

## Homework 2

**Task 1.** Write a Fortran program that:

- 1) Declares two integers of different kinds, e.g. `integer(kind=4)` and `integer(kind=8)`.
- 2) Computes the factorial  $n!$  in a loop from  $n = 1$  upwards.
- 3) Stops when an overflow occurs (e.g. when the value becomes negative or smaller than the previous one).
- 4) Prints the last correct value and the  $n$  where overflow happened.

### Observations

Every data type, in this case integer has a specified bit sized memory allocated for its value. An **Integer overflow** occurs when a calculation exceeds this specified allocation it causes the value to wrap around to a smaller number or result in undefined behavior (like **negative numbers in this case**). In this task integer overflow was demonstrated by taking factorials of two kinds of integer, namely integer **kind=4** and **kind=8**, each of them has **4 bytes (32 bits)** and **8 bytes (64)** of memory respectively.

For **kind=4**, program overflows at 14 by repeating the same result, for **kind=8**, overflows at 21 by resulting in negative numbers. Therefore, precisely demonstrating how memory allocation affects overflow.

---

```
1  program overflow_factorial
2      implicit none
3
4      integer(kind=4):: fact4, previous_fact4
5      integer(kind=8):: fact8, previous_fact8
6      integer :: n
7
8      !initializing to avoid error
9      fact4 = 1
10     fact8 = 1
11     previous_fact4 = 1
12     previous_fact8 = 1
13
14     do n =1,50
15         if(n>1) then
16             previous_fact4 = fact4
17             fact4 = fact4*n
18             if(fact4 < previous_fact4) then
19                 print*, "Oveflow Fact4 at n =", n
20                 exit
21             end if
22         end if
23     end do
24
25     do n=1,50
26         if(n>1) then
27             previous_fact8 = fact8
28             fact8 = fact8*n
29             if(fact8<previous_fact8) then
30                 print*, "Overflow Fact8 at n=", n
31                 exit
32             end if
33         end if
```

```

34     end do
35 end program overflow_factorial

```

LISTING 1. Code to demonstrate Integer Overflow

n	fact4	n	fact8
2	2	2	2
3	6	3	6
4	24	4	24
5	120	5	120
6	720	6	720
7	5040	7	5040
8	40320	8	40320
9	362880	9	362880
10	3628800	10	3628800
11	39916800	11	39916800
12	479001600	12	479001600
13	1932053504	13	6227020800
14	1278945280	14	87178291200
Overflow Fact4 at n = 14		15	1307674368000
		16	20922789888000
		17	355687428096000
		18	6402373705728000
		19	121645100408832000
		20	2432902008176640000
		21	-4249290049419214848
		Overflow Fact8 at n= 21	

FIGURE 1. Demonstration of integer overflow

**Task 2.** Write a program similar to the in-class example:

- 1) Start with `big = 1.0d0` and `small = 1.0d0`.
- 2) In each loop iteration:
  - compute `sum = big + small`,
  - print `small`, `big`, and `sum`,
  - divide `small` by 2.
- 3) Stop when adding `small` no longer increases `big`

## Observations

Loss of significance is an undesirable effect in calculations using floating-point arithmetic. Computer systems use floating point arithmetic to express fractional numbers. In this code we try to demonstrate the loss of significance or precision. To be exact, the program shows that when adding a very small number(`small`) to a much larger one(`big`), the result of their addition stops changing as the system cannot represent the difference in numbers after a certain limit called **machine epsilon**. This happens due to the finite number of bits available to store floating point numbers which limits precision of real numbers.

In this case, the limit reaches at , therefore, we can say that it is our machine epsilon.

---

```

1  program significance
2      implicit none
3
4
5      real(kind=8) :: small,big, summ
6      integer :: i,d
7
8      big = 1.0d0
9      small = 1.0d0
10
11     do i=1,1000
12         summ = big+small
13         print*, small, big, summ
14         if(summ<= big) then
15             print *, 'Adding small has no effect on big at iteration', i
16             exit
17         end if
18         small = small/2.0d0
19     end do
20 end program significance

```

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LISTING 2. Code to demonstrate Integer Overflow

### Task 3. Reflect

#### 1) What numerical property limits integer types?

Integer types are limited by their **range**, which is determined by the number of bits used to store them. For example, a signed 32-bit integer can represent values from -2,147,483,648 to 2,147,483,647.

#### 2) What numerical property limits real types?

The numerical properties that limit real types include their **maximum values and precision**, which are determined by the specific implementation and the number of bits allocated for storage. Typically, single precision real numbers can represent about seven decimal digits, while double precision can represent around fifteen decimal digits.

#### 3) Which error (overflow or precision loss) is more dangerous for scientific simulations, and why?

Precision loss is generally more dangerous for scientific simulations because it can lead to significant inaccuracies in calculations, especially when small changes in input can result in large discrepancies in output. Overflow can often be detected and handled, while precision loss may go unnoticed, compromising the reliability of the results.

**Task 4. Bonus**

Implement a single program that performs both experiments sequentially: first integer factorial overflow, then real precision loss. Print the number of iterations before overflow and before significance loss. Comment how these limits correspond to the bit length of the data types used.

integer overflow		loss of significance (kind =4)		
n	factorial(kind=4)	small	big	sum
2	2	1.00000000	1.00000000	2.00000000
3	6	0.50000000	1.00000000	1.50000000
4	24	0.25000000	1.00000000	1.25000000
5	120	0.12500000	1.00000000	1.12500000
6	720	6.25000000E-02	1.00000000	1.06250000
7	5040	3.12500000E-02	1.00000000	1.03125000
8	40320	1.56250000E-02	1.00000000	1.01562500
9	362880	7.81250000E-03	1.00000000	1.00781250
10	3628800	3.90625000E-03	1.00000000	1.00390625
11	39916800	1.95312500E-03	1.00000000	1.00195312
12	479001600	9.76562500E-04	1.00000000	1.00097656
13	1932053504	4.88281250E-04	1.00000000	1.00048828
14	1278945280	2.44140625E-04	1.00000000	1.00024414
overflow at n =		14	1.22070312E-04	1.00012207
			6.10351562E-05	1.00006104
			3.05175781E-05	1.00003052
			1.52587891E-05	1.00001526
			7.62939453E-06	1.00000763
			3.81469727E-06	1.00000381
			1.90734863E-06	1.00000191
			9.53674316E-07	1.00000095
			4.76837158E-07	1.00000048
			2.38418579E-07	1.00000024
			1.19209290E-07	1.00000012
			5.96046448E-08	1.00000000
			Addition has no effect	25

FIGURE 2. Demonstration of integer overflow and loss of significance