resource 5	Resource 6	Binary	Linking
Product 1	177	$510.00	2000	2	6	2	7	5	10	1	-99.9
Product 2	116	$300.00	4000	10	3	3	6	6	3	1	-132.7
Product 3	0	$510.00	8000	2	6	10	5	3	5	0	0
Product 4	0	$270.00	16000	3	3	6	4	10	3	0	0
Product 5	162	$810.00	1000	6	10	2	3	2	4	1	-141
	Total profits-->	249,290.000	USED-->	2486	3030	1026	2421	1905	2766		
			AVAILABLE-->	2487	3030	5217	4000	4999	2769		

Minimum Production	300
Maximum Production	450
Minimum Production Constraint	0
Maximum Production Constraint	0
Multiple of 50 Constraint	0

Solver Paraneters
Maximise L18 by changing K13:K17
and
Constraints K13:K17 >= 0 (The production quantity has to be greater than or equal to zero as negative production doesnt make sense) &
N18:S18 <= N19:S19 (The resource used up fpr production has to be less than the resources available for production)
&
K13:K17 = Integer (As required by the question)
&
T13:T17 are set to binary (This is explained in detail in the PDF document)
&
U13:U17 <= 0 (The equation comes from the lnking constraint equation explained in the PDF)
&
P24 >= 0(The equation comes from the question's condition explained in the PDF)
&
P25 >= 0(The equation comes from the question's condition explained in the PDF)
&
P27 = 0 (The formula in the cell is mod of Xi and 50, explianed in the PDF)

Hence, we add two more equations in cell P24 and P25.
So, to set a maximum limit we can put the LHS of $X_3 <= 450Y_3$ in cell P25.

Or

$$X_3 - 450Y_3 <= 0$$

Similarly,
To set minimum limit we can put the LHS of $X_3 >= 300Y_3$ in cell P24.

Or

$$X_3 - 300Y_3 >= 0$$

To incorporate the constraint of multiple of 50, we create another cell which is fed with the formula of Mod(X_i, 50).
This formula in excel gives the value of the remainder of the division of X_i/ 50.

This cell P27 is then fed in the constraints of worksheet '1(p)' as cell P27 = 0, which means the remainder after dividing X_i with 50 should be 0 hence making it a multiple of 50 and therefore satisfying the condition of the question.

PART 1

PRODUCTS	OPTIMAL PRODUCTION QUANTITY
Product 1	177
Product 2	116
Product 3	0
Product 4	0
Product 5	162

This means we do not produce Product 3 and Product 4. In worksheet '1(p)' we can see in cells T13:T17 that the corresponding binary value for product 3 and 4 is 0 hence the production quantities are 0 whereas, others have binary value 1 and hence have a positive production quantity value.

The production value of product 3 is 0 because if you compare from the question 1(n), the X_3 value there was 423. Here in question '1(p)' the value is 0 even after the maximum constraint being 450 because the previous optimal solution value of X_i doesn't satisfy the constraint which require X_3 to be a multiple of 50.

Linear Programming Model

										Constraints	
Products	Production Quantity	Unit Profits	Fixed Cost	Resource 1	Resource 2	Resource 3	Resource 4	Resource 5	Resource 6	Binary	Linking
Product 1	0	$510.00	2000	2	6	2	7	5	10	0	0
Product 2	164	$300.00	4000	10	3	3	6	6	3	1	-84.7
Product 3	423	$510.00	8000	2	6	10	5	3	5	1	-82
Product 4	0	$270.00	16000	3	3	6	4	10	3	0	0
Product 5	0	$810.00	1000	6	10	2	3	2	4	0	0
	Total profits-->	252,930.000	USED-->	2486	3030	4722	3099	2253	2607		
			AVAILABLE-->	2487	3030	5217	4000	4999	2769		

Solver Paraneters
Maximise L18 by changing K13:K17
and
Constraints K13:K17 >= 0 (The production quantity has to be greater than or equal to zero as negative production doesnt make sense) &
N18:S18 <= N19:S19 (The resource used up fpr production has to be less than the resources available for production)
&
K13:K17 = Integer (As required by the question)
&
T13:T17 are set to binary (This is explained in detail in the PDF document)
&
U13:U17 <= 0 (The equation comes from the lnking constraint equation explained in the PDF)

PART 2

In the worksheet '1(p)' we can see in cell L18 the objective function which the solver has maximized.

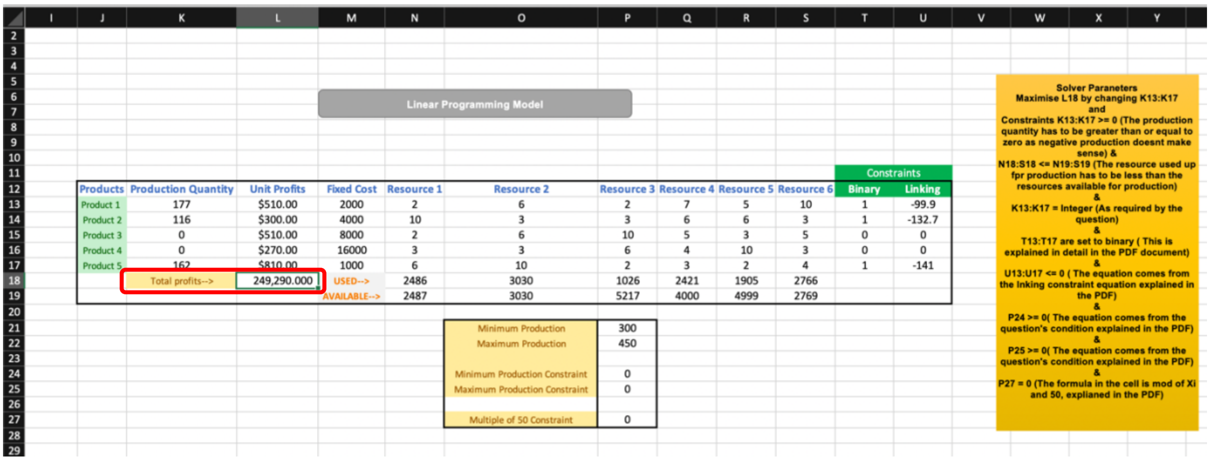
The optimum value of the objective function is $ 249290.000.

In other ways we can calculate it though the objective function as well by substituting values of X_i from the cells K13:K17 and the value of Y_i from the cells T13:T17.

Objective function = $510x_1 + 300x_2 + 510x_3 + 270x_4 + 810x_5 – 2000Y_1 - 4000Y_2 – 8000Y_3 – 16000Y_4 – 1000Y_5$

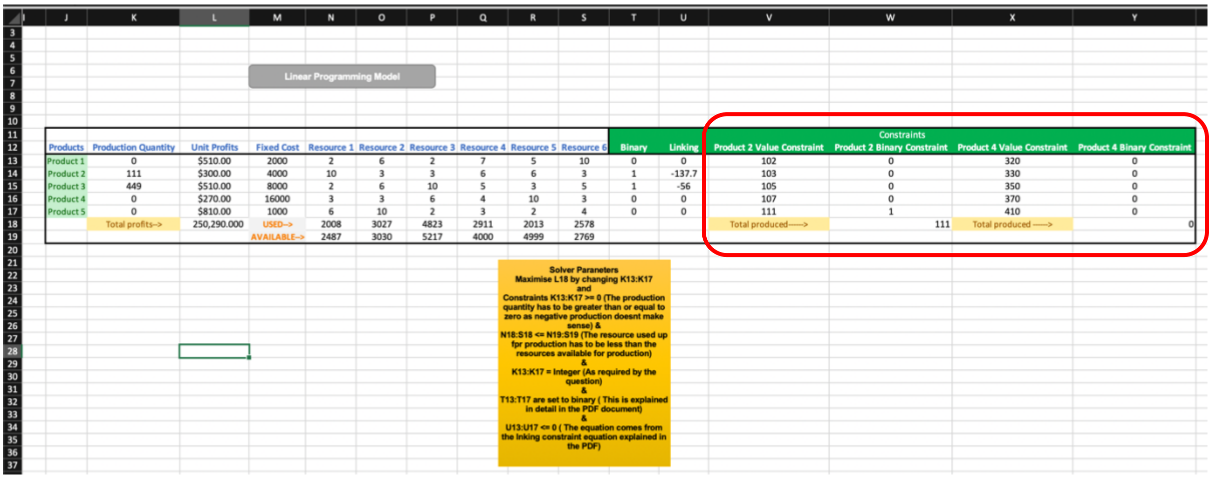
= 510*177+ 300*116 + 510*0 + 270*0 + 810*162 – 2000*0 -4000*1 – 8000*1 – 16000*0 – 1000*1

= 249290.000



1(q)

The question asks us to put constraints on values of product 2 and 4.
In worksheet '1(q)' the condition of the specific X_2 and X_4 values are put in cells V13:V17 and X13:X17 respectively. Now, since only one of the various values can be selected, we have also added a binary selection method in cells W13:W17 and Y13:Y17 respectively.



Also, we have to remove the binary selection of product 2 and product 4 in cell T14 and T16 from the constraints list in the solver.
All the other constraints are explained in question 1(n) in the constraint table.

