Anmol Singh | Biostatistics - Internal 1 | 27.04.23

M.Sc 12

Roll no: 225MSBB004

1

1. Given: $n = 20$, 10 boys & 10 girls

To draw without replacement:

First girl = $\frac{\text{No. of girls}}{\text{Total no}} = \frac{10}{20}$

Now: Total no. = Tot. no. − 1 = 19 ; no. of girls = 9

Second boy = $\frac{\text{No. of boys}}{\text{Total no.}} = \frac{10}{19}$

Tot no. = 18

Third is girl = $\frac{\text{Girls}}{\text{total}} = \frac{9}{18}$

Prob. of successive draw = $\frac{10}{20} \times \frac{10}{19} \times \frac{9}{18}$ (Product Rule)

Prob. of draws = $\frac{900}{6840} = 0.1315$

Ans

2. $p = 0.05$, $n = 8$, $x = 5$

Given that the question asks for an event with 2 mutually exclusive outcome i.e. side effect or no side effect consider side effect as success. We are using Bernoulli Trials:

$P(x \text{ success}, n \text{ events}, 0.05 \text{ prob}) = p^x (1-p)^{n-x}$

$= 0.05^5 \times (1-0.05)^{8-5}$

$P(5, 8, 0.05) = 2.679 \times 10^{-7}$

Ans

Thus for more than 5 to suffer side effect prob is less than $\approx 2.679 \times 10^{-7}$

~~3) Given the time interval 8 to 10 AM~~

~~$\mu = 9$ birds / 2 hrs~~

2. This is the case of Binomial distribution as 2 mutually exclusive events. ~~x~~ $n = 8$, $p = 0.05$

$$P_{Bi}(x, n, p) = \frac{n!}{x!(n-x)!} p^x (1-p)^{n-x}$$

For 6 facing side effect

$$P_{Bi}(6, 8, 0.05) = \frac{8!}{6!(8-6)!} \times 0.05^6 \times (0.95)^2$$

$$= 3.94 \times 10^{-7}$$

For $x = 7$;

$$P_{Bi}(7, 8, 0.05) = \frac{8!}{7!(1)!} \times 0.05^7 \times 0.95$$

$$= 8 \times 0.05^7 \times 0.95$$

$$= 5.937 \times 10^{-9}$$

For $x = 8$

$$P_{Bi}(8, 8, 0.05) = \frac{8!}{8! \times 0!} \times 0.05^8 \times 1 = 3.90 \times 10^{-11}$$

$$P(x > 5) = 3.94 \times 10^{-7} + 5.93 \times 10^{-9} + 3.90 \times 10^{-11}$$

$$= 3.997 \times 10^{-7}$$

Ans

3. Time interval 8 - 10 Am - Poisson Distribution

$\mu = 9$ birds

For finding 14 birds

$$P_P(x, \mu) = \frac{\mu^x e^{-\mu}}{x!}$$

$$P_p(14, 9) = \frac{9^{14} e^{-9}}{14!} = \underline{\underline{0.0323}}$$

For finding less than 4 birds : ~~x = 0~~

1) $x = 0$

$$P_{P_0}(\cancel{4}\,0, 9) = \frac{\mu^x e^{-\mu}}{x!}$$

$$= \frac{\mu^0 e^{-\mu}}{0!} = 1.23 \times 10^{-4}$$

2) $x = 1$

$$P_{P_1}(1, 9) = \frac{\mu\, e^{-\mu}}{1!} = 1.11 \times 10^{-3}$$

3) $x = 2$

$$P_{P_2}(2, 9) = \frac{\mu^2 e^{-\mu}}{2!} = 4.99 \times 10^{-3}$$

4) $x = 3$ $$P_{P_3}(3, 9) = \frac{\mu^3 e^{-\mu}}{3!} = 0.01499$$

$$P_p(>4, 9) = P_{P_0} + P_{P_1} + P_{P_2} + P_{P_3} = \underline{\underline{0.021213}} \text{ Ans}$$

