COMPUTATIONAL HEAT AND FLUID FLOW (ME605)

Assignment 4: Convection and Diffusion

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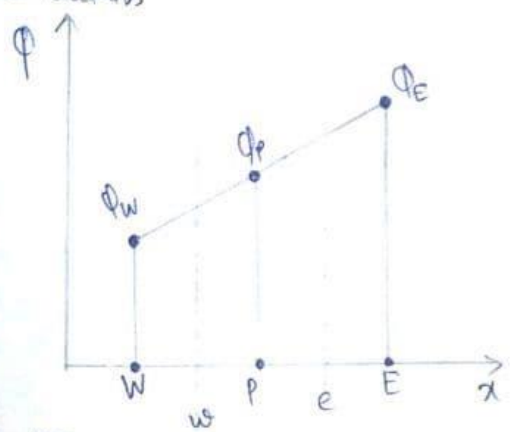
Ques1 We have used a piecewise linear profile assumption for the transported variable ϕ.

i) Grid details



Using the above prescribed profile, we estimate ϕ inside each cell. We have used finite volume method.
The entire domain is partitioned into $(n+1)$ equal intervals with each interior interval being of length Δx & volume $\Delta x \times 1 \times 1$ (corresponding cell) and both the boundary intervals of length $\frac{\Delta x}{2}$ & corresponding cells of volume $\frac{\Delta x}{2} \times 1 \times 1$.

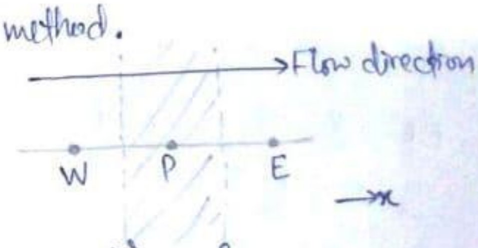
The control volume boundaries are located at the mid-points of each cell length & the boundaries for the boundary cells are located on the respective ends

ϕ_A 1 2 3 4 ... $(n-1)$ n ϕ_B

ii) The discretization is done using finite volume method.

The governing equation is:

$$\frac{\partial}{\partial t}(\rho\phi) + \frac{\partial}{\partial x}(\rho u\phi) = \frac{\partial}{\partial x}\left(\Gamma\frac{\partial\phi}{\partial x}\right)$$



Assumptions:

a) No source term e) Constant properties
b) $(\delta x)_e = (\delta x)_w = \Delta x$ f) 1-Dimensional
c) at $t = t_0$, $\phi_P = \phi_P^0$ & $t = t_0 + \Delta t$, $\phi_P = \phi_P$
d) flow is in +ve x-direction.

Integrating the governing equation over time & domain of interest

$$\int_w^e \int_t^{t+\Delta t} \frac{\partial}{\partial t}(\rho\phi)\, dt\, dx \times 1 \times 1 + \int_t^{t+\Delta t}\int_w^e \frac{\partial}{\partial x}(\rho u\phi)(dx \times 1 \times 1) \times dt = \int_t^{t+\Delta t}\int_w^e \frac{\partial}{\partial x}\left(\Gamma\frac{\partial\phi}{\partial x}\right)(dx \times 1 \times 1)\, dt$$

$$(\rho_P\phi_P - \rho_P^o\phi_P^o)\Delta x + \int_t^{t+\Delta t}\{(\rho u\phi)_e - (\rho u\phi)_w\}dt = \int_t^{t+\Delta t}\left\{\left(\Gamma\frac{d\phi}{dx}\right)_e - \left(\Gamma\frac{d\phi}{dx}\right)_w\right\}dt$$

Now, approximating $\int_t^{t+\Delta t}\phi_P dt$ as $[f\phi_P' + (1-f)\phi_P^o]\Delta t$ & employing fully implicit approach by putting $f=1$, we get the following form: for all internal points: (2 to (n-1))

$$(\rho_P\phi_P - \rho_P^o\phi_P^o)\Delta x + \left((\rho u\phi)_e - (\rho u\phi)_w\right)\Delta t = \left(\left(\Gamma\frac{d\phi}{dx}\right)_e - \left(\Gamma\frac{d\phi}{dx}\right)_w\right)\Delta t$$

① CDS scheme

$$(\rho_P\phi_P - \rho_P^o\phi_P^o)\Delta x + (\rho u)_e\Delta t\left[\frac{\phi_P+\phi_E}{2}\right] - (\rho u)_w\Delta t\left[\frac{\phi_P+\phi_W}{2}\right] = \frac{\Gamma_e}{\Delta x}(\phi_E-\phi_P) - \frac{\Gamma_w}{\Delta x}(\phi_P-\phi_W)$$

$$\phi_P\left(\frac{\rho_P\Delta x}{\Delta t} + \frac{F_e}{2} - \frac{F_w}{2} + D_e + D_w\right) = \phi_E\left(D_e - \frac{F_e}{2}\right) + \phi_W\left(D_w + \frac{F_w}{2}\right) + \frac{\rho_P^o\Delta x}{\Delta t}\phi_P^o$$

$\therefore a_E = D_e - \frac{F_e}{2} \longrightarrow$ let it be α $\qquad a_P^o = \frac{\rho_P\Delta x}{\Delta t} \longrightarrow$ let it be μ

$a_W = D_w + \frac{F_w}{2} \longrightarrow$ let it be β $\qquad a_P = a_E + a_W + a_P^o \longrightarrow$ let it be δ

from continuity equation, we get that $F_e = F_w$, this is a result that we will follow everywhere & will not mention again in this report.

② Upwind scheme (flow is in +ve x direction $\Rightarrow$ -ve x is the upstream)

$$(\rho_P\phi_P - \rho_P^o\phi_P^o)\Delta x + (\rho u)_e\Delta t\,\phi_P - (\rho u)_w\Delta t\,\phi_W = \frac{\Gamma_e}{\Delta x_e}(\phi_E-\phi_P)\Delta t - \frac{\Gamma_w}{\Delta x_w}(\phi_P-\phi_W)$$

$$\phi_P\left(\frac{\rho_P\Delta x}{\Delta t} + F_e + D_e + D_w\right) = \phi_E(D_e) + \phi_W(F_w + D_w) + \frac{\rho_P^o\Delta x}{\Delta t}\phi_P^o$$

$a_E = D_e \longrightarrow \alpha$ $\qquad a_P^o = \frac{\rho_P^o\Delta x}{\Delta t} \longrightarrow \mu$

$a_W = F_w + D_w \longrightarrow \beta$ $\qquad a_P = a_E + a_W + a_P^o \longrightarrow \delta$

③ Hybrid scheme

$a_E = -F_e$ & $a_W = 0$; $P_e < -2$

$a_E = D_e - \frac{F_e}{2}$ & $a_W = D_w + \frac{F_w}{2}$; $P_e \in [-2, 2]$

$a_E = 0$ & $a_W = F_w$; $P_e > 2$

where P_e is the Peclet number, which signifies the strength of convection to diffusion, defined by:

$$P_e = \frac{F}{D} = \frac{(\rho u)}{\Gamma/L} = \frac{\text{strength of convection}}{\text{strength of diffusion}}$$

let $a_E \longrightarrow \alpha$, $a_W \longrightarrow \beta$

$a_P^\circ = \frac{\rho_P^\circ \Delta x}{\Delta t} \longrightarrow \mu$ & $a_P = a_E + a_W + a_P^\circ \longrightarrow \delta$

all the above equations/system of equations are of the form:

for all interior points:

$$-a_W \phi_W^{k+1} + a_P \phi_P^{k+1} - a_E \phi_E^{k+1} = a_P^\circ \phi_P^k$$

where k = previous time step & $k+1$ = current time step

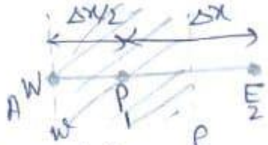
or we can write it as:

$$-\beta \phi_W^{k+1} + \delta \phi_P^{k+1} - \alpha \phi_E^{k+1} = \mu \phi_P^k$$

iii) Boundary condition implementation details

For the expressions at the boundary points, we can again integrate the governing equations over the boundary points. We will get the following expressions for all schemes:

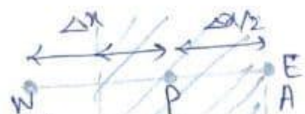
for left boundary:



$$-\phi_W^{k+1}(a_W + D_W) + \phi_P^{k+1}(a_P + D_W) - \phi_E^{k+1} a_E = a_P^0 \phi_P^k$$

Here $\phi_W^{k+1} = \phi_A$ & the extra D_w is due to the western volume being halved.

for right boundary



$$-\phi_W^{k+1} a_W + \phi_P^{k+1}(a_P + D_E) - \phi_E^{k+1}(a_E + D_E) = a_P^0 \phi_P^k$$

$\phi_E = \phi_B$ & extra D_E is due to eastern volume being halved.