PART 1

PRODUCTS	OPTIMAL PRODUCTION QUANTITY
Product 1	0
Product 2	111
Product 3	449
Product 4	0
Product 5	0

This means we do not produce Product 1, Product 4, and Product 5. In worksheet '1(q)' we can see in cells T13:T17 that the corresponding binary value for Product 1, Product 4, and Product 5 is 0 hence the production quantities are 0 whereas, others have binary value 1 and hence have a positive production quantity value.

The production value of product 2 is 111 because if you compare from the question 1(n), the X_2 value there was 164. Here in question '1(q)' the value is 111, This is because in the question 1(n) there is no constraint for X_2 to have specific values hence, it chose its optimum quantity as 164. Since the previous number was higher than the set of numbers (102, 103, 105, 107, and 111), the binary constraint in cell W17 changes to 1 to select the optimal order quantity as 111. This is chosen because 111 is closest to the previous optimum value amongst the other quantity choices given in the question.

Linear Programming Model

										Constraints	
Products	Production Quantity	Unit Profits	Fixed Cost	Resource 1	Resource 2	Resource 3	Resource 4	Resource 5	Resource 6	Binary	Linking
Product 1	0	$510.00	2000	2	6	2	7	5	10	0	0
Product 2	164	$300.00	4000	10	3	3	6	6	3	1	-84.7
Product 3	423	$510.00	8000	2	6	10	5	3	5	1	-82
Product 4	0	$270.00	16000	3	3	6	4	10	3	0	0
Product 5	0	$810.00	1000	6	10	2	3	2	4	0	0
	Total profits-->	252,930.000	USED-->	2486	3030	4722	3099	2253	2607		
			AVAILABLE-->	2487	3030	5217	4000	4999	2769		

Solver Paraneters
Maximise L18 by changing K13:K17
and
Constraints K13:K17 >= 0 (The production quantity has to be greater than or equal to zero as negative production doesnt make sense) &
N18:S18 <= N19:S19 (The resource used up fpr production has to be less than the resources available for production)
&
K13:K17 = Integer (As required by the question)
&
T13:T17 are set to binary (This is explained in detail in the PDF document)
&
U13:U17 <= 0 (The equation comes from the lnking constraint equation explained in the PDF)

Linear Programming Model

										Constraints					
Products	Production Quantity	Unit Profits	Fixed Cost	Resource 1	Resource 2	Resource 3	Resource 4	Resource 5	Resource 6	Binary	Linking	Product 2 Value Constraint	Product 2 Binary Constraint	Product 4 Value Constraint	Product 4 Binary Constraint
Product 1	0	$510.00	2000	2	6	2	7	5	10	0	0	102	0	320	0
Product 2	111	$300.00	4000	10	3	3	6	6	3	1	-137.7	103	0	330	0
Product 3	449	$510.00	8000	2	6	10	5	3	5	1	-56	105	0	350	0
Product 4	0	$270.00	16000	3	3	6	4	10	3	0	0	107	0	370	0
Product 5	0	$810.00	1000	6	10	2	3	2	4	0	0	111	1	410	0
	Total profits-->	250,290.000	USED-->	2008	3027	4823	2911	2013	2578			Total produced---->	111	Total produced ---->	0
			AVAILABLE-->	2487	3030	5217	4000	4999	2769						

Solver Paraneters
Maximise L18 by changing K13:K17
and
Constraints K13:K17 >= 0 (The production quantity has to be greater than or equal to zero as negative production doesnt make sense) &
N18:S18 <= N19:S19 (The resource used up fpr production has to be less than the resources available for production)
&
K13:K17 = Integer (As required by the question)
&
T13:T17 are set to binary (This is explained in detail in the PDF document)
&
U13:U17 <= 0 (The equation comes from the lnking constraint equation explained in the PDF)

PART 2

In the worksheet '1(q)' we can see in cell L18 the objective function which the solver has maximized.
The optimum value of the objective function is $ 250290.000.

In other ways we can calculate it though the objective function as well by substituting values of X_i from the cells K13:K17 and the value of Y_i from the cells T13, T15, T17, W13:W17 and Y13:Y17.

Objective function = $510x_1 + 300x_2 + 510x_3 + 270x_4 + 810x_5 – 2000Y_1 - 4000Y_2 – 8000Y_3 – 16000Y_4 – 1000Y_5$

= 510*0+ 300*111 + 510*449 + 270*0 + 810*0 – 2000*0 - 4000*1 – 8000*1 – 16000*0 – 1000*0

= 250290.000

1(r)

In worksheet '1(r)' there are additions of 4 more conditions. Here is how we have applied them on the excel sheet:

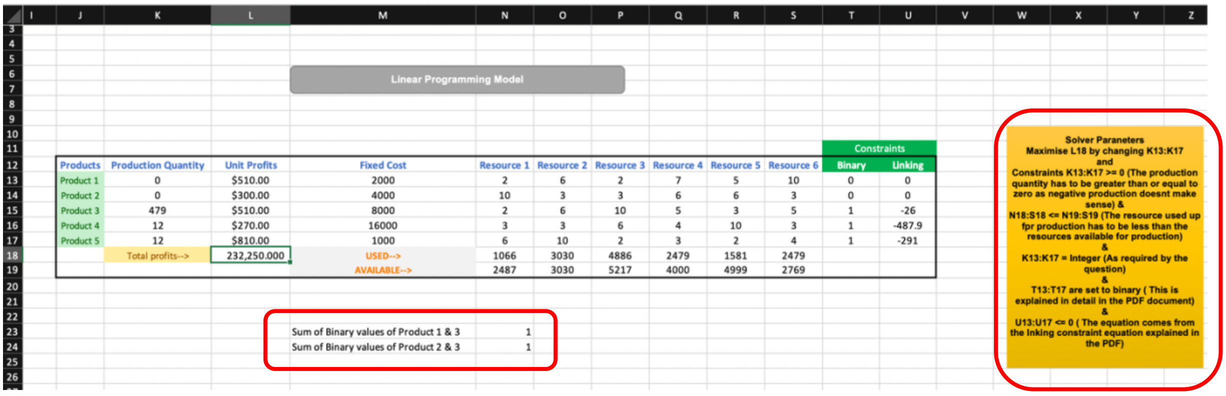
1. Produce product 1 and 2 in equal quantities: Here we add a constraint to the solver which equates cell K13 and K14.

2. Produce product 4 and 5 in equal quantities: Here we add a constraint to the solver which equates cell K16 and K17.

3. If product 3 is produced, 1 and 2 shouldn't be produced: We make 2 cell N23 which have addition of Y_1 and Y_3 and in cell N24 we have addition of Y_2 and Y_3 . In the solver we put a constraint which says values of these 2 cells should be 1. This ensures that if product 3 is produced (i.e. Y_3 is 1) then Y_2 and Y_3 will have to be 0 (i.e. Product 2 and 3 will not be produced).

4. If product 3 is produced, then X_5 should be between 10 and 100 or $10 <= X_5 <= 100$. Therefore, to ensure this we again add three constraints in the solver, one being $X_5 <= 100$, one being $X_5 >= 10$ and other being $Y_3 = Y_5$.

**Assumption: We assume that if product 3 is not produced, then product 5 will also not be produced. This is because we have equated Y_3 and Y_5, so if Y_3 is 0, then Y_5 will also be 0.



Linear Programming Model

										Constraints	
Products	Production Quantity	Unit Profits	Fixed Cost	Resource 1	Resource 2	Resource 3	Resource 4	Resource 5	Resource 6	Binary	Linking
Product 1	0	$510.00	2000	2	6	2	7	5	10	0	0
Product 2	0	$300.00	4000	10	3	3	6	6	3	0	0
Product 3	479	$510.00	8000	2	6	10	5	3	5	1	-26
Product 4	12	$270.00	16000	3	3	6	4	10	3	1	-487.9
Product 5	12	$810.00	1000	6	10	2	3	2	4	1	-291
	Total profits-->	232,250.000	USED-->	1066	3030	4886	2479	1581	2479		
			AVAILABLE-->	2487	3030	5217	4000	4999	2769		

Sum of Binary values of Product 1 & 3	1
Sum of Binary values of Product 2 & 3	1

PART 1

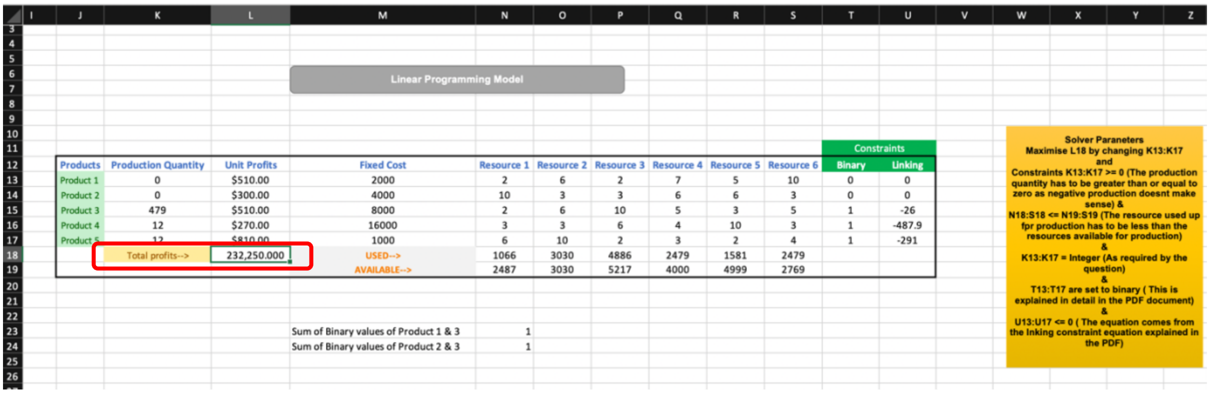
PRODUCTS	OPTIMAL PRODUCTION QUANTITY
Product 1	0
Product 2	0
Product 3	479
Product 4	12
Product 5	12

This means we do not produce Product 1 and Product 2. In worksheet ‘1(r)’ we can see in cells T13:T17 that the corresponding binary value for Product 1 and Product 2 is 0 hence the production quantities are 0 whereas, others have binary value 1 and hence have a positive production quantity value.

