Trigonometric Identities & Formulas

Tutorial Services – Mission del Paso Campus

Reciprocal Identities

$$\sin x = \frac{1}{\csc x}$$

$$\csc x = \frac{1}{\sin x}$$

$$\cos x = \frac{1}{\sec x}$$
 $\sec x = \frac{1}{\cos x}$

$$\sec x = \frac{1}{\cos x}$$

$$\tan x = \frac{1}{\cot x}$$

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Ratio or Quotient Identities

$$\tan x = \frac{\sin x}{\cos x}$$

$$\cot x = \frac{\cos x}{\sin x}$$

$$\sin x = \cos x \tan x$$

$$\cos x = \sin x \cot x$$

Pythagorean Identities

$$\sin^2 x + \cos^2 x = 1$$

$$1 + \tan^2 x = \sec^2 x$$

$$1 + \cot^2 x = \csc^2 x$$

Note: there are only three, basic Pythagorean identities, the other forms are the same three identities, just arranged in a different order.

Confunction Identities

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x$$

$$\sin\left(\frac{\pi}{2} - x\right) = \cos x \qquad \qquad \cos\left(\frac{\pi}{2} - x\right) = \sin x$$

$$\tan\left(\frac{\pi}{2} - x\right) = \cot x$$

$$\tan\left(\frac{\pi}{2} - x\right) = \cot x$$
 $\cot\left(\frac{\pi}{2} - x\right) = \tan x$

$$\sec\left(\frac{\pi}{2} - x\right) = \csc x$$
 $\csc\left(\frac{\pi}{2} - x\right) = \sec x$

$$\csc\left(\frac{\pi}{2} - x\right) = \sec x$$

Pythagorean Identities in Radical Form

$$\sin x = \pm \sqrt{1 - \cos^2 x}$$

$$\tan x = \pm \sqrt{\sec^2 x - 1}$$

$$\cos x = \pm \sqrt{1 - \sin^2 x}$$

Odd-Even Identities

Also called negative angle identities

$$Sin(-x) = -sin x$$

$$Sin(-x) = -sin x$$
 $Csc(-x) = -csc x$

$$\cos(-x) = \cos x$$

$$Cos(-x) = cos x$$
 $Sec(-x) = sec x$

$$Tan(-x) = -tan$$
.

$$Tan(-x) = -tan x$$
 $Cot(-x) = -cot x$

Phase Shift =
$$\frac{-c}{h}$$

Period =
$$\frac{2\pi}{h}$$

Sum and Difference Formulas/Identities

$$\sin(u+v) = \sin u \cos v + \cos u \sin v$$

$$\sin(u-v) = \sin u \cos v - \cos u \sin v$$

$$cos(u+v) = cosucosv - sinusinv$$

$$\cos(u-v) = \cos u \cos v + \sin u \sin v$$

$$\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$$

$$\tan(u-v) = \frac{\tan u - \tan v}{1 + \tan u \tan v}$$

How to Find Reference Angles

Step 1: Determine which quadrant the angle is in

Step 2: Use the appropriate formula

Quad I = is the angle itself

Quad II = $180 - \theta$

 $\pi - \theta$ or

Ouad III = $\theta - 180$

or

Quad IV = $360 - \theta$

θ - π $2\pi - \theta$

Reciprocal Identities

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$$\csc x = \frac{1}{\sin x}$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\cot x = \frac{\cos x}{\sin x}$$

$$\cos x = \frac{1}{\sec x}$$

$$\cos x = \frac{1}{\sec x} \qquad \qquad \sec x = \frac{1}{\cos x}$$

$$\sin x = \cos x \tan x$$

$$\cos x = \sin x \cot x$$

$$\tan x = \frac{1}{\cot x}$$

$$\tan x = \frac{1}{\cot x} \qquad \cot x = \frac{1}{\tan x}$$

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$$\tan\left(\frac{\pi}{2} - x\right) = \cot x$$
 $\cot\left(\frac{\pi}{2} - x\right) = \tan x$

$$\sec\left(\frac{\pi}{2} - x\right) = \csc x$$
 $\csc\left(\frac{\pi}{2} - x\right) = \sec x$

$$(\pi)$$

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 $Sec(-x) = sec x$

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$$Tan(-x) = -tan x$$
 $Cot(-x) = -cot x$

Sum and Difference Formulas - Identities

$$\sin(u+v) = \sin u \cos v + \cos u \sin v$$

$$\sin(u-v) = \sin u \cos v - \cos u \sin v$$

$$\cos(u+v) = \cos u \cos v - \sin u \sin v$$

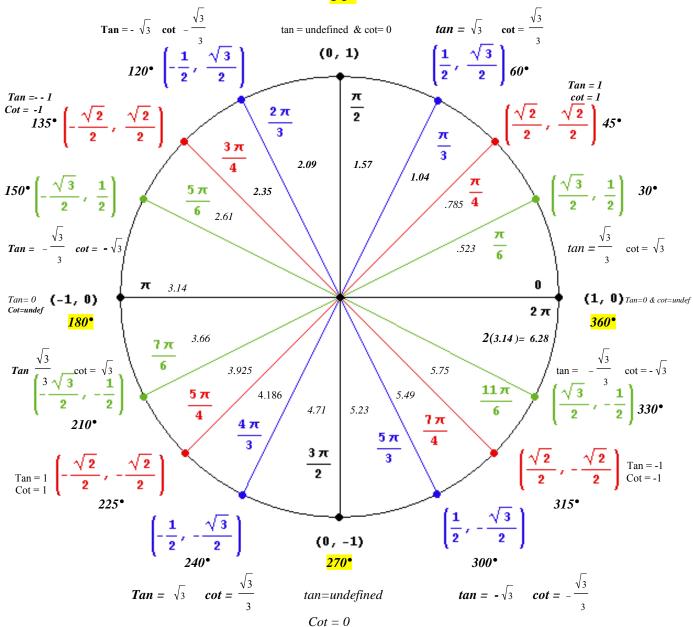
$$\cos(u-v) = \cos u \cos v + \sin u \sin v$$