$\therefore$ The complete system of equations is:

for left boundary: $a_1\phi_1^{k+1} - a_2\phi_2^{k+1} = a_1^0\phi_1^k + a_A\phi_A$

for intermediate points, $i = 2$ to $(n-1)$: $-\phi_{i-1}^{k+1}a_{i-1} + a_i\phi_i^{k+1} - a_{i+1}\phi_{i+1}^{k+1} = a_i^0\phi_i^k$

for right boundary: $-\phi_{n-1}^{k+1}a_{n-1} + a_n\phi_n^{k+1} = a_n^0\phi_n^k + a_B\phi_B$

putting $a_E = \alpha$, $a_W = \beta$, $a_P^0 = \mu$ & $a_P = \delta$ and writing as a matrix equation:

$$\begin{bmatrix} \delta & -\alpha & 0 & \cdots & & 0 \\ -\beta & \delta & -\alpha & 0 & \cdots & \\ 0 & -\beta & \ddots & \ddots & & \vdots \\ \vdots & & \ddots & \ddots & -\alpha & 0 \\ & \cdots & 0 & -\beta & \delta & -\alpha \\ 0 & \cdots & & 0 & -\beta & \delta \end{bmatrix} \begin{bmatrix} \phi_1^{k+1} \\ \phi_2^{k+1} \\ \vdots \\ \vdots \\ \phi_{n-1}^{k+1} \\ \phi_n^{k+1} \end{bmatrix} = \begin{bmatrix} \mu\phi_1^k + a_A\phi_A \\ \mu\phi_2^k \\ \vdots \\ \vdots \\ \mu\phi_{n-1}^k \\ \mu\phi_n^k + a_B\phi_B \end{bmatrix}$$

Required output (plots/any other means)

Peclet number is a dimensionless number that quantifies the strength of convection to diffusion. It is defined as:

$$Pe = \frac{Convective\ transport\ rate\ \varphi}{Diffusive\ transport\ rate\ of\ \varphi}$$

Or we can define Peclet number in terms of the convective and diffusive coefficient,

$$\boxed{\boldsymbol{Pe = F/D}}$$

Where F = ρu and D = Γ/L, therefore,

$$\boxed{\boldsymbol{Pe = \rho u L/\Gamma}}$$

The above formulation is used for continuous problems, and this number directly influences the analytical solution for steady-state,1D convection-diffusion equation with no source:

$$\boxed{\boldsymbol{\frac{\varphi - \varphi_o}{\varphi_L - \varphi_o} = \frac{e^{xPe/L} - 1}{e^{Pe} - 1}}}$$

But for our purpose, we will define the Peclet number in terms of the grid-length/cell-length Δx as it then can be treated as property, exclusive to the cell, and we will call it the cell Peclet number:

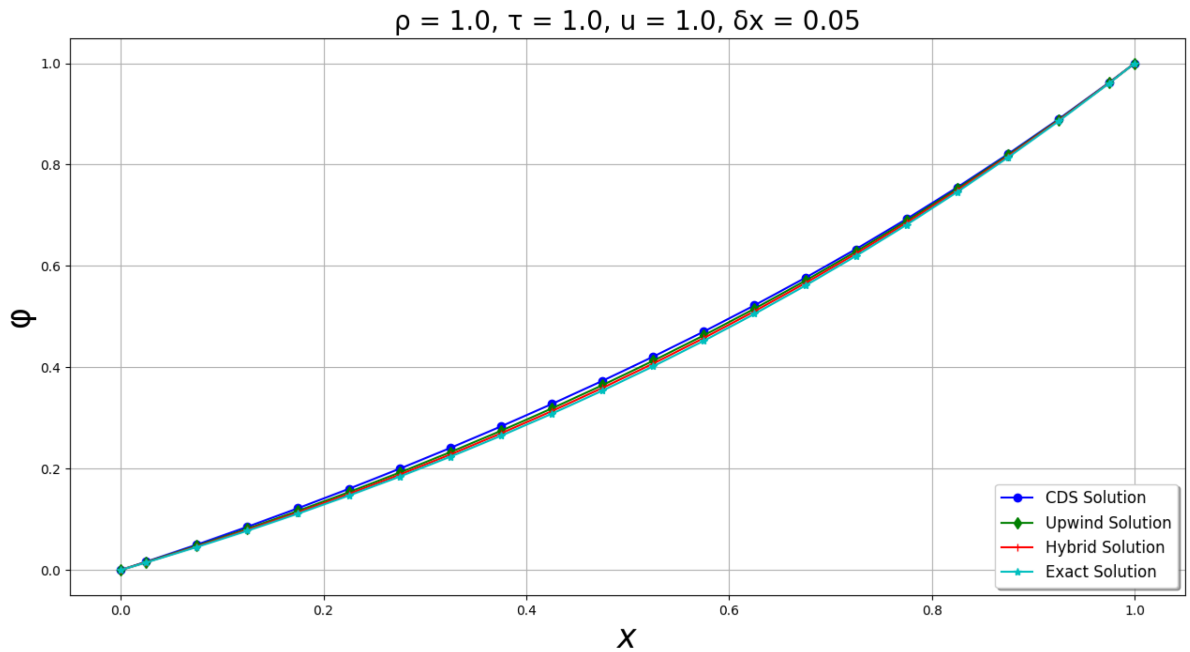
$$\boxed{\boldsymbol{Pe = \rho u \Delta x/\Gamma}}$$

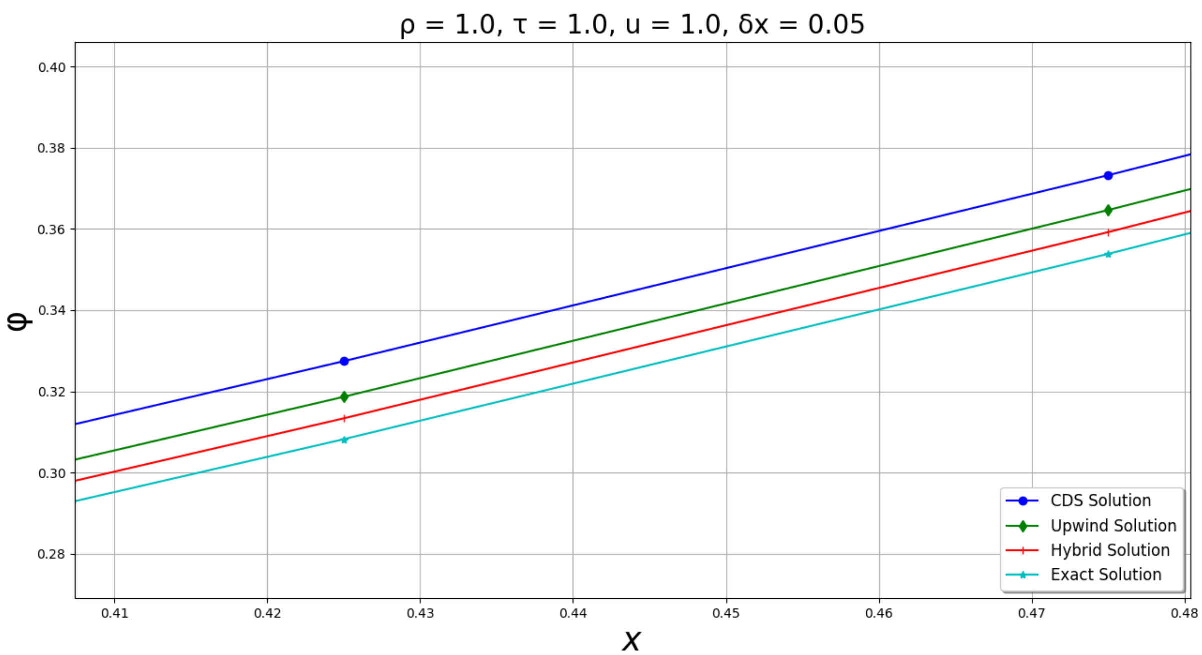
And $\boldsymbol{\Delta x = L/n}$, where L is the length of the domain of interest and n is the number of iterations. Therefore, the final, usable form of Peclet number in our case is:

$$\boxed{\boldsymbol{Pe = \rho u L/n\Gamma}}$$

From the latest equation, we can observe that the Peclet number depends on several factors. To keep the Peclet number the same and change other properties, we have to simultaneously change at least two properties to maintain the constant Peclet number. Therefore, for a computational scheme, the Peclet number depends on the fluid properties, flow conditions, and grid discretization properties.

i) $\rho = 1, \Gamma = 1, u = 1$

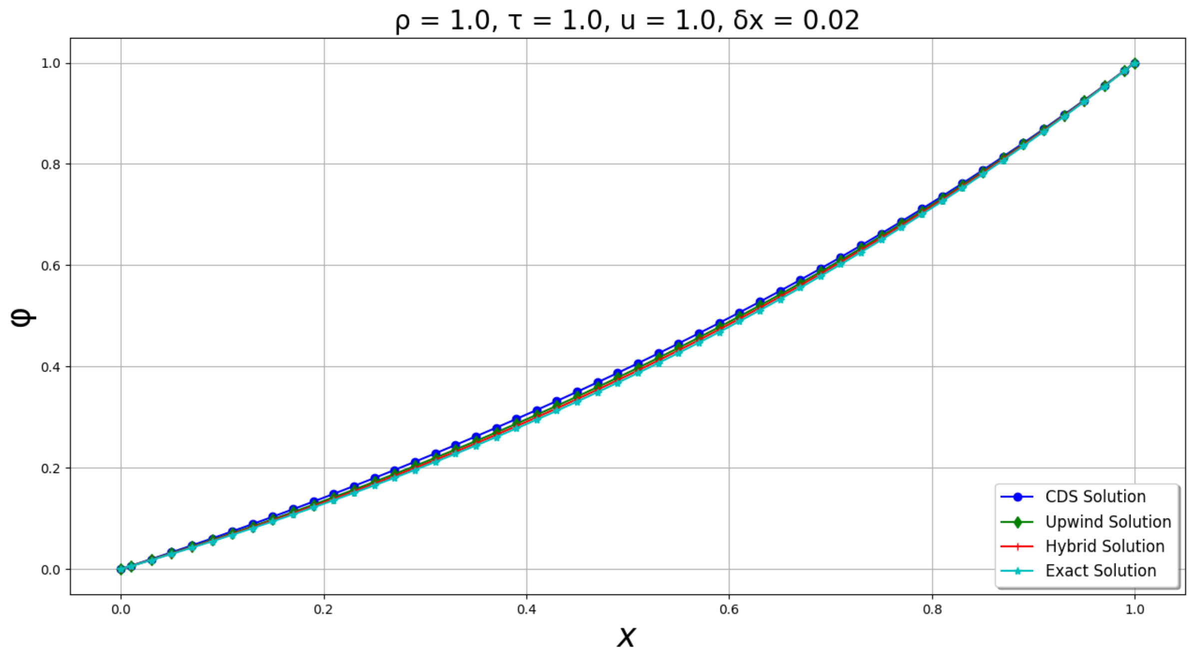


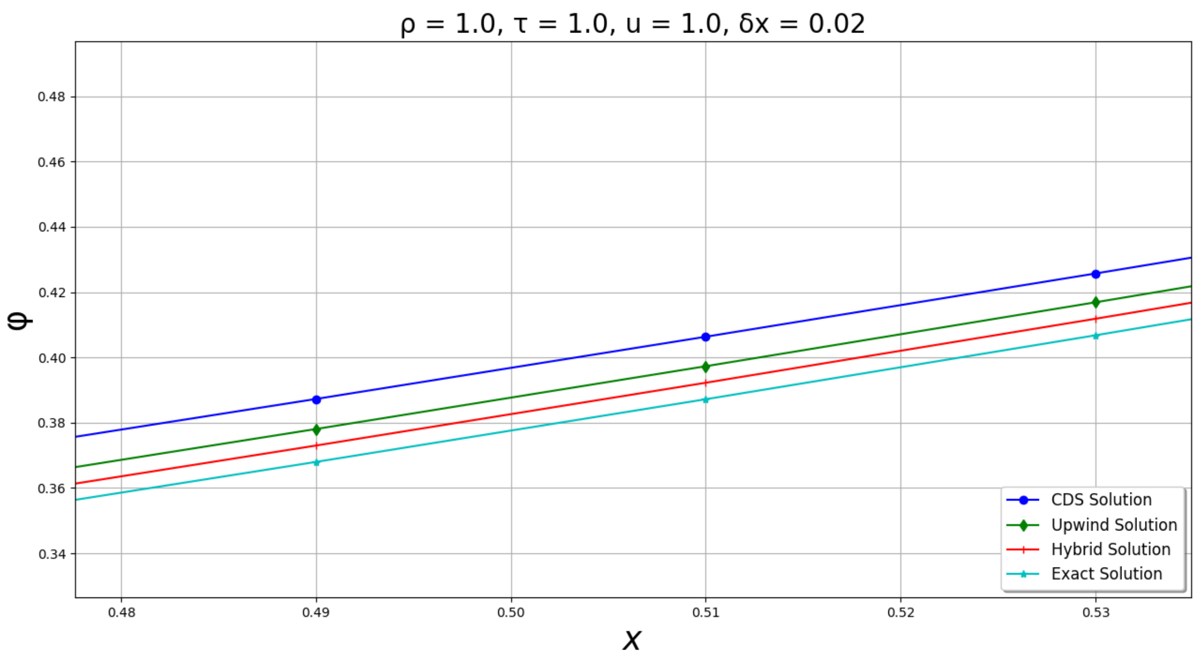


In the above case, the Peclet number is:

$$Pe = \frac{\rho u L}{n\Gamma} = \frac{1 * 1 * 1}{20 * 1} = 0.05$$

The above graphs show the variation of φ over the domain. From the graphs, we can easily infer that the Hybrid scheme is close to the exact solution while the CDS scheme is farthest away from the solution.