4. Gaussian distribution :

$\mu = 40, \quad \sigma = 5$

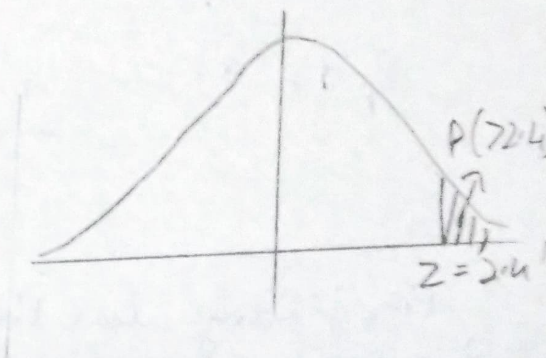
To compute probability we will do Z transformation

1) $x = 52$

$$z = \frac{x-\mu}{\sigma} = \frac{52-40}{5} = \frac{12}{5} = \underline{\underline{2.4}}$$

From Z-value table

$P_Z(>2.4) = \underline{\underline{0.0081}}$ Ans

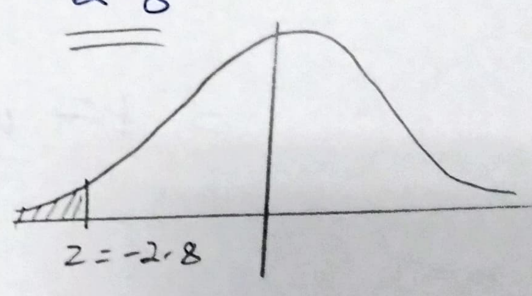


2) $x = 26$

$$z = \frac{x-\mu}{\sigma} = \frac{26-40}{5} = \underline{\underline{-2.8}}$$

From Z-value table

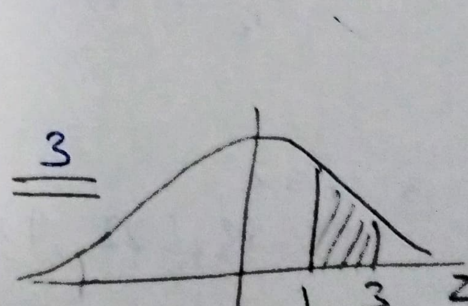
$P_Z(<-2.8) = \underline{\underline{0.00255}}$ Ans



3) $x_1 = 45, \quad x_2 = 55$

$$z_1 = \frac{45-40}{5} = \underline{\underline{1}} \qquad z_2 = \frac{55-40}{5} = \underline{\underline{3}}$$

From Z-value table



$P_{z_1}(<1) = 0.8413 \qquad P_{z_2}(<3) = 0.99865$

$P(z_2 > x > z_1) = P_{z_2} - P_{z_1} = \underline{\underline{0.1573}}$

Ans

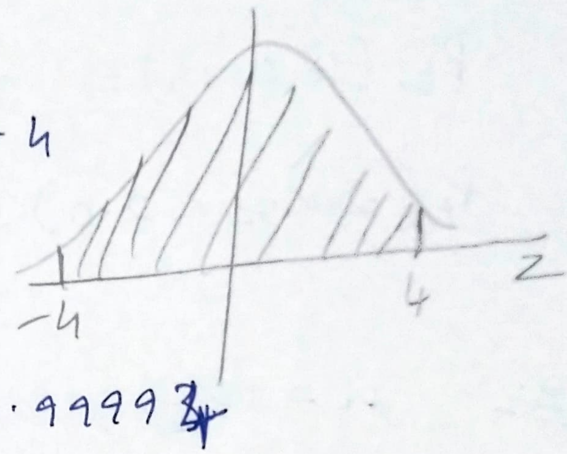
4 cont.

4) B/w 20 to 60

$x_1 = 20 \qquad x_2 = 60$

$$Z_1 = \frac{x_1 - \mu}{\sigma} = \frac{20-40}{5} = -4$$

$$Z_2 = \frac{x_2 - \mu}{\sigma} = \frac{60-40}{5} = +4$$



$P_2(<+4) = 0.99997$

$P_2(<-4) = 0.00003$

$P(-4 < x < 4) = P_2 - P_1 = 0.99994$

5. $\Rightarrow$ t-distribution $\rightarrow$ compute probability of random deviate with a value, assuming $n = 16$

degree of freedom $= n-1$
$= 15$

(1) $> 2.3 \quad t = 2.3$

for $P(2.3, 15) = 0.9818$ (From R : pt(t, n-1))