The production value of product 1 and product 2 is equal and hence satisfies the first condition. Similarly, X_4 and X_5 are equal as well.
Moreover, since product 3 is being produced, Products 1 and 2 aren’t.

PART 2

In the worksheet ‘1(r)’ we can see in cell L18 the objective function which the solver has maximized.
The optimum value of the objective function is $ 232250.000.



Linear Programming Model

Products	Production Quantity	Unit Profits	Fixed Cost	Resource 1	Resource 2	Resource 3	Resource 4	Resource 5	Resource 6	Binary	Linking
Product 1	0	$510.00	2000	2	6	2	7	5	10	0	0
Product 2	0	$300.00	4000	10	3	3	6	6	3	0	0
Product 3	479	$510.00	8000	2	6	10	5	3	5	1	-26
Product 4	12	$270.00	16000	3	3	6	4	10	3	1	-487.9
Product 5	12	$810.00	1000	6	10	2	3	2	4	1	-291
	Total profits-->	232,250.000	USED-->	1066	3030	4886	2479	1581	2479		
			AVAILABLE-->	2487	3030	5217	4000	4999	2769		

(Binary and Linking columns under the heading "Constraints".)

Sum of Binary values of Product 1 & 3 1
Sum of Binary values of Product 2 & 3 1

In other ways we can calculate it though the objective function as well by substituting values of X_i from the cells K13:K17 and the value of Y_i from the cells U13:U17.

Objective function = $510x_1 + 300x_2 + 510x_3 + 270x_4 + 810x_5 – 2000Y_1 - 4000Y_2 – 8000Y_3 – 16000Y_4 – 1000Y_5$

= 510*0+ 300*0 + 510*479 + 270*12 + 810*12 – 2000*0 - 4000*0 – 8000*1 – 16000*1 – 1000*1

= 232250.000

2.

Transshipment Problem statement:

The problem statement has the following table:

FROM	TO	UNIT COST (IN $)
1	3	50
1	4	80
2	3	70
2	4	40
3	5	70
3	6	50
4	5	40
4	6	80
5	7	80
5	8	40
6	7	60
6	8	70

The following table and their unit costs can also be represented diagrammatically using nodes and edges. The diagrammatic representation of the problem statement is given below.

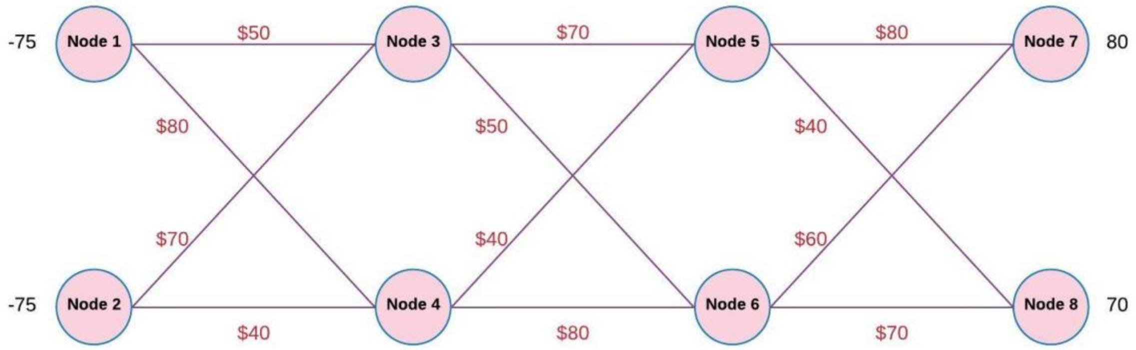


Figure: Problem representation - diagrammatical

In the above diagram, the circular objects are the nodes and the lines connecting the nodes are the edges. The values of unit costs are given on the edges which tells us the cost to move products between those nodes.
Node 1 and node 2 here are supply nodes, i.e. the nodes FROM where products are 'sent'. Whereas, node 7 and node 8 are the demand nodes which 'receives' products from supply nodes (1 and 2).

Here, in this scenario the supply nodes have an equal supply of 75 each which is denoted by the text '-75' besides node 1 and 2. The minus sign signifies that the products 'leave' from that node.
Also, the demand (80+70) is equal to the supply (75+75). Hence balance of flow at each node is:

Inflow – Outflow = Demand or Supply

2(a)

Step 1: Defining Variables (Decision variables)

X_{ij} = No of products that flow between node 'i' and 'j'
&
X_{jk} = No of products that flow between node 'j' and 'k'

Hence, the intermediary nodes are denoted by j, the supply nodes by i, and the demand nodes by k

Therefore, i= Node 1 and 2
j = Node 3, node 4, node 5, node 6
k= Node 7 and node 8

Therefore, the below table summarizes the decision variables

FROM	TO	DECISION VARIABLES(X_{ij})	UNIT COST (IN $)
1	3	X_{13}	50
1	4	X_{14}	80
2	3	X_{23}	70
2	4	X_{24}	40
3	5	X_{35}	70
3	6	X_{36}	50
4	5	X_{45}	40
4	6	X_{46}	80
5	7	X_{57}	80
5	8	X_{58}	40
6	7	X_{67}	60
6	8	X_{68}	70

Now, the objective function here is a minimization problem. It is defined an equation of sumproduct of the decision variables and the unit costs associated to it. The unit cost can also be referred to as shipping cost later in the report.

Therefore,

Objective Function = $50X_{13} + 80X_{14} + 70X_{23} + 40X_{24} + 70X_{35} + 50X_{36} + 40X_{45} + 80X_{46} + 80X_{57} + 40X_{58} + 60X_{67} + 70X_{68}$

Constraint definition:

NODE	CONSTRAINT
1	$X_{13} + X_{14} = 75$
2	$X_{23} + X_{24} = 75$
3	$X_{13} + X_{23} - X_{35} - X_{36} = 0$
4	$X_{14} + X_{24} - X_{45} - X_{46} = 0$
5	$X_{35} + X_{45} - X_{57} - X_{58} = 0$
6	$X_{36} + X_{46} - X_{67} - X_{68} = 0$
7	$X_{57} + X_{58} = 80$
8	$X_{67} + X_{68} = 70$

*In addition to this, a non-negativity constraint is also applied where $X_{ij} > 0$ for all values of 'i' and 'j'.

2(b)

As discussed in question 2(a), there are <u>**12**</u> decision variables in the model. These decision variables are denoted by the convention X_{ij}.

SR.NO.	FROM	TO	DECISION VARIABLES(X_{ij})
1	1	3	X_{13}
2	1	4	X_{14}
3	2	3	X_{23}
4	2	4	X_{24}
5	3	5	X_{35}
6	3	6	X_{36}
7	4	5	X_{45}
8	4	6	X_{46}
9	5	7	X_{57}
10	5	8	X_{58}
11	6	7	X_{67}
12	6	8	X_{68}

X_{ij} = No of products that flow between node 'i' and 'j'
&
X_{jk} = No of products that flow between node 'j' and 'k'

Hence, the intermediary nodes are denoted by j, the supply nodes by i, and the demand nodes by k

Therefore, i= Node 1 and 2

j = Node 3, node 4, node 5, node 6
k= Node 7 and node 8

With respect to the network flow, X_{ij} will always be equal to the number of edges in the model. Edges are the lines which connect 2 nodes. X_{ij} corresponds to the No of products that flow between two nodes i and j.