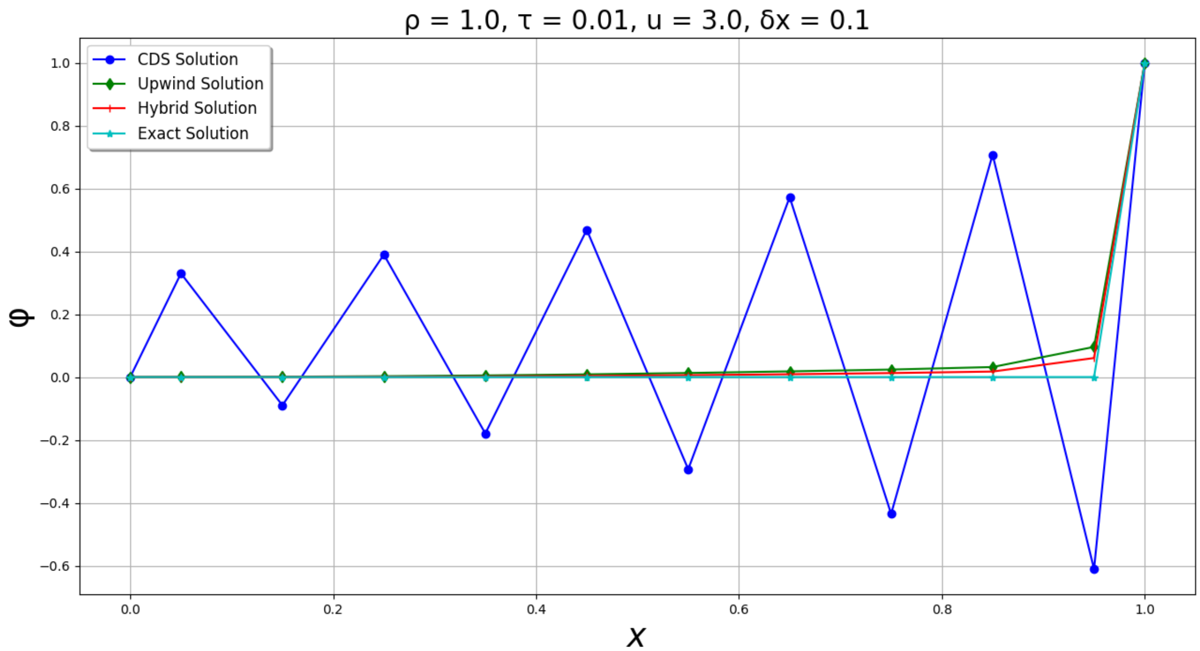


For this setting, the Peclet number is:

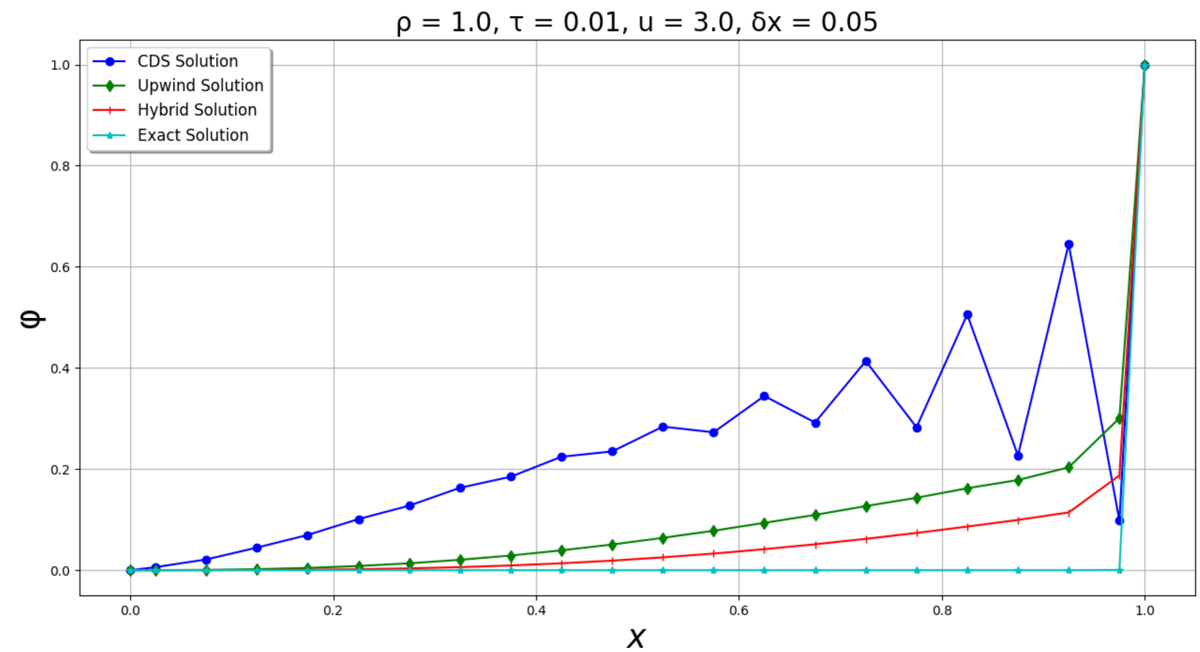
$$Pe = \frac{\rho u L}{n\Gamma} = \frac{1 * 1 * 1}{50 * 1} = 0.02$$

For both the above cases, the results agree with the Exact solution with varying accuracy, the highest being for the Hybrid scheme, then Upwind, and lastly, CDS. The Peclet number is also very small for these cases.