For $P(>2.3, 15) = 1 - 0.9818$

$P(>2.3, 15) = \underline{0.0182}$

2) < -2.65, $t = -2.65$

For $P(<-2.65, 15) = \sim \underline{0.0091}$

3) Between 2.1 to 3.1, $t_1 = 2.1$, $t_2 = 3.1$

$P_{t_1}(2.1, 15) = \underline{0.9734} \quad P_{t_2}(3.1, 15) = \underline{0.99634}$

$P_t(2.1 < x < 3.1) = 0.99364 - 0.9734 = \underline{0.02024}$

Ans

5. cont

~~8~~4. B/W -2.0 to 2.0, $df = 16-1 = 15$

$t_1 = -2.0$, $t_2 = 2.0$

$P_{t_1}(-2.0, 15) = 0.03197$

$P_{t_2}(2.0, 15) = 0.96803$

$P_t(-2.0 < x < 2.0) = P_{t_2} - P_{t_1} = 0.93606$

Ans

6. $\mu = 100$, $s = 20$, $n = 25$

$\bar{x}_1 = 96$ $\bar{x}_2 = 108$, $df = n-1 = 24$

$$t_1 = \frac{\bar{x}_1 - \mu}{\cancel{20/\sqrt{25}}\ s/\sqrt{n}} = \frac{96-100}{\frac{20}{\sqrt{25}}} = \frac{-\cancel{4}^{1}}{\frac{\cancel{20}^{4}}{\cancel{5}_{1}}} = -1$$

$$t_2 = \frac{\bar{x}_2 - \mu}{s/\sqrt{n}} = \frac{108-100}{\frac{\cancel{20}^{4}}{\cancel{5}_{1}}} = \frac{+\cancel{8}^{2}}{\cancel{4}_{1}} = +2$$

$P_{t_1}(-1, 24) = \cancel{0.83636}$ 0.16364

$P_{t_2}(+2, 24) = \cancel{0.0284}$ 0.97153

$P_t(96 \leq \bar{x} \leq 108) = P_{t_2} - P_{t_1} = 0.8079$

Ans

4

7. Given = $\bar{x} = 29.0$, $\sigma = 3.8$

Let us assume $n = 16$

1) $9\cancel{0}\% \ CI = \bar{x} \pm Z_{1-\alpha/2} \frac{\sigma}{\sqrt{n}}$

$Z_{1-\alpha/2}$

For 90% CI

$\alpha = 0.1 \quad \alpha/2 = 0.05$

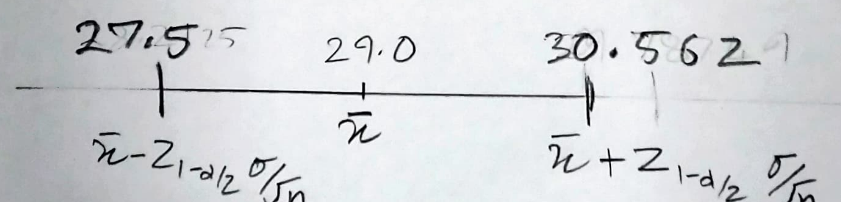
$1-\alpha/2 = 0.95$

$Z_{\cancel{0.}1-\alpha/2} = Z_{0.95} = 0.8289$ ($\because$ R: pnorm( ))

$90\% \ CI = \bar{x} \pm Z_{1-\alpha/2} \frac{\sigma}{\sqrt{n}}$

$= 29.0 \pm 0.8284 \times \frac{3.8}{4}$

$\underline{90\% \ CI = 29.0 \pm \overset{1.562}{\cancel{0.78749}}}$



2. $95\% \ CI = \bar{x} \pm Z_{1-\alpha/2} \frac{\sigma}{\sqrt{n}}$

$Z_{1-\alpha/2} =$

$\alpha = 0.05$

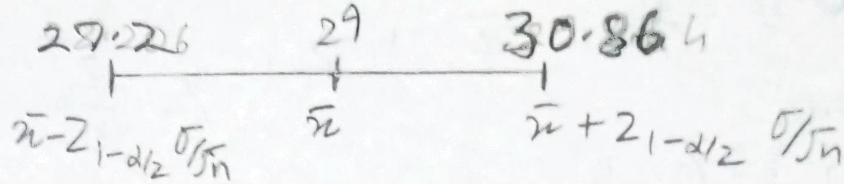
$\alpha/2 = 0.025$, $1-\alpha/2 = 0.975$

$Z_{0.975} = 0.8352$; $Z_{0.975}\,\sigma/\sqrt{n} = 0.79344$

$95\% \ CI = 29.0 \pm \overset{1.86}{\cancel{0.79344}}$

57. cont.

