Anmol Singh
M.Sc 12
Roll no. 04

Biostatistics
Assignment 1

1. % success in selling policies = 10% or 0.1

To sell 20 policies :

no. of people × % = no. of policies sold

$$x = \frac{20}{0.1}$$

$$\underline{\underline{x = 200}}$$

2. a) Prob. to draw ace out of 52 cards = $\frac{4}{52}$

Prob. to draw king from 51 cards = $\frac{4}{51}$

Using multiplicative rule :

P(first ace, second king) = P(first ace) × P(Second King)

$$= \frac{4}{52} \times \frac{4}{51}$$

$$= \frac{16}{2652}$$

$$\boxed{P(\text{first ace, second king}) = 0.006033}$$

b) From previous question we know

$P(\text{ace first, king second}) = \frac{16}{2652}$

$P(\text{one ace, one king}) = P(\text{ace first, king second}) + P(\text{king first, ace second})$

$P(\text{one ace, one king}) = \frac{16}{2652} + \frac{16}{2652}$ $(\because P(A,K) = P(K,A))$

$= \frac{32}{2652}$

$P(\text{one ace, one king}) = 0.01206$

3. No. of seq's for 6 nt. seq $= 4^6$

If we fix first 3 as 'ATG' we are left with 3 nt.

No. of seq. when first 3 is 'ATG' $= 4^3 = 64$

4. No. of 8 digits numbers can be formed $= 8!$

We have 2 3's, 3 4's and 3 5's

Thus unique numbers that can be formed

$$\boxed{\text{No. of 8 digit nos.} = \frac{8!}{2!\,3!\,3!} = 560}$$

5. $P(A) = 0.32$, $P(B) = 0.41$,

$P(A \cup B) = ?$

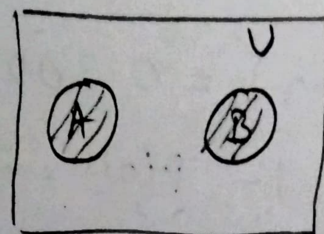
We know;

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

thus we need info. about $P(A \cap B)$ to find $P(A \cup B)$

But if we assume events A and B are <u>mutually exclusive</u>.

$P(A \cap B) = 0$



We can conclude

$$P(A \cup B) = P(A) + P(B) = 0.32 + 0.41 = 0.73$$

6. $P(C|M) = 5/100$; $P(C|F) = 25/10000$

$P(M|C) = ?$

We know the formula: From Bayes theorem

$$P(M|C) = \frac{P(C|M)\,P(M)}{P(C|M)\,P(M) + P(C|F)\,P(F)}$$

As the population is not defined, neither is $P(M)$ & $P(F)$ we can assume that

$P(M) = P(F) = 0.5$

$$P(M|C) = \frac{0.05 \times 0.5}{0.05 \times 0.5 + 0.0025 \times 0.5}$$

$$= \frac{0.05 \times \cancel{0.5}}{(0.05 + 0.0025)\cancel{0.5}}$$

$$\boxed{P(M|C) = \frac{0.05}{0.0525} = 0.9523}$$

7. The no. of ways to choose 5 unique chocolates from the 5 bags containing 10 unique chocolates

$= {}_{10}P_5 = 30,240$

Total events $= 10^5$

Prob of all unique $= \dfrac{30,240}{10^5}$

P(all uniq) $= 0.3024$

P(at least 2 identical) = 1 − P(all uniq)

P(2 identical) $= 0.6976$

8. a) No. of round $= n(r \cap y) + n(r \cap g)$

$n(r) = 315 + 108 = 423$

No. of wrinkled $= n(w \cap y) + n(w \cap g)$

$= 133$

Ratio $(r/w) = \dfrac{423}{133} = 3.180$

b) no. of yellow $= n(r \cap y) + n(w \cap y)$

$= 315 + 101 = 416$

no. of green $= 556 - 416 = 140$, $R(y/g) = \dfrac{416}{140}$

$= 2.971$

c) $P(y) = \dfrac{416}{556} = 0.748$

d) $P(g|r) = \dfrac{n(g \cap r)}{n(r)} = \dfrac{108}{423} = 0.255$

9. a) Chance to pass on recessive allele

$= \frac{1}{2}$ for both.