2(c)

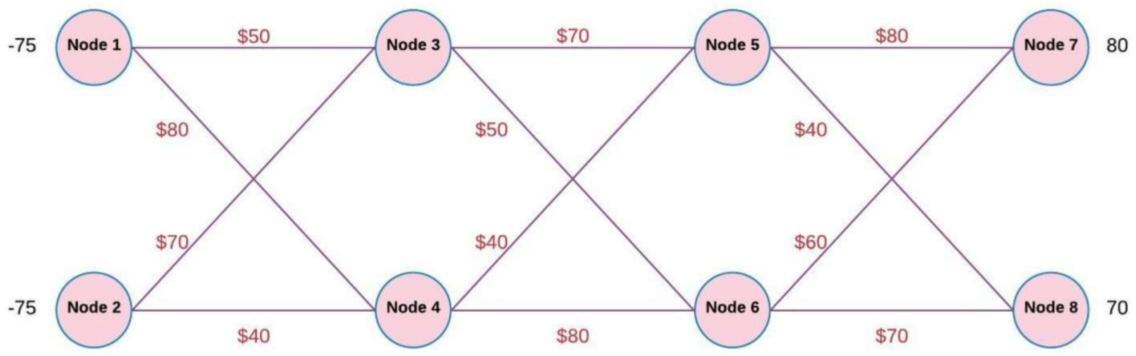
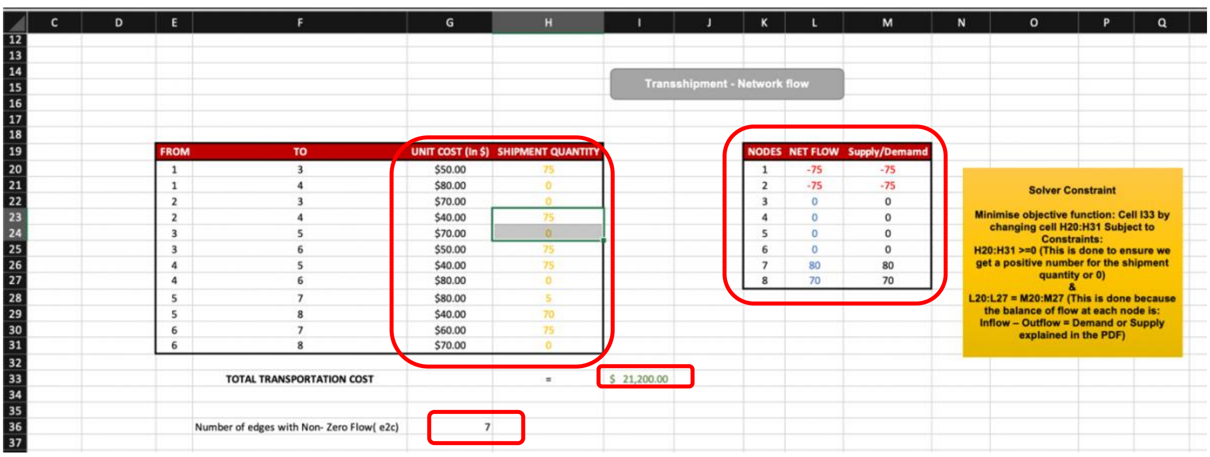


Figure: Network diagram

In worksheet '2(c)' the net flow and the supply/demand are fed cells L20:L27 and M20:M27. In the net flow column, the formula of Inflow – outflow is inputted using the 'SUMIF' function.

Also, for the objective function, the sumproduct of cells H20:H31 and G20:G31 is taken which basically multiplies and adds each shipment quantity with its unit cost to give the total cost.
Therefore, what cell I33 denotes is the objective function. The amount of flow is denoted in cells H20:H31 in column 'Shipment quantity'.


Transshipment - Network flow

FROM	TO	UNIT COST (In $)	SHIPMENT QUANTITY
1	3	$50.00	75
1	4	$80.00	0
2	3	$70.00	0
2	4	$40.00	75
3	5	$70.00	0
3	6	$50.00	75
4	5	$40.00	75
4	6	$80.00	0
5	7	$80.00	5
5	8	$40.00	70
6	7	$60.00	75
6	8	$70.00	0

TOTAL TRANSPORTATION COST = $ 21,200.00

Number of edges with Non- Zero Flow(e2c) 7

NODES	NET FLOW	Supply/Demamd
1	-75	-75
2	-75	-75
3	0	0
4	0	0
5	0	0
6	0	0
7	80	80
8	70	70

Solver Constraint

Minimise objective function: Cell I33 by changing cell H20:H31 Subject to Constraints:
H20:H31 >=0 (This is done to ensure we get a positive number for the shipment quantity or 0)
&
L20:L27 = M20:M27 (This is done because the balance of flow at each node is: Inflow – Outflow = Demand or Supply explained in the PDF)

The value of the objective function from the worksheet is in cell I33. The value is stated as $21200.000.

In terms of the objective function, substitute values of cell H20:H31 as the values of X_{ij},

We get,

Objective function = 50*75 + 80*0 + 70*0 + 40*75 + 70*0 + 50*75 + 40*75 + 80*0 + 80*5 + 40*70 + 60*75 + 70*0

= 21200

From the excel worksheet '2(c)' we can see that there are 7 non-zero values of X_{ij} (Values in the cells H20:H31).

This denotes that there are 7 edges through which the products will flow between nodes.

To do path with least cost manually, we do the following steps:

1. Start with a starting node and assign it the value of 0.
2. Mark all the modes which are corresponding to this starting node
3. Check the cost along that path and compare it with other paths to the same node.
4. Mark the path which has the shortest cost associated to it and shade the evaluated node to show completion
5. Do all steps until you reach the destination node

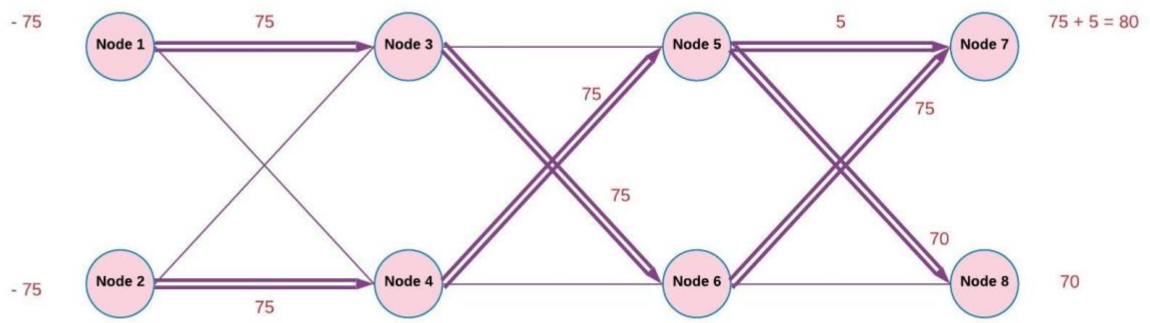


Figure: Amount of flow shown as texts over the edges, as solved by the solver in worksheet '2(c)'

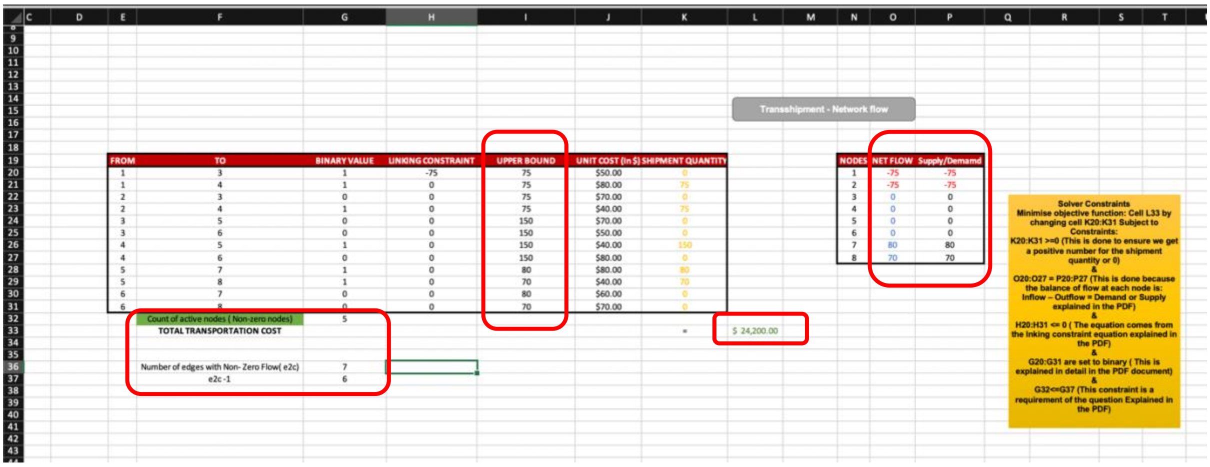
2(d)

Now, the value for e_{2c} in question '2(c)' corresponds to the value of all non-zero edges calculated in worksheet '2(d)' in cell G36 with cross worksheet formulation.

We have set upper bounds for each decision variable to maximize the number of products flowing in the network flow. Upper bounds are mentioned in cells I20:I31. They are calculated by checking what is the maximum quantity that can flow between nodes. For instance, for decision variable X_{13}, the maximum flow can be of 75 as the supply from node 1 can be a maximum of only 75. Similarly, with X_{14} will also have an upper bound of 75. However, for the intermediary nodes, the maximum flow which can take place can be 75

+75 = 150 (The total net flow). Therefore, for decision variables like X_{35}, which have intermediary nodes, will have upper bound of 150. Lastly, for decision variables in the demand nodes/destination nodes, will have the maximum flow of their demand, like X_{68} will have upper bound of 70.

We also add <u>linking constraints</u> which help to find the maximum load size that can traverse trough the network flow.