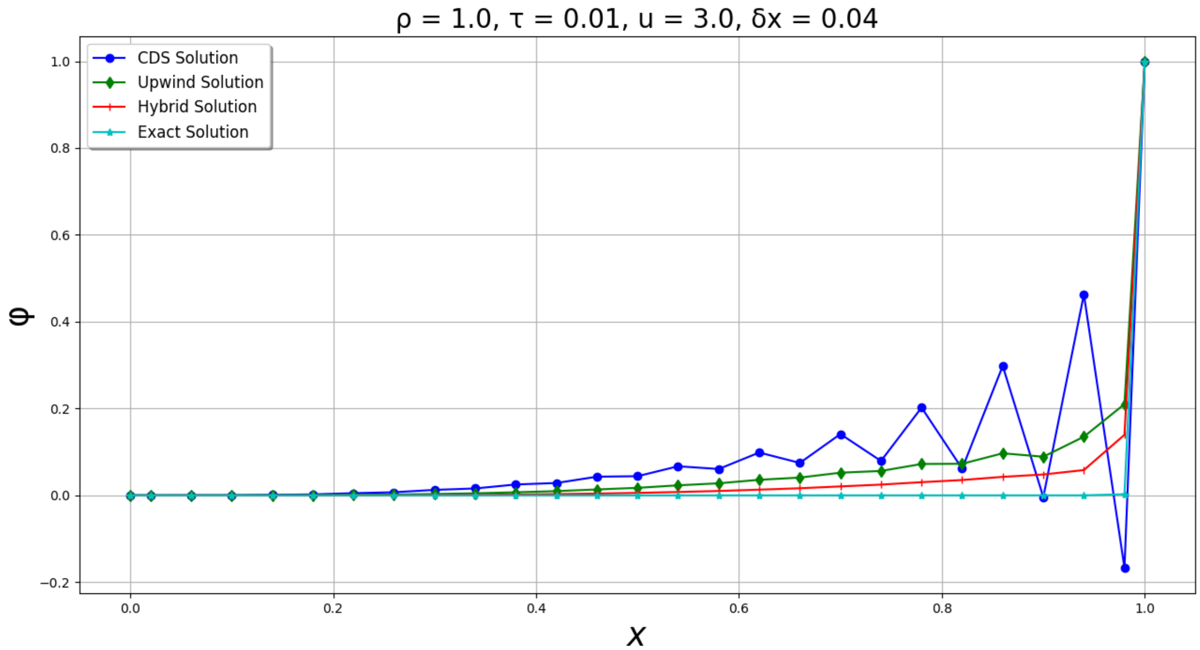
ii) $\rho = 1, \Gamma = 0.01, u = 3$



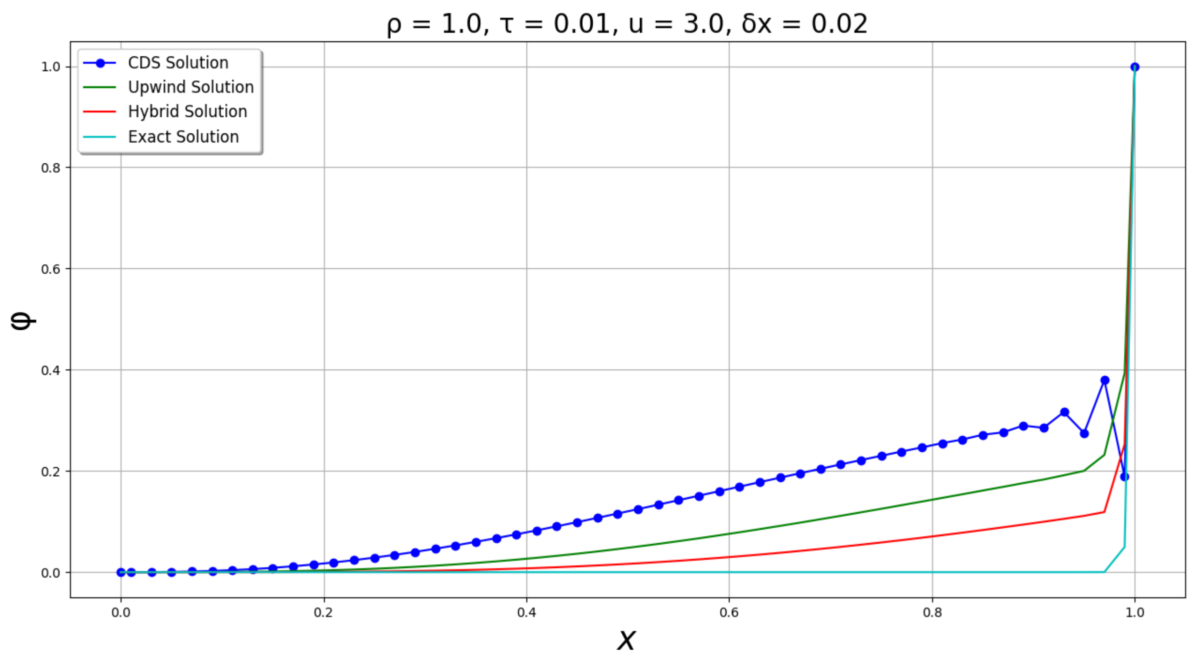
$$Pe = \frac{\rho u L}{n\Gamma} = \frac{1 * 3 * 1}{10 * 0.01} = 30$$



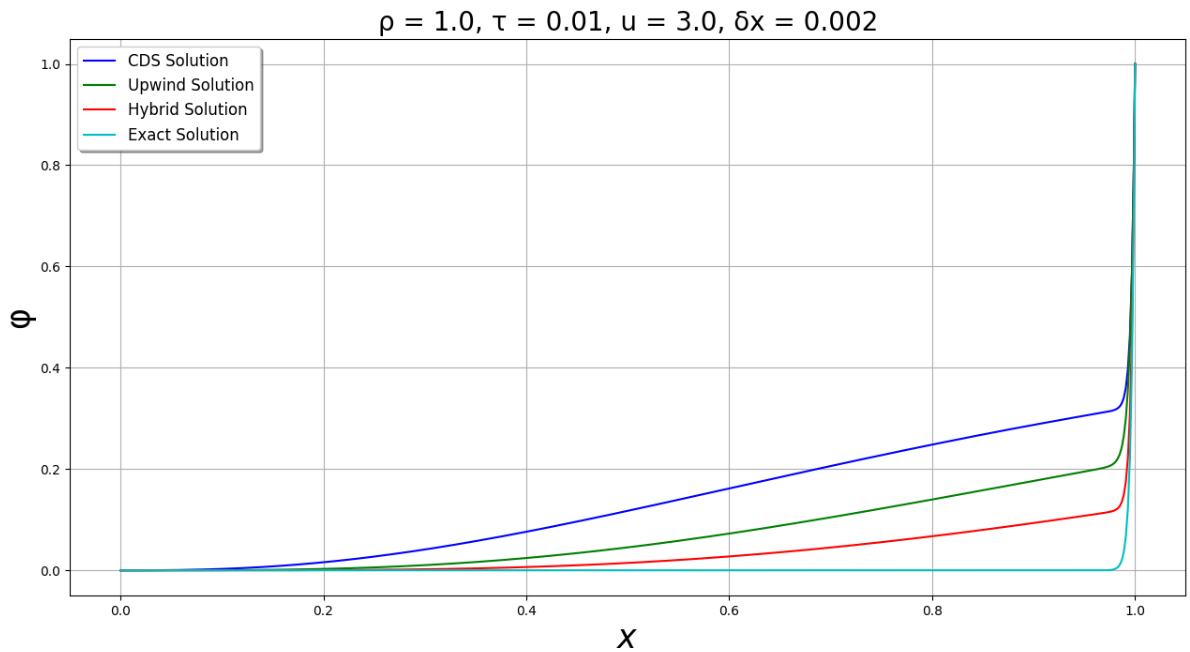
$$Pe = \frac{\rho u L}{n\Gamma} = \frac{1 * 3 * 1}{20 * 0.01} = 15$$

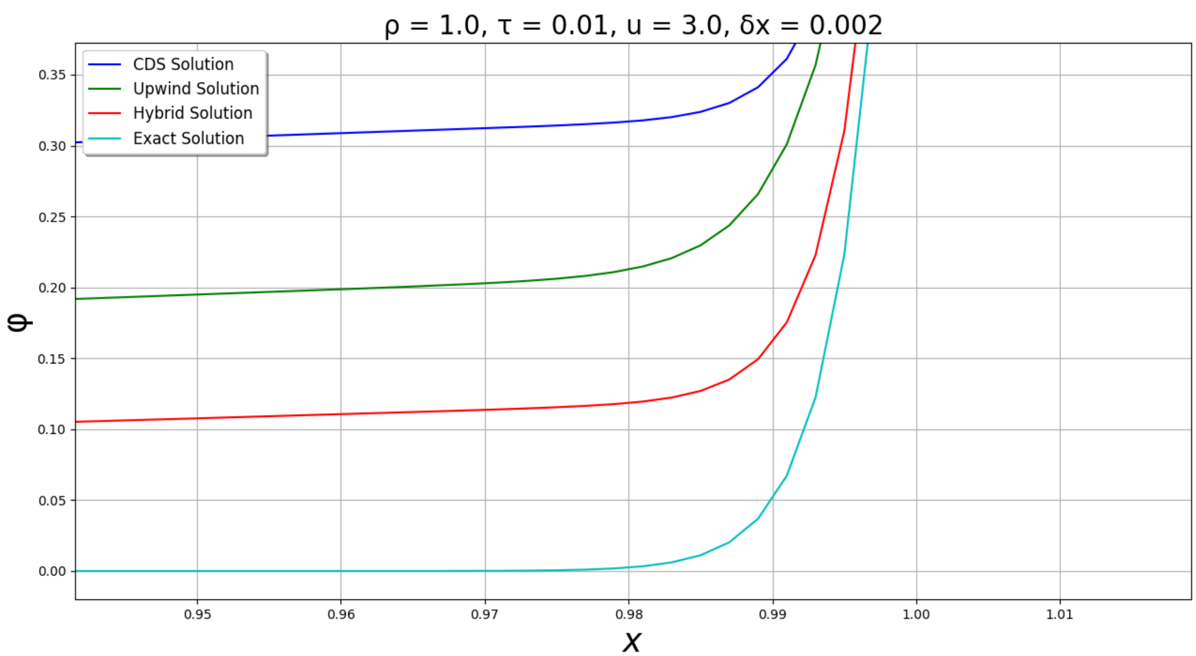


$$Pe = \frac{\rho u L}{n\Gamma} = \frac{1 * 3 * 1}{25 * 0.01} = 12$$



$$Pe = \frac{\rho u L}{n\Gamma} = \frac{1 * 3 * 1}{50 * 0.01} = 6$$





$$Pe = \frac{\rho u L}{n\Gamma} = \frac{1 * 1 * 1}{50 * 1} = 0.02$$

From the above plots, we can draw three inferences:

i. CDS scheme is highly unstable at large Peclet numbers
ii. Hybrid scheme and Upwind scheme provides very close answers at large Peclet numbers but grow farther from exact solution at smaller Peclet numbers
iii. The Hybrid scheme is always more comparable to the Exact solution than Upwind scheme

Analysis of point i) The CDS solution oscillates about the exact solution for large Peclet number values. These oscillations, also known as wiggles, arise because any of the coefficients has become negative or of opposite sign than others, which means that the coefficient matrix is not diagonally dominant. It causes enormous overshoots and undershoots about the exact solution. Grid refinement decreased the Peclect number from 30 to 0.02. The Peclet number must be smaller than two for CDS to give meaningful answers.