2) $95\%\ CI = 29 \pm \cancel{0.79544}\ 1.86$



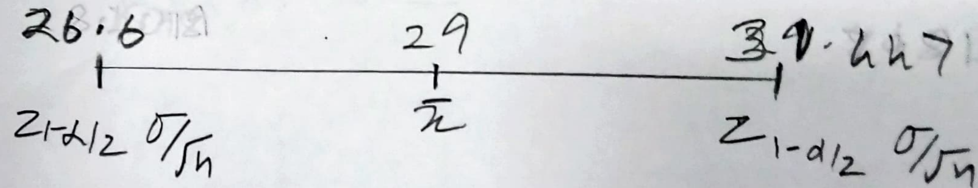
3) $99\%\ CI \Rightarrow \alpha = 0.01 \qquad \alpha/2 = 0.005$

$1 - \alpha/2 = 0.995$

$Z_{0.995} = 0.84013$

$99\%\ CI = \bar{x} \pm Z_{0.995}\, \sigma/\sqrt{n}$

$99\%\ CI = 29 \pm \cancel{0.84013}\ 2.447$



8: Given : $CaCO_3$ conc (mg/L) $n = 20$

i) $X = \{130.8, 129.9, 131.5, 131.2, 129.5, 132.7, 131.5, 127.8, 133.7, 132.2, 134.8, 131.7, 133.9, 129.8, 131.4, 128.8, 132.7, 132.8, 131.4, 131.3\}$

Sorting in ascending order:

$X = \{127.8, 128.8, 129.5, 129.8, 129.9, 130.8, 131.2, 131.3, 131.4, 131.4, 131.5, 131.5, 131.7, 132.2, 132.7, 132.7, 132.8, 133.7, 133.9, 134.8\}$

Mean $\bar{x} = \dfrac{127.8 + 128.8 + 129.5 + \cdots + 134.8}{20}$

$\bar{x} = 131.47$ mg/L

$$S^2 = \frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2$$

$$= \frac{1}{19} \left((127.8 - 131.47)^2 + (128.8 - 131.47)^2 + \cdots + (134.8 - 131.47)^2 \right)$$

$S^2 = 3.03378$

$S = \sqrt{3.0337} = 1.~~74~~ 7417$ mg/L

For even values ⇒

$$\text{Median} = \frac{\frac{n}{2}^{th}\text{ term} + (\frac{n}{2}+1)^{th}\text{ term}}{2} = \frac{10^{th}\text{ term} + 11^{th}\text{ term}}{2}$$

$$= \frac{131.\cancel{4} + 131.5}{2} = 131.\cancel{4}5 \text{ mg/L}$$

1st Quartile = $\left(\frac{1}{4} \times (n+1)\right)^{th}$ term

$= \left(\frac{21}{4}\right)^{th} = 5.25^{th}$; thus we will take avg. of 5th and 6th term

1st Quartile = $\frac{5^{th}\text{ term} + 6^{th}\text{ term}}{2} = \frac{129.9 + 130.8}{2} = 130.35$ mg/L

3rd Quartile = $\left(\frac{3}{4} \times (n+1)\right)^{th}$ term $= 15.75^{th}$; ⇒ Avg of 15th & 16th term