Transshipment - Network flow

FROM	TO	BINARY VALUE	LINKING CONSTRAINT	UPPER BOUND	UNIT COST (in $)	SHIPMENT QUANTITY
1	3	1	-75	75	$50.00	0
1	4	1	0	75	$80.00	75
2	3	0	0	75	$70.00	0
2	4	1	0	75	$40.00	75
3	5	0	0	150	$70.00	0
3	6	0	0	150	$50.00	0
4	5	1	0	150	$40.00	150
4	6	0	0	150	$80.00	0
5	7	1	0	80	$80.00	80
5	8	1	0	70	$40.00	70
6	7	0	0	80	$60.00	0
6	8	0	0	70	$70.00	0

Count of active nodes (Non-zero nodes)	5
TOTAL TRANSPORTATION COST =	$ 24,200.00
Number of edges with Non- Zero Flow(e2c)	7
e2c -1	6

NODES	NET FLOW	Supply/Demamd
1	-75	-75
2	-75	-75
3	0	0
4	0	0
5	0	0
6	0	0
7	80	80
8	70	70

Solver Constraints
Minimise objective function: Cell L33 by changing cell K20:K31 Subject to Constraints:
K20:K31 >=0 (This is done to ensure we get a positive number for the shipment quantity or 0)
&
O20:O27 = P20:P27 (This is done because the balance of flow at each node is: Inflow – Outflow = Demand or Supply explained in the PDF)
&
H20:H31 <= 0 (The equation comes from the inking constraint equation explained in the PDF)
&
G20:G31 are set to binary (This is explained in detail in the PDF document)
&
G32<=G37 (This constraint is a requirement of the question Explained in the PDF)

The formula used for calculating this value is:

$$X_{ij} <= M_{ij}Y_{ij}$$

OR

$$X_{ij} - M_{ij}Y_{ij} <= 0$$

X_{ij} = Decision variable
M_{ij} = Upper bound
Y_{ij} = Binary value

The new path given by the optimal solution is given below:

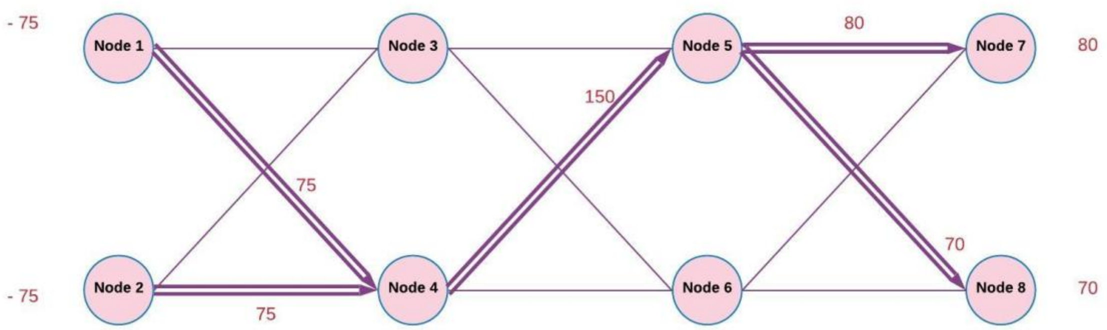


Figure: Path for question '2(d)'

The value of the objective function from the worksheet is in cell I33. The value is stated as $24200.000.

In terms of the objective function, substitute values of cell H20:H31 as the values of X_{ij},

We get,

Objective function = 50*0 + 80*75 + 70*0 + 40*75 + 70*0 + 50*0 + 40*150 + 80*0 + 80*80 + 40*70 + 60*0 + 70*0

= 24200

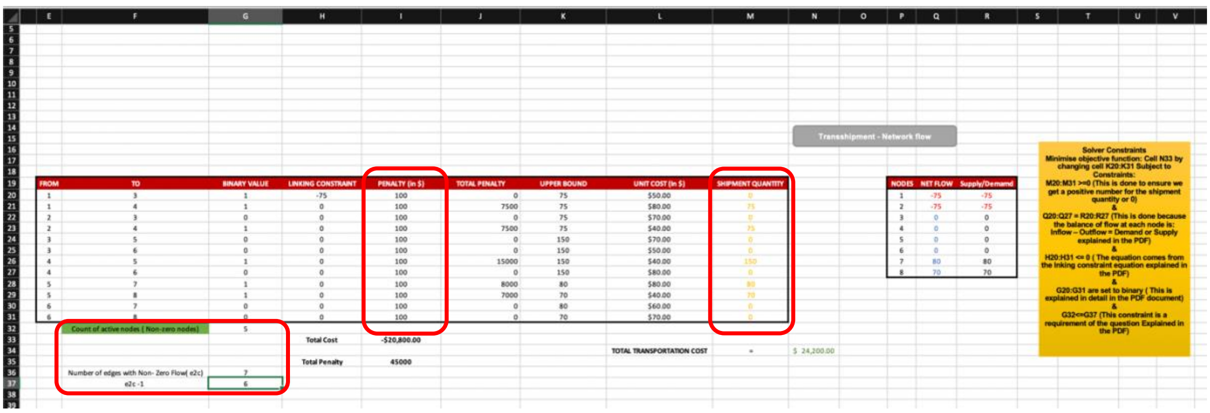
From the excel worksheet '2(d)' we can see that there are 5 non-zero values of X_{ij} (Values in the cells K20:K31).

There are only 5 such shipment quantities which traverse through the network path. This denotes that there are 5 edges through which the products will flow between nodes.
Function Used: Countif (K20:K31,"<>0") in cell G32

This counts the number of rows which have any other value other than 0, and hence gives us the count 5.

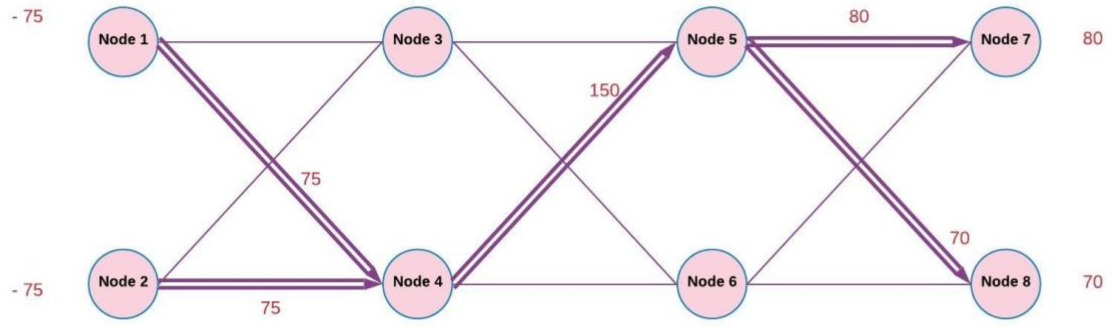
2(e)

In this question we have been given a condition where we have to apply a penalty as hinted in the question.
The question wants us to find a solution with the least non-zero nodes, therefore we have applied a penalty for paths which is active or have a binary value 1. In the workbook '2(e)' the penalties have been multiplied with the shipment quantities. The value for shipment quantities are only > 0 when the node is active. Hence, the ones which are non-zero edges will be penalized; hence the model forces the least number of non-zero edges to be selected by the solver.



The solver gives 5 as the number of non-zero edges in this situation which can be seen in cell G32(Formula used- countif)

Below is the diagrammatic representation of the network flow in this question. It is important to note that it is the same as question 2(d) because the first question is also the optimal solution which chooses the least number of non-zero edges because that is the least costly way to reach the destination point.

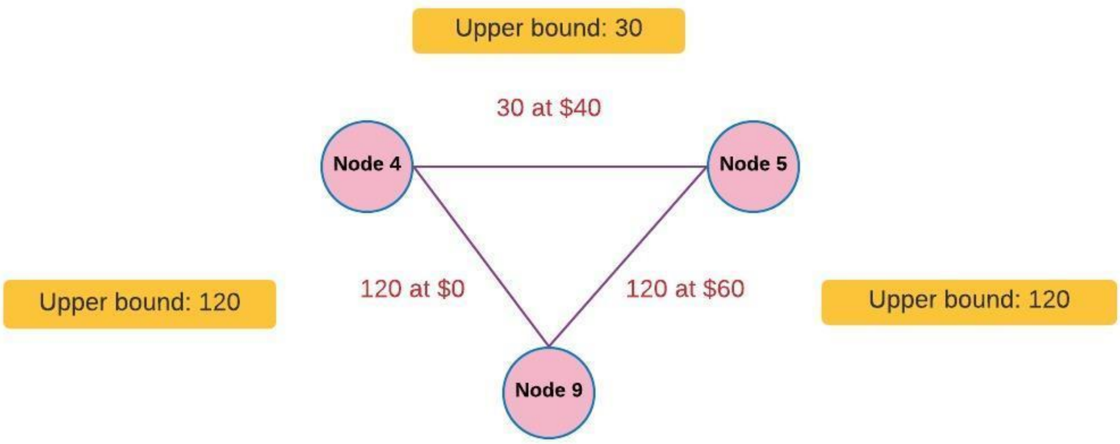


2(f)

Between node 4 and node 5, the unit cost is different depending on the flow quantity between those nodes.

- $40 up to 30 units
- $60 for 30+ units

Therefore, to solve this problem, we will introduce another node, called node 9. This node is introduced to solve the issue of separate costing structure.
What this does is essentially break the edge into 2 edges with different costs associated to it. This solves our problem by isolating each of the two conditions in 2 different scenarios, thus taking care of both.



Th upper bound between nodes 4 and 5 is 30 as that is the maximum it can send, as per the condition stated by the question and at a unit cost of $40. The quantity in excess of 30 will be sent through node 9. The total upper bound between node 4 and node 5 is 150 in total, since these nodes are intermediary nodes. Hence, the upper bound between node 4 – node 9 – node

5 arc is whatever is left of 150. Therefore, 150 – 30 = 120. The upper bound of edges between node 4, 9 and 5 is 120. It is important to note that the value of unit cost between node 4 and 9 is 0, to avoid taking double cost in our calculation, which can spoil our optimal solution.