$P(\text{both recessive } aa) = \overset{p(a)}{\frac{1}{2}} \times \overset{p(a)}{\frac{1}{2}} = \frac{1}{4} = \underline{\underline{0.25}}$

b) Chance of it being a healthy carrier (Aa)

$p(Aa) = P(A) \times P(a) + P(a) \times P(A)$

$= \frac{1}{2} \times \frac{1}{2} + \frac{1}{2} \times \frac{1}{2}$

$p(Aa) = \frac{1}{4} + \frac{1}{4} = \frac{2}{4}$ or $\frac{1}{2}$ or $\underline{\underline{0.50}}$

c) $p(AA) = p(A) \times p(A)$

$p(AA) = \frac{1}{2} \times \frac{1}{2} = \frac{1}{4} = \underline{\underline{0.25}}$

d) For $Aa \times aa$

$P(Aa) = p(A|Aa) \times p(a|aa)$

$p(Aa) = \frac{1}{2} \times 1 = \underline{\underline{\frac{1}{2}}}$ or $\underline{\underline{0.50}}$

50% or 1/2 their children will have recessive disorder.

10. (1) When order doesn't matter we use

$nCr = \underline{_{12}C_3} = \underline{\underline{220}}$

(2) When order matters $= \underline{nPr} = \underline{\underline{1{,}320}}$

11. a) For the case first 5 female, last male

Ⓕ Ⓕ Ⓕ Ⓕ Ⓕ Ⓜ ~ Bernoulli trials

$P_B(5, 6, 0.5) = p^{x}(1-p)^{n-x}$

$= 0.5^{5}(1-0.5)^{6-5}$

$= 0.5^{5} \times 0.5$

$\underline{P_B(5,6,0.5) = 0.015625}$

b) If order does not matter i.e. Binomial prob.

$P_{Bi}(\underset{x,\,n,\,p}{5, 6, 0.5}) = {}^6C_5\, p^{5}(1-p)^{1}$

$= 6 \times 0.5^{6}$

$\underline{P_{Bi}(5,6,0.5) = 0.09375}$

12. (1) Given first ballon is yellow, also total
10 yellow, 7 red, 8 green = 25

Second also yellow $= \dfrac{9}{24}$

(2) First 2 are yellow, Prob. (3rd is red) = ?

$P(3^{rd} \text{ red}) = \dfrac{7}{23}$

13.

4.

P(

5.

n(

n(

3. we have 5 3's, 3 6's, 2 9's

10 digit uniq. nos. out of these = $\frac{10!}{5!\,3!\,2!}$

$\boxed{\text{No. of 10 digit uniq. nos.} = 2520}$

14.

$p(H) = 0.35$

$p(NH) = 0.65$

$P(S|H) = 0.86$

$$P(S|H) = \frac{P(S \cap H)}{P(H)}$$

$$p(S \cap H) = p(S|H)\,p(H)$$

$$= 0.86 \times 0.35$$

$$\underline{p(S \cap H) = 0.301}$$

15.

$n(P) = 15$

$n(C) = 25$

$n(P \cap C) = 10$

$n(P-C) = n(P) - n(P \cap C) = 15 - 10 = 5$

$n(C-P) = n(C) - n(P \cap C) = 15$

$n(P-C) + n(C-P) = \underline{20}$ 20 people with only one disease

16. 16 people, 5 people to be selected

10 women, 6 men. Total ways $= {}_{16}C_5$

a) 3 women & 2 men

To select 3 women out of 10 $= {}_{10}C_3$

" 2 men out of 6 $= {}_6C_2$

$$\text{Prob}(3W, 2M) = \frac{{}_{10}C_3 \times {}_6C_2}{{}_{16}C_5}$$

$$\boxed{P(3W, 2M) = \frac{120 \times 15}{4368} = \frac{1800}{4368} = 0.412}$$

b) $$P(4W, 1M) = \frac{{}_{10}C_4 \times {}_6C_1}{{}_{16}C_5}$$

$$P(4W, 1M) = \frac{1260}{4368} = 0.288$$

c) $$P(5W) = \frac{{}_{10}C_5}{{}_{16}C_5} = \frac{252}{4368} = 0.0576$$

d) $P(\text{at least 3 women}) = P(3W, 2M) + P(4W, 1M) + P(5W)$

$= 0.7576$

17

18.

13.