Analysis of point ii) The fact that Upwind and Hybrid schemes do not wiggle; in other words, they always have diagonally dominant coefficient matrices. Thus, all the coefficients are positive.

Analysis of point iii) The Hybrid scheme is always closer because the Hybrid scheme employs the best of both CDS and Upwind to get the best result. The Hybrid scheme switches to Upwind, when the Peclet number is large, e.g., less than -2 or greater than 2, and it switches to CDS when the Peclet number is small, i.e., between -2 and 2. Thus, the entire solution as a whole is better than both of its predecessors.

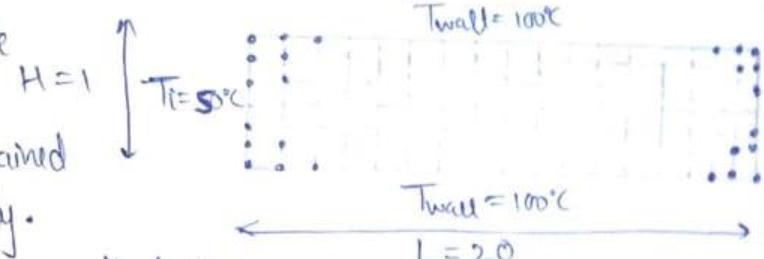
Accuracy and suitability of CDS: The Taylor series truncation error is of second-order, and the requirement of positive coefficients gives us the condition that $|Pe| < 2$. Thus, it is not used in real life due to severe restriction on Peclet number. We can apply it when we know for sure that the value of Peclet number lies between -2 and 2. Higher values of Pe can lead to oscillations which will result in wiggles and unphysical solutions. Therefore, it can be applied but with utmost care to keep all the coefficients positive.

Analysis of Upwind: Upwind scheme is based on the backward differencing formula, making the accuracy only first-order due to truncation error. This scheme's significant drawback is the introduction of false diffusion. Even when the flow is pre-dominantly convective, it retains the diffusive terms and thus produces false diffusion. It causes the distributions of the transported properties to become spread. The resulting error resembles diffusion and is thus known as false diffusion. Since all the coefficients are positive, there is no issue of oscillations. We can employ this scheme when we have sizeable Peclet number values.

Analysis of Hybrid: This scheme is based on the best parts of both schemes. The CDS is employed when dealing with low Peclet ($|Pe| < 2$) number conditions, and Upwind is called when dealing with large Peclet ($|Pe| \geq 2$). The Hybrid scheme uses a piecewise linear method which is based on the local value of the Peclet number. Since this scheme is a combination of the above two, it is the best and can be employed with both the Peclet conditions.

Ques2 We assume that the temperature at the nodes prevails + remains constant over the whole control volume. We get a system of num_x*num_y linear equations in num_x*num_y variables where num_x + num_y are the number of nodes in a column + row respectively.

i) We assume that the temperature at the outmost nodes for the top + bottom + left wall are maintained constant, Twall, Twall + Ti respectively.



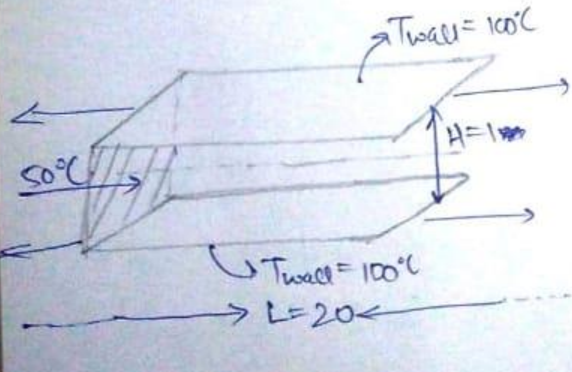
For the right boundary, we are given that the $\frac{\partial T}{\partial x}=0$, which, when discretized, translates to the following form:

$$T[i][num\_x-1] = T[i][num\_x-2] \quad , \; i \in [1, num\_y-1]$$

which means that the value temperature in the last column of the grid will be same as the value in immediate left column.

Now we employ our finite volume method + create a volume of $\Delta x \times \Delta y \times 1$ units^3 around each node, + the temperature in each of these ~~nodes~~ volumes or cells remain constant.

It is to be taken care that all the control volumes will be having the same volume as the boundary nodes because the boundary nodes have a constant temperature being applied to them. Thus, we have a system of ~~num_y*n~~ (numy − 1)*(num_x − 1) system of equations to solve.



$u = 1.5(1-4y^2)$ + $v=0$

$\rho=1$, $C=100$, $k=1$

$\frac{\partial T}{\partial x}=0$ at outflow → Neumann Boundary Condition

Rest are Dirichlet Boundary Conditions

$\Delta x = \Delta y$

ii) The governing equation is

$$\nabla \cdot (\rho c \vec{V} T) = \nabla \cdot (k \nabla T)$$

where the equation is 2-Dimensional

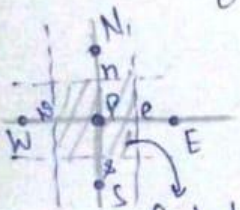
$\therefore$ It becomes:

$$\frac{\partial}{\partial x}(\rho c u T) + \frac{\partial}{\partial y}(\rho v c T) = \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right) + \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)$$

Here ρc are constant in LHS + k is constant in RHS

Thus:

$$\rho c \frac{\partial}{\partial x}(uT) + \rho c \frac{\partial}{\partial y}(vT) = k\frac{\partial}{\partial x}\left(\frac{\partial T}{\partial x}\right) + k\frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)$$



Control Volume

The discretization is done using finite volume method.

Assumptions:

a) No sources

b) $(\delta x)_e = (\delta x)_w = (\delta x)_n = (\delta x)_s$

c) constant properties

d) flow is in +ve directions

e) No viscous dissipation

Integrating the governing equation over the domain of interest

$$\int_s^n\int_w^e \frac{\partial}{\partial x}(c\rho u T)\,dx\,dy + \int_w^e\int_s^n \frac{\partial}{\partial y}(c\rho v T)\,dy\,dx = \int_s^n\int_w^e \frac{\partial}{\partial x}\left(k\frac{\partial T}{\partial x}\right)dx\,dy + \int_w^e\int_s^n \frac{\partial}{\partial y}\left(k\frac{\partial T}{\partial y}\right)$$

$$\Rightarrow \left[(\rho u T c)_e - (\rho u T c)_w\right]\Delta y + \left[(\rho v T c)_n - (\rho v T c)_s\right]\Delta x = \left[\left(k\frac{\partial T}{\partial x}\right)_e - \left(k\frac{\partial T}{\partial x}\right)_w\right]\Delta y + \left[\left(k\frac{\partial T}{\partial y}\right)_n - \left(k\frac{\partial T}{\partial y}\right)_s\right]\Delta x$$

$\Rightarrow$ Let $c_e(\rho u)_e \Delta y = F_e$

$c_w(\rho u)_w \Delta y = F_w$

$c_n(\rho v)_n \Delta x = F_n$

$c_s(\rho v)_s \Delta x = F_s$

$D_e = (k/\Delta x)\Delta y$

$D_w = (k/\Delta x)\Delta y$

$D_n = (k/\Delta y)\Delta x$

$D_s = (k/\Delta y)\Delta x$

our equation is of the form:

$a_P T_P = a_E T_E + a_W T_W + a_S T_S + a_N T_N$

where $a_P = a_E + a_W + a_S + a_N$

The hybrid scheme is defined as:

if $Pe < -2 \longrightarrow$ upwind scheme (convection dominates)

$a_E = -F_e \quad a_W = 0 \quad a_N = -F_n \quad a_S = 0$

if $Pe \in [-2, 2] \longrightarrow$ CDS scheme (Diffusion is considerable)

$a_E = D_e - \frac{F_e}{2} \quad a_W = D_w + \frac{F_w}{2} \quad a_N = D_n - \frac{F_n}{2} \quad a_S = D_s + \frac{F_s}{2}$

if $Pe > 2 \longrightarrow$ upwind scheme (convection dominates)

$a_E = 0 \quad a_W = F_w \quad a_N = 0 \quad a_S = F_s$

from the continuity we will obtain that $F_e + F_N - F_w - F_s = 0$

we can observe that the system of equations takes the following form:

$$-a_E T_E^{k+1} - a_W T_W^{k+1} + a_P T_P^{k+1} - a_S T_S^{k+1} - a_N T_N^{k+1} = 0$$

where k is the iteration level.