3rd Quartile = $\frac{132.7 + 132.8}{2} = 132.75$ mg/L

ii) 95% CI = $\bar{x} \pm Z_{1-\alpha/2}\, \sigma/\sqrt{n}$

$= 131.47 \pm Z_{1-\alpha/2} \frac{1.7417}{\sqrt{20}}$

$Z_{1-\alpha/2}$ ⇒ $\alpha = 0.05$, $\alpha/2 = 0.025$

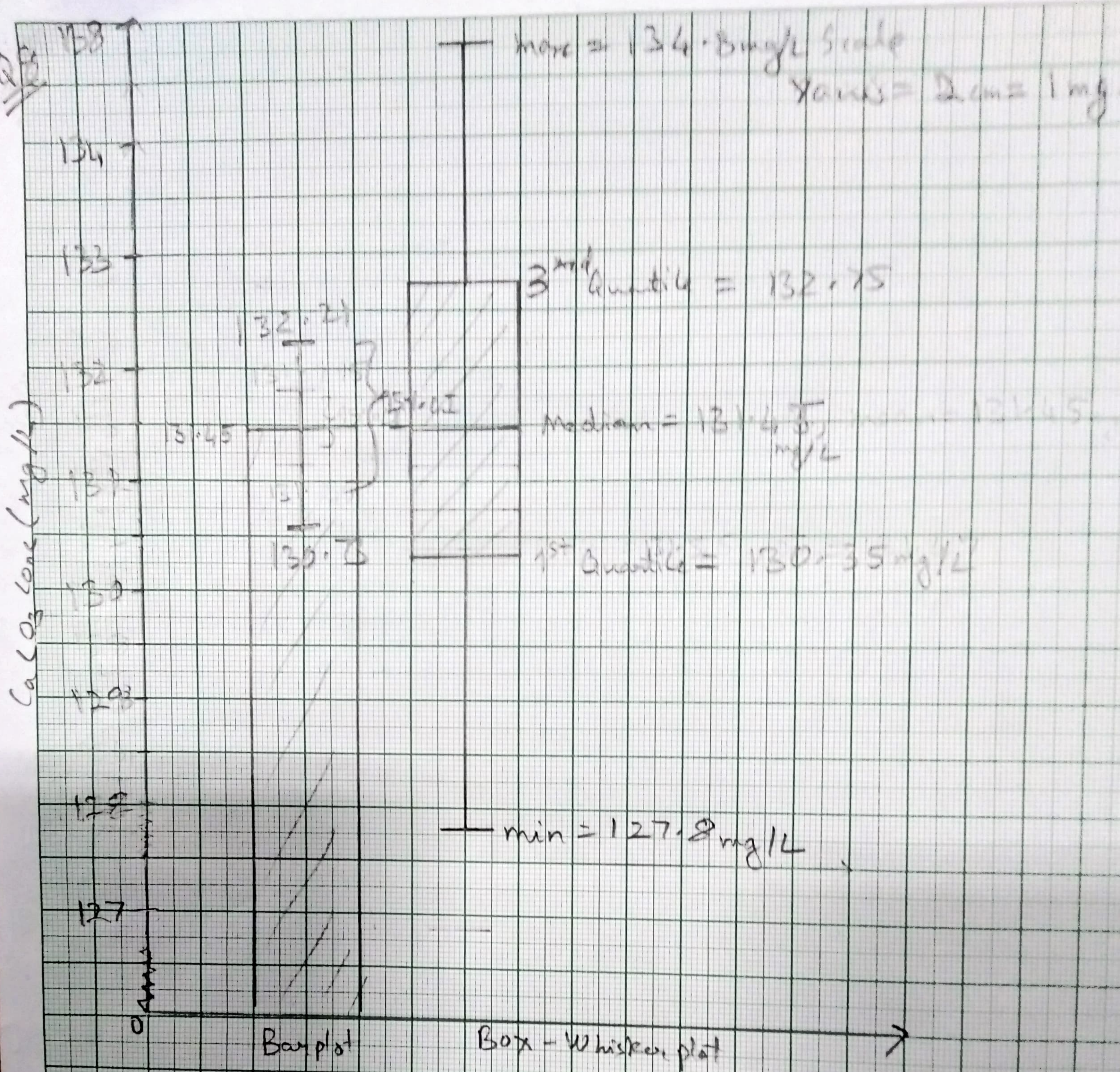
$1-\alpha/2 = 0.975$

$Z_{0.975} = 0.8352$; $Z_{1-\alpha/2}\, \sigma/\sqrt{n} = 0.7450$

95% CI = 131.47 ± 0.7450

= ~~(131.145 mg/L, 131.795 mg/L)~~

= (130.73 , 132.21)



max = 134.8 mg/L
Scale
Y axis = 2cm = 1 mg/l
3rd Quartile = 132.75
Median = 131.45 mg/L
1st Quartile = 130.35 mg/L
min = 127.8 mg/L
132.21
131.45
130.7
Barplot
Box - Whisker plot
CaCO3 conc (mg/L)