The optimal solution can be seen in worksheet ‘2(f)’ in cell N34, i.e. $22,100/-

Transshipment - Network flow

FROM	TO	UNIT COST (In $)	SHIPMENT QUANTITY	UPPER BOUND
1	3	$50.00	75	75
1	4	$80.00	0	75
2	3	$70.00	5	75
2	4	$40.00	70	75
3	5	$70.00	0	150
3	6	$50.00	80	150
4	5	$40.00	30	30
4	6	$80.00	0	150
5	7	$80.00	0	80
5	8	$40.00	70	70
6	7	$60.00	80	80
6	8	$70.00	0	70
4	9	$0.00	40	120
9	5	$60.00	40	120

TOTAL TRANSPORTATION COST = $ 22,100.00

NODES	NET FLOW	Supply/Demamd
1	-75	-75
2	-75	-75
3	0	0
4	0	0
5	0	0
6	0	0
7	80	80
8	70	70
9	0	0

Minimise objective function: Cell J35 by changing cell H20:H33 Subject to Constraints:
H20:H33 >=0 (This is done to ensure we get a positive number for the shipment quantity or 0)
&
M20:M28 = N20:N28 (This is done because the balance of flow at each node is: Inflow – Outflow = Demand or Supply explained in the PDF)
&
H20:H33 <= I20:I33 (The value of decision varibales or shipping quantity cannot be greater than the upper bound)

2(g)

Between node 4 and node 5, the unit cost is different depending on the flow quantity between those nodes.

- $40 up to 30 units
- $60 for 30- 55 units
- 110 for 55+

Therefore, to solve this problem, in worksheet ‘2(g)’ we will introduce another node, called node 9 and node 10. These nodes are introduced to solve the issue of separate costing structures.
What this does is essentially break the edge into 3 edges with different costs associated to it. This solves our problem by isolating each of the three conditions in 3 different scenarios, thus taking care of it.

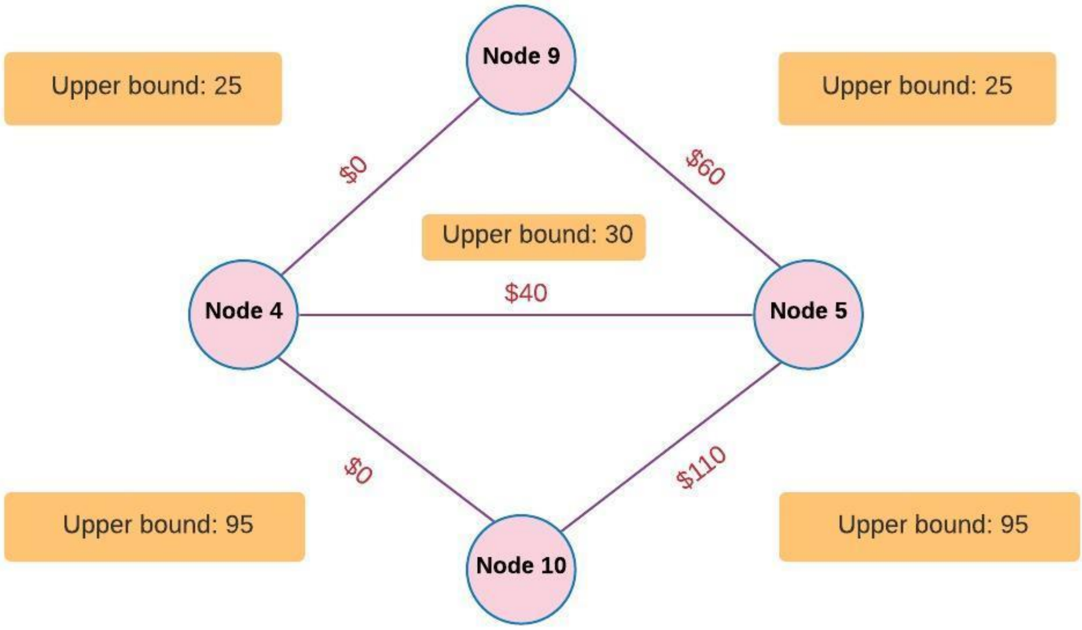
Transshipment - Network flow

FROM	TO	UNIT COST (In $)	SHIPMENT QUANTITY	UPPER BOUND
1	3	$50.00	75	75
1	4	$80.00	0	75
2	3	$70.00	20	75
2	4	$40.00	55	75
3	5	$70.00	15	150
3	6	$50.00	80	150
4	5	$40.00	30	30
4	6	$80.00	0	150
5	7	$80.00	0	80
5	8	$40.00	70	70
6	7	$60.00	80	80
6	8	$70.00	0	70
4	9	$0.00	25	25
9	5	$60.00	25	25
4	10	$0.00	0	95
10	5	$110.00	0	95

TOTAL TRANSPORTATION COST = $ 22,700.00

NODES	NET FLOW	Supply/Demamd
1	-75	-75
2	-75	-75
3	0	0
4	0	0
5	0	0
6	0	0
7	80	80
8	70	70
9	0	0
10	0	0

Minimise objective function: Cell J37 by changing cell H20:H35 Subject to Constraints:
H20:H35 >=0 (This is done to ensure we get a positive number for the shipment quantity or 0)
&
M20:M29 = N20:N29 (This is done because the balance of flow at each node is: Inflow – Outflow = Demand or Supply explained in the PDF)
&
H20:H35 <= I20:I35 (The value of decision varibales or shipping quantity cannot be greater than the upper bound)



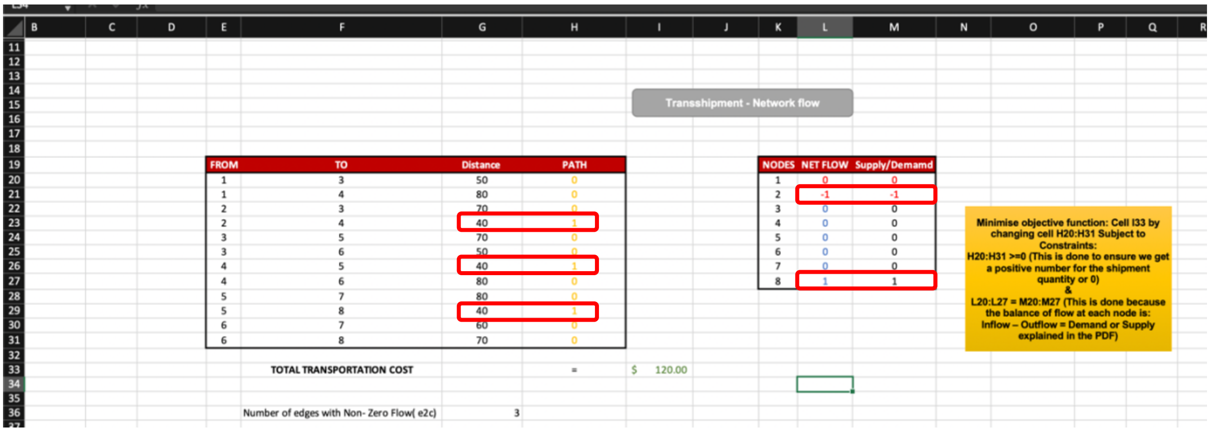
Th upper bound between nodes 4 and 5 is 30 as that is the maximum it can send, as per the condition stated by the question and at a unit cost of $40. Any shipment in excess of 30 but less than 55 goes through node 9. Now, since till 30, the cost is $40, then if 55 products are to be moved between nodes, then the upper bound of edge between node 4 and 9 will be 55-30 = 25. Hence, the upper bound of edge between node 4 and node 9 is 25.
Similarly, any product movement above the quantity of 55 will traverse through node 10. Therefore, the remaining from the upper bound is 150 – 55 = 95.

The optimal solution can be seen in worksheet ‘2(g)’ in cell J37, i.e. $22,700/-.

2(h)

According to the question, the unit costs given for each edge is now the distance between the nodes. Also, node 1 is now not a supply node anymore. Hence all the transfer of products between now start from Node 2. Hence, we do not require the supply and demand quantities, we just consider it -1 and 1. -1 for supply node (Node 2 here) and 1 for demand nodes (Node 8 here).

Hence, the same changes have to be reflected in the excel worksheet ‘2(h)’.
To incorporate the -1 and 1 in the excel, we hardcode 1 and -1 for Node 8 and Node 2 in cells M27 and M21 respectively. All the other nodes are now intermediary nodes and hence have to be equated to 0 in the supply/demand cells of M20:M27.