	Men	Women	
Right handed	43	44	87
Left handed	9	4	13
	52	48	100

a) $P(M|L) = \frac{n(M \cap L)}{n(L)} = \frac{9}{13} = \underline{\underline{0.692}}$

b) $P(R|W) = \frac{n(R \cap W)}{n(W)} = \frac{44}{48} = \underline{\underline{0.916}}$

c) $P(L) = \frac{n(L)}{n(T)} = \frac{13}{100} = \underline{\underline{0.13}}$

18. Given:

$P(\text{Forcast}|\text{Rain}) = 0.8$

$P(\text{Forcast}|\text{No rain}) = 0.2$

$P(R) = \frac{20}{365}$; $P(\text{No Rain}) = \frac{345}{365}$

Using Bayes Theorem:

$$P(\text{Rain}|\text{Forcast}) = \frac{P(\text{Forcast}|\text{Rain})\,P(\text{Rain})}{P(F|R)\,P(R) + P(F|NR)\,P(NR)}$$

$$P(R|F) = \frac{0.8 \times \frac{20}{365}}{0.8 \times \frac{20}{365} + \frac{0.2 \times 345}{365}}$$

$$P(R|F) = \frac{\frac{0.8 \times 20}{365}}{\frac{0.8 \times 20 + 0.2 \times 345}{365}}$$

$$P(R|F) = \frac{16}{16+69} = \frac{16}{85} = 0.1882$$

There is 18.8% chance it will rain tomorrow.

19. $m, n \in \mathbb{Z}$ (integer)

$m - n = x$; x is even

We know,

odd − even or even − odd = odd

thus

m, n are either both odd or both even.

Before we move on let us make few things clear:

If m, n = odd

m^2, n^2 = odd

let $q_1 \times q_2 \times \dots q_n$ be prime factor of any odd number x, then x^2 will also have same prime factors where $2 \notin \{q_1, q_2 \dots q_n\}$ thus x^2 is odd.

By same principal if x is even with prime factors $q_1, q_2, \dots q_n$, $2 \in \{q_1, q_2 \dots\}$ x^2 will be even.

We know odd + odd = even;

Let us assume x and y as odd numbers; $2a+1$ and $2b+1$

a)

b)

$x + y = 2a+1 + 2b+1$

$= 2 + 2a + 2b$

$x+y = 2(1+a+b)$

$x+y = 2(z)$

$x+y$ is even

Similarly if x, y are even; $x+y =$ even.

a) m^2+n^2+3;

(i) If m, n are odd, m^2 is odd, n^2 is odd

$m^2+n^2 =$ even

we know

even + odd = odd

$\underbrace{(m^2+n^2)}_{\text{even}} + \underbrace{3}_{\text{odd}} = \underline{\underline{\text{odd}}}$

(ii) m, n are even, $m^2+n^2 =$ even

By the same principal mentioned above

$m^2+n^2+3 =$ odd.

Thus m^2+n^2+3 cannot be even.

b) $n(m-1)$

we know odd $- 1 =$ even, even $- 1 =$ odd

If n, m are both even or odd

$n(m-1) = x$, $q_1, q_2, q_3 \cdots q_n$ are prime factors of $(m-1)$ & $p_1, p_2 \cdots p_n$ of n.

Either n or $(m-1)$ will be even

Thus $2 \in \{q_1, q_2, \cdots q_n, p_1, p_2 \cdots p_n\}$

x will be always even.

c) m^2+n+1

We know odd + odd = even, even + even = even

odd + even = odd.

If m, n are even

$m^2+n \Rightarrow$ even

$(m^2+n)+1 \Rightarrow$ odd

Similarly when m, n are odd

$m^2+n \Rightarrow$ even $\Rightarrow (m^2+n)+1 \Rightarrow$ odd

thus m^2+n+1 will always be odd.

d) $mn = ?$

Let $q_1 \times q_2 \times \cdots q_n$ be prime factors of m

" $p_1 \times p_2 \cdots p_n$ be prime factors of n

If m, n are odd

$2 \notin \{q_1, q_2 \cdots q_n\}$ (Lets call it set M; $2 \notin M$)

$2 \notin N$ (set of prime factors of n)

Thus

$2 \notin M \cup N \Rightarrow mn$ is odd

Thus

mn will be odd,

For m, n be even

$2 \in M$

$2 \in N$

$2 \in M \cup N \Rightarrow mn$ is even

But as for case 1 (m, n is odd) mn is odd

mn will not always be even

20. Given:

$C_F = 10m, \quad C_B = 12m$

Let the no. of revolutions traveled by back wheel be x, revolutions by front wheel $= 6+x$

$$(6+x)\,C_F = x\,C_B$$

$$(6+x)\,10 = x \times 12$$

$$60 + 10x = 12x$$

$$60 = 2x$$

$$x = \frac{\cancel{60}^{30}}{\cancel{2}_{1}}$$

No. of revolutions done by back wheel $= 30$

Distance traveled $= 30 \times 12$

$= \underline{\underline{360\,m}}$ i.e. (C)