$$\tan(u+v) = \frac{\tan u + \tan v}{1 - \tan u \tan v}$$

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The Unit Circle

<mark>90°</mark>



Definition of Trigonometric Functions concerning the Unit Circle

$$\sin \theta = \frac{opp}{hyp} = \frac{y}{r}$$

$$\cos \theta = \frac{hyp}{opp} = \frac{r}{y}$$

$$\cos \theta = \frac{adj}{hyp} = \frac{x}{r}$$

$$\sec \theta = \frac{hyp}{adj} = \frac{r}{x}$$

$$\tan \theta = \frac{opp}{adj} = \frac{y}{x}$$
 $\cot \theta = \frac{adj}{opp} = \frac{x}{y}$

Right Triangle Definitions of Trigonometric Functions

Note: sin & cos are complementary angles, so are tan & cot and sec & cos, and the sum of complementary angles is 90 degrees.

$$\sin \theta = \frac{opp}{hyp} = -\frac{y}{r} \qquad \qquad \csc \theta = \frac{h_0}{op}$$

$$\cos \theta = \frac{adj}{hyp} = \frac{x}{r}$$
 $\sec \theta = \frac{hyp}{adj} = \frac{r}{x}$

opposite

В

$$\tan \theta = \frac{opp}{adj} = \frac{y}{x} \qquad \cot \theta =$$

Adjacent = is the side adjacent to the angle in consideration. So if we are considering Angle A, then the adjacent side is CB

Trigonometric Values of Special Angles

Degrees	0°	30°	45°	60°	90°	180°	270°
Radians	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$
sinθ	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1	0	-1
$\cos\theta$	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0	-1	0
tanθ	0	$\frac{\sqrt{3}}{3}$	1	$\sqrt{3}$	undefined	0	undefined

To Convert <u>Degrees to Radians</u>, Multiply by $\frac{\pi \operatorname{rad}}{180 \operatorname{deg}}$

To Convert <u>Radians to Degrees</u>, Multiply by $\frac{180 \deg}{\pi \operatorname{rad}}$

Vocabulary

- Cotangent Angles are two angles with the same terminal side
- Reference Angle is an acute angle formed by terminal side of $angle(\alpha)$ with x-axis

Double Angle Identities

$\sin 2A = 2\sin A\cos A$

$$\cos 2A = \cos^2 A - \sin^2 A$$

$$\cos 2A = 2\cos^2 A - 1$$

$$\cos 2A = 1 - 2\sin^2 A$$

$$\tan 2A = \frac{2\tan A}{1-\tan^2 A}$$

Half Angle Identities

$$\sin\frac{A}{2} = \pm\sqrt{\frac{1-\cos A}{2}}$$

$$\cos\frac{A}{2} = \pm\sqrt{\frac{1+\cos A}{2}}$$

$$\tan\frac{A}{2} = \frac{1 - \cos A}{\sin A}$$

$$\tan\frac{A}{2} = \frac{\sin A}{1 + \cos A}$$

Power Reducing Formulas

$$\sin^2 u = \frac{1 - \cos 2u}{2}$$

$$\cos^2 u = \frac{1 + \cos 2u}{2}$$

$$\tan^2 u = \frac{1 - \cos 2u}{1 + \cos 2u}$$

Product-to-Sum Formulas

$$\sin u \sin v = \frac{1}{2} \left[\cos(u - v) - \cos(u + v) \right]$$

$$\cos u \cos v = \frac{1}{2} \left[\cos(u - v) - \cos(u + v) \right]$$

$$\sin u \cos v = \frac{1}{2} \left[\sin(u+v) + \sin(u-v) \right]$$

$$\cos u \sin v = \frac{1}{2} \left[\sin(u+v) - \sin(u-v) \right]$$

Law of Sines

Solving Oblique Triangles using sine: AAS, ASA, SSA, SSS, SAS

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad \text{or} \quad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Sum-to-Product Formulas

$$\sin x + \sin y = 2\sin\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\sin x - \sin y = 2\cos\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

$$\cos x + \cos y = 2\cos\left(\frac{x+y}{2}\right)\cos\left(\frac{x-y}{2}\right)$$

$$\cos x - \cos y = -2\sin\left(\frac{x+y}{2}\right)\sin\left(\frac{x-y}{2}\right)$$

Law of Cosines

Cosine: SAS, SSS

Standard Form

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$b^2 = a^2 + c^2 - 2ac\cos B$$

$$c^2 = b^2 + a^2 - 2ab\cos C$$

Alternative Form

$$\cos A = \frac{b^2 + c^2 - a^2}{2bc}$$

$$\cos B = \frac{a^2 + c^2 - b^2}{2ac}$$

$$\cos C = \frac{a^2 + b^2 - c^2}{2ab}$$

Finding the Area of non-90degree Triangles

Area of an Oblique Triangle

$$area = \frac{1}{2}bc\sin A = \frac{1}{2}ab\sin C = \frac{1}{2}ac\sin B$$

Heron's Formula

Step 1: Find "s"
$$s = \frac{(a+b+c)}{2}$$
Step 2: Use the formula $area = \sqrt{s(s-a)(s-b)(s-c)}$