Computational Heat & Fluid Flow (ME 605)

Instructors: Dr. Rudra Narayanan Roy & Dr. Sudhakar Yogaraj

School of Mechanical Sciences, IIT Goa.

Assignment 1 – Finite difference methods

Write a computer program (in any programming language of your choice) to solve the Poisson equation within the domain [0,1]

$$-\frac{\partial^2 u}{\partial x^2} + u = (\pi^2 + 1)\sin(\pi x)$$

The boundary conditions are given as,

$$u(0) = 0$$
$$u(1) = 0$$

Discretize the equation using second order finite difference scheme, and solve the resulting system of linear algebraic equations. Submit a short report containing discretization and the following tasks. Provide interpretation for all graphs.

1. Plot the distribution of the computational solution and the exact solution for 11 and 51 grid points using TDMA as the solver.

2. Solve the above using number of mesh intervals 4, 8, 16, 32, 64, and 128 with TDMA as the solver. Estimate the order of accuracy by plotting the L_∞-norm of solution error against the number of mesh intervals. The L_∞-norm can be computed as,

$$L_\infty = \max_i \left| u_i - u_i^{\text{exact}} \right|$$

3. Plot the convergence (error vs. iteration count) of all three iterative schemes: Jacobi, Gauss-Seidel, and Successive over-relaxation method (with one specific relaxation parameter $1 < \omega < 2$ of your choice.)

Additional talks (voluntary but fetch you additional points)

a) For the above problem, solve the resulting system of equations using V-cycle multigrid method (choose appropriate number of mesh points). Compare the convergence behavior of multigrid method vs Jacobi method for 129 mesh points.

b) Discretize the governing equation using fourth order central difference scheme (Remember to take care of the nodes that are adjacent to the boundary) and solve the resulting system of algebraic equation using Gauss-Seidel method. Demonstrate fourth order accuracy. On the same plot, you can also include the curve from task 2 mentioned above.