The above system of equations in a 5-Diagonal system which can be solved using Line by Line TDMA or Gauss-Seidel Algorithm.

iii) Boundary condition implementation

The general form of the equation is

$$-a_E T_E^{k+1} - a_W T_W^{k+1} + a_P T_P^{k+1} - a_S T_S^{k+1} - a_N T_N^{k+1} = 0$$

calculating the coefficients of the equation,

$F_e = (\rho u c)_e \Delta y = 100 u \Delta y \qquad D_e = k/\Delta x \times \Delta x = k = 1$; similarly

$F_w = (\rho u c)_w \Delta y = 100 u \Delta y \qquad D_w = 1$

$F_n = (\rho v c)_n \Delta x = 0 \qquad D_n = 1$

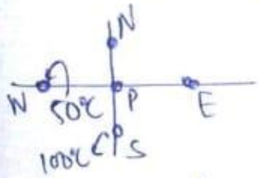
$F_s = (\rho v c)_s \Delta x = 0 \qquad D_s = 1$

The coefficients take value according to the hybrid scheme

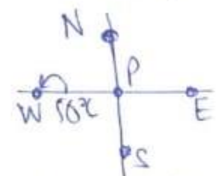
iii) Our system of equations is a five diagonal system, & we will employ Gauss-Seidel method to evaluate the temperature at each point in the domain. Since three of our boundaries are maintained at constant temperatures & for the right-side boundary, the change in T w.r.t x is zero, Therefore, we can employ the following boundary conditions:

for row i & column 1 : i = 0 to num_y-1

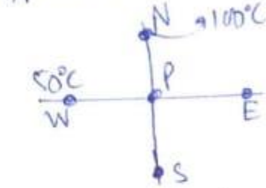
$$-a_W T_{i,0} - a_E T_{i,2} + a_P T_{i,1} - a_N T_{i+1,1} - a_S T_{i-1,1} = 0$$



Lower-most point



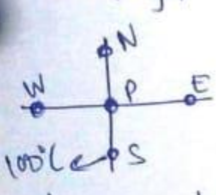
intermediate point



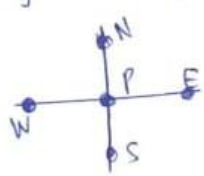
Upper-most point

for row i & column j : i = 1 to num_y-1 & j = 1 to num_x-2

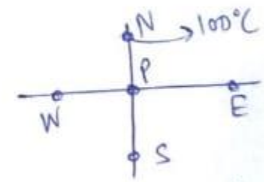
$$-a_W T_{i,j-1} - a_E T_{i,j+1} + a_P T_{i,j} - a_N T_{i+1,j} - a_S T_{i-1,1} = 0$$



Lower most



intermediate

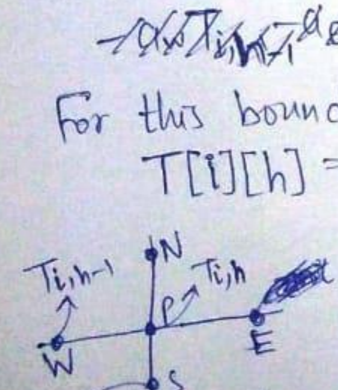


uppermost

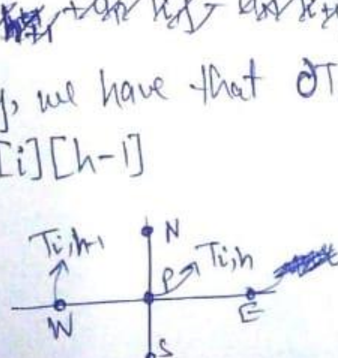
for row i & column num_x-1 : i = 1 to num_y-1 (let num_x-1 = h)