9. Comp. 1 - 252, 240, 205, 200, 170, 170, 320, 148, 214, 270, 265, 203

Ascending → Comp 1 : 148, 170, 170, 200, 203, 205, 214, 240, 252, 265, 270, 320

Comp 2 ⇒ 185, 310, 212, 238, 184, 136, 200, 270, 200, 212, 182, 225

Ascending Comp 2 ⇒ 136, 182, 184, 185, 200, 200, 212, 212, 225, 238, 270, 310

Comp 1 ⇒

$$\bar{x} = \sum_{i=1}^{n} \frac{x_i}{n} = 221.416$$

$$S = \sqrt{\frac{1}{n-1} \sum_{i=1}^{n} (x_i - \bar{x})^2} = \sqrt{2459.9} = 49.597$$

1st Q ⇒ $\left(\frac{1}{4} \times (n+1)\right)^{th}$ term $= \frac{13}{4} = 3.25$, 3rd & 4th term avg

$$= \frac{170 + 200}{2} = 185$$

3rd Q ⇒ 9.75th term ⇒ Avg 9th & 10th term $= \frac{252 + 256}{2} = 254$

Median ⇒ $\frac{\frac{n}{2}^{th} + \frac{n}{2}+1^{th}}{2} = \frac{6^{th} + 7^{th} \text{ term}}{2} = \frac{205 + 214}{2} = 209.5$

95% CI ⇒ $\alpha = 0.05$, $1 - \alpha/2 = 0.975$

$$Z_{0.975} \, \sigma/\sqrt{n} = 28.057$$

95% CI = 221.416 ± 28.057

(193.35, 249.46)