Transshipment - Network flow

FROM	TO	Distance	PATH
1	3	50	0
1	4	80	0
2	3	70	0
2	4	40	1
3	5	70	0
3	6	50	0
4	5	40	1
4	6	80	0
5	7	80	0
5	8	40	1
6	7	60	0
6	8	70	0

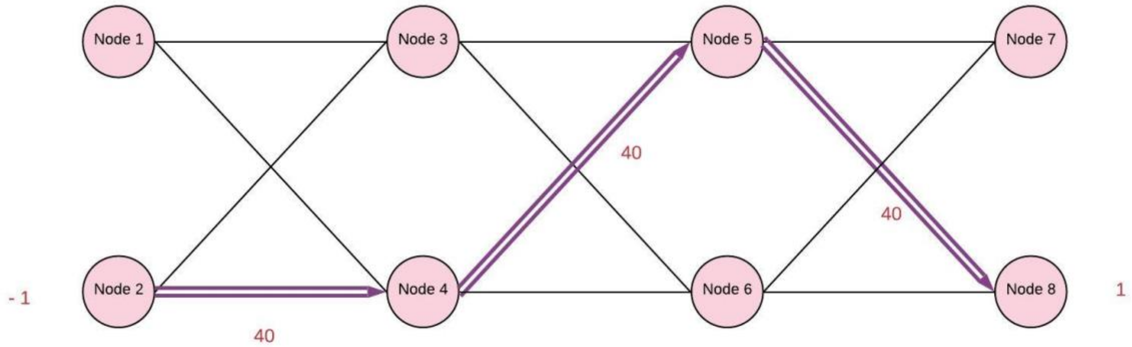
TOTAL TRANSPORTATION COST = $ 120.00

Number of edges with Non- Zero Flow(e2c) 3

NODES	NET FLOW	Supply/Demamd
1	0	0
2	-1	-1
3	0	0
4	0	0
5	0	0
6	0	0
7	0	0
8	1	1

Minimise objective function: Cell I33 by changing cell H20:H31 Subject to Constraints:
H20:H31 >=0 (This is done to ensure we get a positive number for the shipment quantity or 0)
&
L20:L27 = M20:M27 (This is done because the balance of flow at each node is: Inflow – Outflow = Demand or Supply explained in the PDF)

The shortest path given by the excel in the solver is shown diagrammatically below:



The 40 is the distance between the nodes, which here is coincidently constant among the edges of the shortest path.

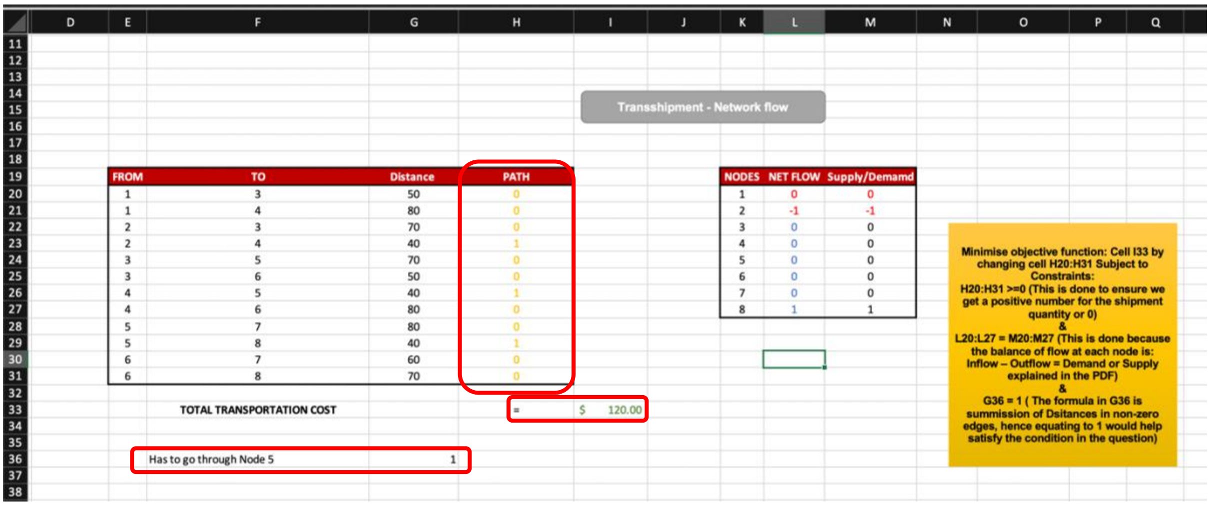
Hence, the total length of the path will be 40+40+40 = 120. That is the same as the answer from the solver, shown in cell I33.

2(i)

PART 1

In this question, the condition awarded by the question is that node 2 and node 8 are the supply/start and demand/destination nodes and the path should go through node 5. In excel term, in workbook '2(i) P1' we can see that in order to incorporate this, the sum of binary values in the Path column in cells which have either 'To' or 'From' having node value of 5. I have taken reference of 'From' column, but we can also do it through the 'To' column.

In cell G36, the formula ensures that at least once the path will pass through node 5. Constraint says that value of G36 has to be one. So it ensures that path goes through node 5 once and only once.



Transshipment - Network flow

FROM	TO	Distance	PATH
1	3	50	0
1	4	80	0
2	3	70	0
2	4	40	1
3	5	70	0
3	6	50	0
4	5	40	1
4	6	80	0
5	7	80	0
5	8	40	1
6	7	60	0
6	8	70	0

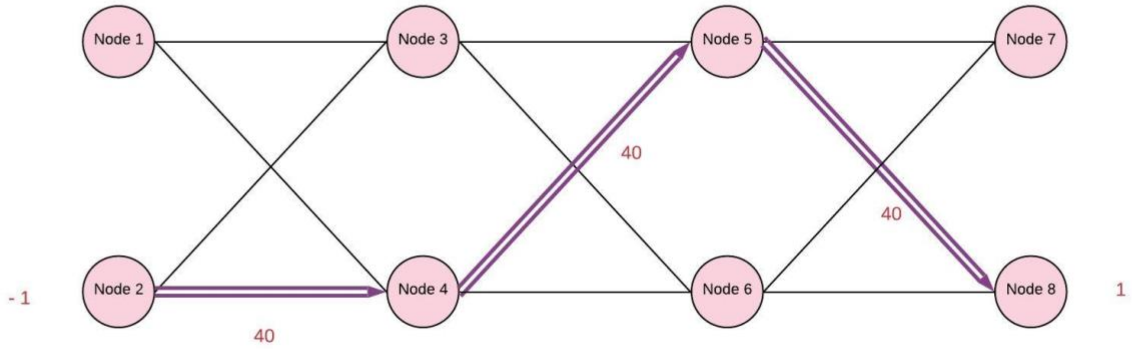
TOTAL TRANSPORTATION COST = $ 120.00

Has to go through Node 5 1

NODES	NET FLOW	Supply/Demamd
1	0	0
2	-1	-1
3	0	0
4	0	0
5	0	0
6	0	0
7	0	0
8	1	1

Minimise objective function: Cell I33 by changing cell H20:H31 Subject to Constraints:
H20:H31 >=0 (This is done to ensure we get a positive number for the shipment quantity or 0)
&
L20:L27 = M20:M27 (This is done because the balance of flow at each node is: Inflow – Outflow = Demand or Supply explained in the PDF)
&
G36 = 1 (The formula in G36 is summission of Dsitances in non-zero edges, hence equating to 1 would help satisfy the condition in the question)

Below is the diagrammatic representation of the shortest path between node 2 and 8 which passes through node 5:



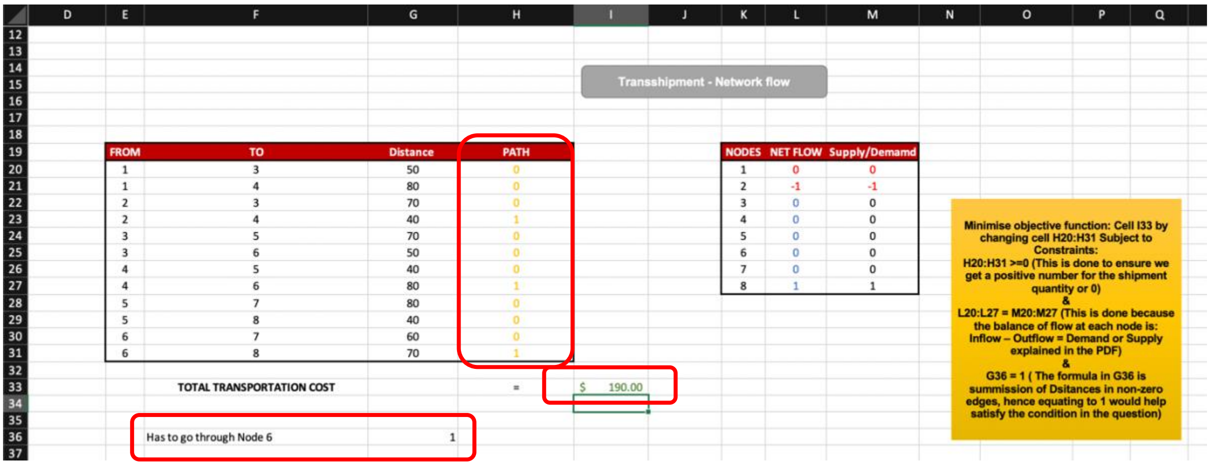
It's important to note that the network path is same as in question 2(h), because 2(h) was an optimal path which did pass node 5 in its solution anyway. Hence, we notice no change.

Hence, the total length of the path will be 40+40+40 = 120. That is the same as the answer from the solver, shown in cell I33.

PART 2

In this question, the condition awarded by the question is that node 2 and node 8 are the supply/start and demand/destination nodes and the path should go through node 6. In excel term, in workbook '2(i) P2' we can see that in order to incorporate this, the sum of binary values in the Path column in cells which have either 'To' or 'From' having node value of 6. I have taken reference of 'From' column, but we can also do it through the 'To' column.

In cell G36, the formula ensures that at least once the path will pass through node 6. Constraint says that value of G36 has to be one. So it ensures that path goes through node 6.