For this boundary, we have that $\partial T/\partial x = 0$ which means that

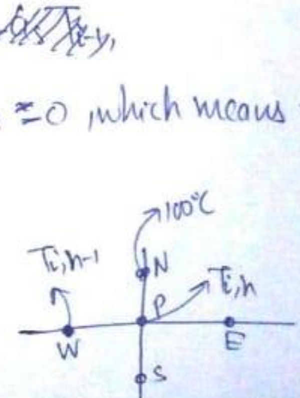
$T[i][h] = T[i][h-1]$



Lower most



intermediate

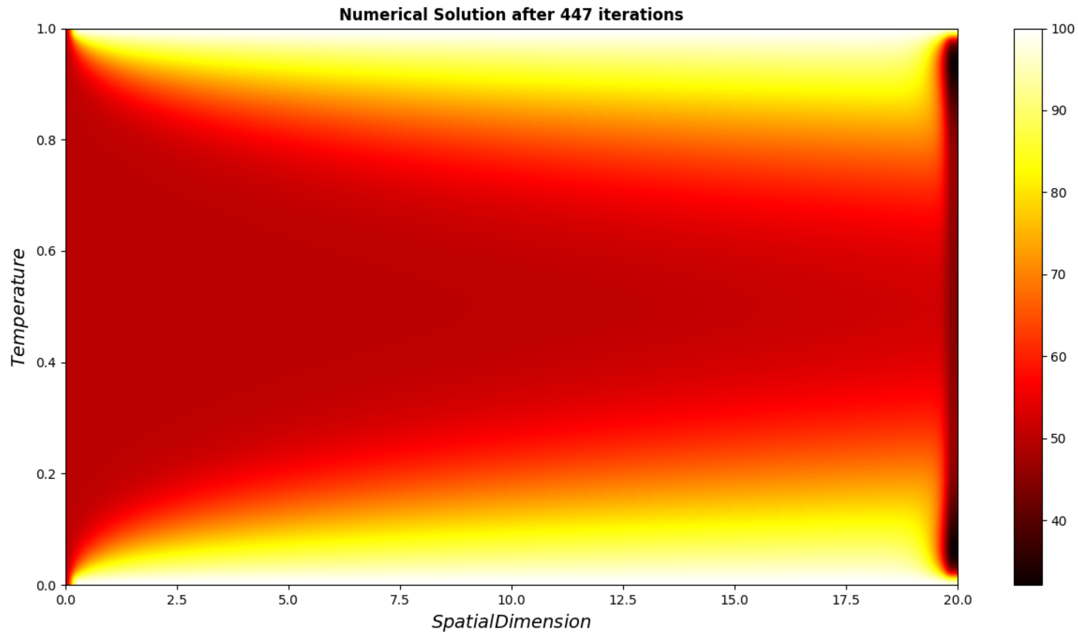


uppermost

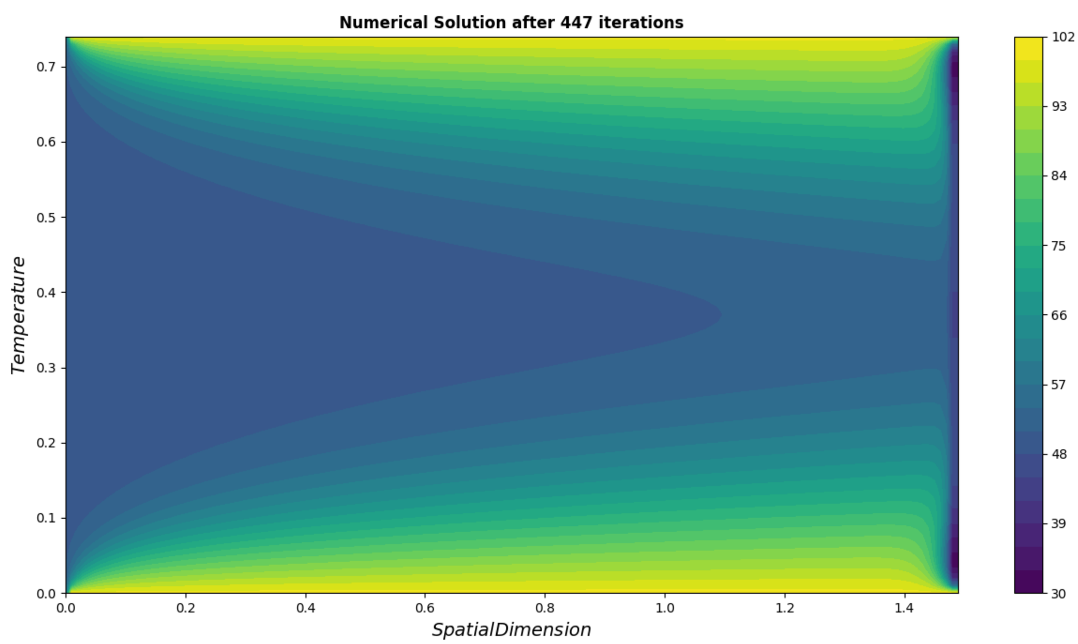
Ques2. Required output (plots/any other means)

The plot showing the variations and development of temperature profile at various axial locations is shown below:

This plot shows the temperature profile in the entire length of interest.



Since we are trying to plot a 3D plot on a 2D plot, it is better to have the contour plot as well. It shows the values of Temperature along the axial direction.



a) Determining the bulk mean temperature

(b) Determining the bulk mean temp.

$$T_{mean} = \frac{\int_0^H V_x T x\,dx}{\int_0^R V_x x\,dx} = \frac{2}{U_m H^2}\int_0^H (V_x T x)\,dx$$

$\because U_m = \frac{2}{3} U_{max}$

$u = 1.5(1-4y^2)$

$\frac{du}{dy} = 1.5(-8y)$;

putting $\frac{du}{dy} = 0 \Rightarrow y=0$ is the point of maxima/minima

$\frac{d^2u}{dy^2} < 0 \longrightarrow$ maxima

$\therefore y=0$ is the ~~xxxx~~ maxima point for $1.5(1-4y^2)$

$\therefore u_{max} = 1.5(1-4(0^2)) = 1.5$

$\therefore U_{mean} = \frac{2}{3} \times \frac{3}{2} = 1$ $\because H=1$

$\therefore T_{mean} = 2\int_0^H (V_x T x)\,dx$

For calculating the T_{mean} we have used the trapezoidal rule for integration. We first store the values of the product of temperature and velocity at each node and then use it and multiply it with the axial location and integrate it over the cross-sectional area to calculate the T_{mean} at that location and store this temperature in a different array.

b) Calculating the heat transfer coefficient at each axial location and estimating the corresponding value of Nusselt number

b) Determining the heat transfer coefficient :

$$h = \frac{\left(-k\frac{\partial T}{\partial y}\right)_w}{T_w - T_b}$$

The prescription $\left.\frac{\partial T}{\partial y}\right|_w$ is taken as a linear piecewise profile assumption, thus $\left.\frac{\partial T}{\partial y}\right|_w$ can be approximated as:

$$\left.\frac{\partial T}{\partial y}\right|_w = \frac{T_{i,j} - T_{i-1,j}}{\Delta y} \quad \text{where } i = numy-1 \ \& \ j = 0 \text{ to } num\_x-1$$

therefore, we can write h as:

$$\boxed{h_j = \frac{\left(-k\left\{\frac{T_{i,j} - T_{i-1,j}}{\Delta y}\right\}\right)}{T_w - T_{b_j}}}$$

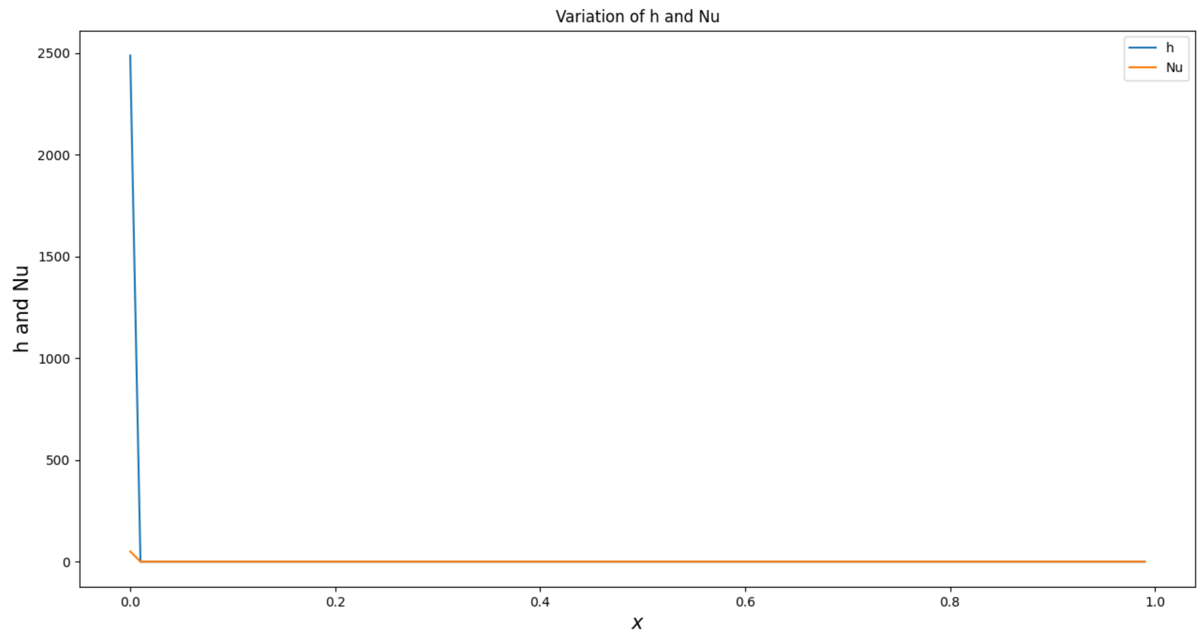
c) Nusselt numbe can be calculated as follows:

$$Nu_j = \frac{h_j D_h}{k}, \quad j = 0 \text{ to } num\_x-1$$

$\boxed{D_h = 2H = 2}$ & $\boxed{k = 1}$

$$\boxed{Nu_j = 2h_j}$$

The plots for h and Nu are shown below:



From the plot we can see that the value of h decreases very rapidly as we move forward in length and then becomes constant throughout the length L. This is in accordance with the fact that in fully developed region h is not depended on L. By performing order of magnitude analysis, we can come up with the equation that, during the thermal entrance, the order of h is given by the following equation:

$$\boldsymbol{h \approx k/\delta_T}$$

Thus, during the entrance region, $\boldsymbol{\delta_T}$ is changing very rapidly as the thermal boundary layer develops and after the entrance length, it becomes constant and thus the value of $\boldsymbol{h}$ also becomes constant.

During entrance region, $\boldsymbol{\delta_T}$ is very small and consequently $\boldsymbol{h}$ is very large.