Comp 2 →

$\bar{x} = 212.833$ pounds

$S = 45.006$ pounds

Median = $\cancel{206}$ $\frac{6^{th} + 7^{th} \text{ term}}{2}$ = 206 pounds

$1^{st}Q \Rightarrow \frac{3^{rd} + 4^{th} \text{ term}}{2}$ = 184.5 pounds

$2^{nd}Q \Rightarrow \frac{9^{th} + 10^{th} \text{ term}}{2}$ = 231.5 pounds

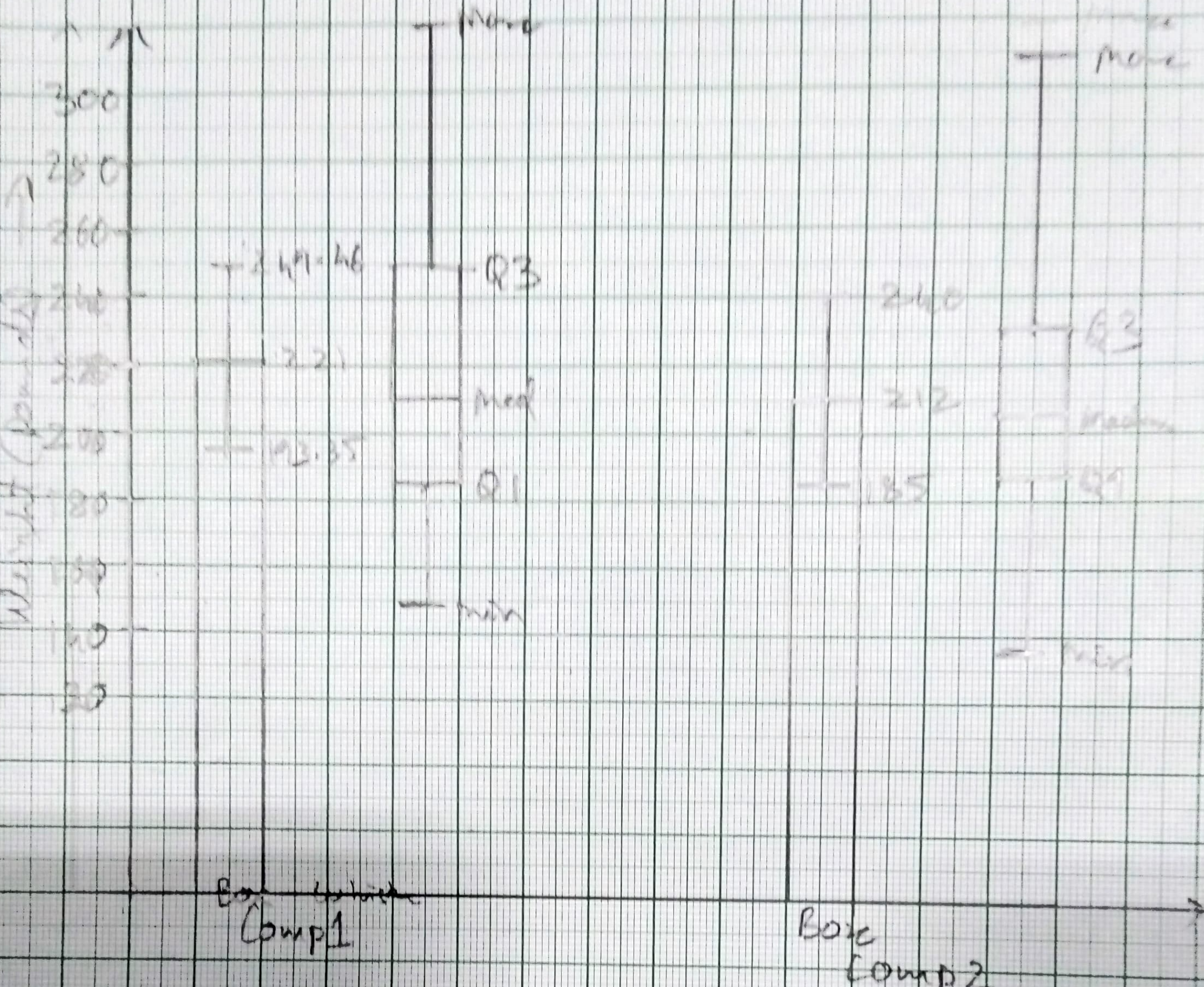
95% CI ⇒ $\cancel{28.057}$ $212.8833 \pm \cancel{28.057}$ 27.7238

(185.1 , 240.5)

By the graph → no significant difference b/w comp 1 & comp 2

Q9 ->

Scale
Yaxis = 1 unit = 10 pound



10. Given $S = ax^2 + by^2$

a) $x = 10, \; dx = 2.6$ $\quad a = 5 \quad b = 12$

$y = 15.1, \; dy = 2.9$

$$dS = \frac{\partial S}{\partial x} dx + \frac{\partial S}{\partial y} dy \quad \text{—} ① \quad ; \; dS \text{ is uncertainity in } S$$

$$\frac{\partial S}{\partial x} = 2ax \; \text{—} ② \qquad \frac{\partial S}{\partial y} = 2by \; \text{—} ③$$

Substituting 2 & 3 on ①

$$dS = (2ax)\,dx + (2by)\,dy$$

$$dS = (2 \times 5 \times 10 \times 2.6) + 2 \times 12 \times 15.1 \times 2.9$$

$$= 260 + 1050.96$$

$$dS = 1310.96$$

Ans

b: $x = 10.5, \; \sigma_x = 2.2$ $\quad a = 5.0$

$y = 14.9, \; \sigma_y = 2.6$ $\quad b = 12.0$

$$\frac{\partial S}{\partial x} = 2ax \; \text{—} ① \qquad \frac{\partial S}{\partial y} = 2by \; \text{—} ②$$

$$\sigma_S^2 = \sigma_x^2 \left(\frac{\partial S}{\partial x}\right)^2 + \sigma_y^2 \left(\frac{\partial S}{\partial y}\right)^2$$

$$= (2.2)^2 (2 \times 5 \times 10.5)^2 + (2.6)^2 (2 \times 12.0 \times 14.9)^2$$

$$\sigma_S^2 = 53361 + 864453.6576 = 917814.6576$$

$$\sigma_S = \sqrt{917814.6576} = 958.0264 \text{ Ans}$$