Transshipment - Network flow

FROM	TO	Distance	PATH
1	3	50	0
1	4	80	0
2	3	70	0
2	4	40	1
3	5	70	0
3	6	50	0
4	5	40	0
4	6	80	1
5	7	80	0
5	8	40	0
6	7	60	0
6	8	70	1

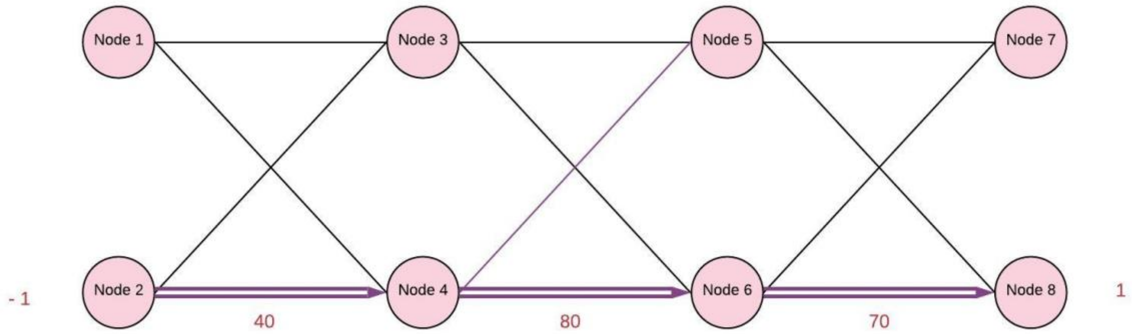
TOTAL TRANSPORTATION COST = $ 190.00

Has to go through Node 6 1

NODES	NET FLOW	Supply/Demamd
1	0	0
2	-1	-1
3	0	0
4	0	0
5	0	0
6	0	0
7	0	0
8	1	1

Minimise objective function: Cell I33 by changing cell H20:H31 Subject to Constraints:
H20:H31 >=0 (This is done to ensure we get a positive number for the shipment quantity or 0)
&
L20:L27 = M20:M27 (This is done because the balance of flow at each node is: Inflow – Outflow = Demand or Supply explained in the PDF)
&
G36 = 1 (The formula in G36 is summission of Dsitances in non-zero edges, hence equating to 1 would help satisfy the condition in the question)

Below is the diagrammatic representation of the shortest path between node 2 and 8 which passes through node 6:



Hence, the total length of the path will be 40+80+70 = 190. That is the same as the answer from the solver, shown in cell I33.

3. Given:

A: 1000
k: 21
h: 25%

c:

Cost of Goods	Discount %	Minimum Quantity	Maximum Quantity
4	0	1	794
3.8	5	795	1099
3.68	8	1100	1859
3.4	15	1860	+

<u>EOQ (Economical Order Quantity) for c = 4</u>:

sqrt(2Ak/ch) = √2(1000)(21)/(0.25)(4) = 204.9390

Total Cost : Ak/Q(ordering Cost) +Qch/2(holding cost) + Ac(Cost of purchase)

Cases:

1. For C=4

Formula Q*(EOQ) = sqrt (2Ak/(ch))
Q* = sqrt (2*1000*21/ (0.25*4))
= 204.93

Total Cost:
Total Annual cost = Ordering Cost + Holding Cost + Cost of Purchase
= Ak/Q* + Q*ch/2 + Ac
= 1000*21/204.93 + 204.93*4*0.25/2 + 1000*4
Total Annual Cost = 4204.94

Similarly,

2. For C= 3.8

Formula Q*(EOQ) = sqrt(2Ak/(ch))
Q* = sqrt (2*1000*21/ (0.25*3.8))
=210.26
Note: But there is a constraint in play here. A minimum order constraint of 795, as per the question.
Hence Q*= 795

Total Cost:
Total Annual cost = Ordering Cost + Holding Cost + Cost of Purchase
= Ak/Q* + Q*ch/2 + Ac
= 1000*21/795 + 795*3.8*0.25/2 + 1000*3.8
Total Annual Cost = 4204.04

3. For C= 3.68

Formula Q*(EOQ) = sqrt(2Ak/(ch))
Q* = sqrt (2*1000*21/ (0.25*3.68))
=213.66
Note: But there is a constraint here in play here. A minimum order constraint of 1100, as per the question.
Hence Q*= 1100

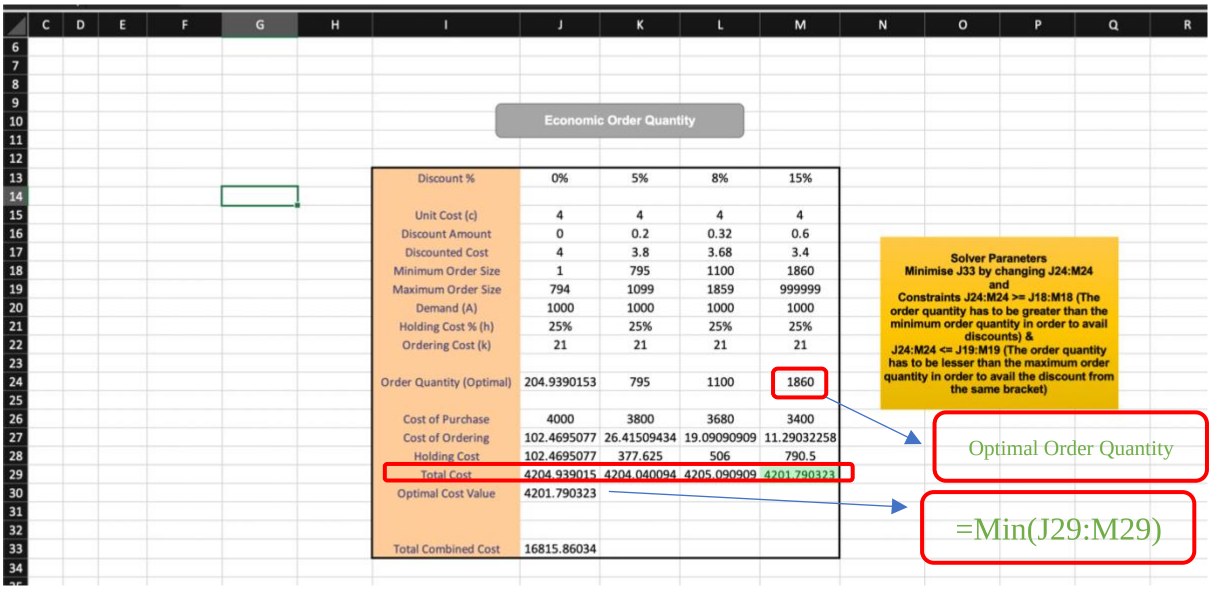
Total Cost:
Total Annual cost = Ordering Cost + Holding Cost + Cost of Purchase
= Ak/Q* + Q*ch/2 + Ac

= 1000*21/795 + 795*3.68*0.25/2 + 1000*3.68
Total Annual Cost = 4205.091

4. For C= 3.4

Formula Q*(EOQ) = sqrt(2Ak/(ch))
Q* = sqrt (2*1000*21/ (0.25*3.4))
=222.28
Note: But there is a constraint here in play here. A minimum order constraint of 1860, as per the question.
Hence Q*= <u>1860</u>

Total Cost:
Total Annual cost = Ordering Cost + Holding Cost + Cost of Purchase
= Ak/Q* + Q*ch/2 + Ac
= 1000*21/795 + 795*3.4*0.25/2 + 1000*3.4
Total Annual Cost = 4201.790



Economic Order Quantity

Discount %	0%	5%	8%	15%
Unit Cost (c)	4	4	4	4
Discount Amount	0	0.2	0.32	0.6
Discounted Cost	4	3.8	3.68	3.4
Minimum Order Size	1	795	1100	1860
Maximum Order Size	794	1099	1859	999999
Demand (A)	1000	1000	1000	1000
Holding Cost % (h)	25%	25%	25%	25%
Ordering Cost (k)	21	21	21	21
Order Quantity (Optimal)	204.9390153	795	1100	1860
Cost of Purchase	4000	3800	3680	3400
Cost of Ordering	102.4695077	26.41509434	19.09090909	11.29032258
Holding Cost	102.4695077	377.625	506	790.5
Total Cost	4204.939015	4204.040094	4205.090909	4201.790323
Optimal Cost Value	4201.790323			
Total Combined Cost	16815.86034			

Now, the total Costs from the calculations are summarised in the table below:

Cost of Goods	Total Cost
4	**4204.94**
3.8	**4204.04**
3.68	**4205.091**
3.4	**4201.790**

The least value for total cost

From the above table it's clear that the cost is least when the goods are bought at $3.4 after availing 15% discount.

It also makes sense that we buy the minimum order quantity to avail that discount as the actual economical order quantity at full price is just 204.93. But since we do not have an issue with inventorying large quantities, the maximum discount should be availed.

Hence,
Optimal Order Quantity: 1860 (The minimum order quantity to avail the maximum discount of 15%)

Optimal Total Annual Cost: $4201.790

Similarly, in the excel file, under the worksheet 'Question 3', we can see the same results through the solver in cell J33

Working of excel sheet:
All the given details (A, k, c, h, discounts, minimum order quantity, and maximum order quantity) from the question are put in.
The total costs of the different discounted costs are calculated by formulas.

In cell J33, addition of all total costs from J29:M29 are taken. Using solver, we minimize the value of cell J33 by changing the order quantity cells of J24:M24. There are 2 constraints which are mentioned in the excel sheet.