MSc-12, Systems Biology First Internal Test

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Answer All three questions. (3 × 10 **Marks)**

Problem-1 Consider the logistic model for the density dependent growth:

$$\frac{dN}{dt} = rN(1 - N/K)$$

where, N(t) is population as a function of time, r and K are parameters of the model.

The exact solution to the above logistic equation is given by,

$$N(t) = \frac{N(0)e^{rt}}{1 + \frac{N(0)}{K}(e^{rt} - 1)}$$

The data on the United states population(in millions) from census taken every 10 years from 1790 to 1930 is given below:

year	t	population(millions)
1790	0	3.9
1800	10	5.3
1810	20	7.2
1820	30	9.6
1830	50	12.9
1840	60	17.8
1850	70	23.2
1860	80	31.4
1870	90	38.5
1880	100	50.2
1890	110	62.9
1900	120	76.2
1910	130	92.2
1920	140	106.2
1930	150	123.2
1940	160	132.2
1950	170	151.3
1960	180	179.3
1970	190	203.3
1980	200	226.5
1990	210	248.7
2000-	220	281.42
2010	230	308.7

(a) Plot time versus population in a graph

(b) For the values $r = 0.0312 \quad and \quad K = 198.6$, compute N(t) as a function of t using above solution formula. Plot this curve on the same plot along with data points. Upto which year, the fit is good? Is the logistic euation is good model for this population growth data?. Comment.

Problem-2 Before a patient undergoes a minor operation, a certain anaesthesia is injected in the muscle of the upper arm. From there it slowly flows into the blood where it exerts its sedating effect. From the blood it is picked up by the liver, where it is ultimately degraded. We write the following model for the amount of anaesthesia in the muscle M, blood B and liver L:

$$\frac{dM}{dt} = -eM, \quad \frac{dB}{dt} = eM - cB, \text{ and } \frac{dL}{dt} = cB - \delta L$$

where the parameter e is the efflux from the muscle, c is the clearance from the blood, and δ is the degradation in the liver. All parameters are rates per hour. The degradation in the muscle and blood is assumed to be negligible.

The initial amount of anaesthesia injected is M (0): 315 mg, the amount in the muscle at time zero.

Let the parameter values of e = 0.5, c = 0.3 and $\delta = 0.4$.

(a) Sketch the amounts of anaesthesia in the muscle, M (t), and in the blood, B (t), as a function of time.

(b) How long does it take before half of the injected amount has flown from the muscle to the blood?

(c) Suppose the degradation rate is slow, i.e., if δ tends to 0, how much anaesthesia will ultimately end up in the liver?

Problem-3 Consider a population in which a predator and its prey co-exist. For prey, the growth rate per capita = 3.5, death rate per capita = 0.5 and the rate at which a prey is eaten by predator is 1. The growth and death rate per capita of the predator is 1 and 3.2. The equations for this simple predator and prey model are as follows:

$$\begin{aligned}\frac{d[prey]}{dt} &= (growth\ rate\ for\ prey)(prey) - (death\ rate\ for\ prey)(prey) \\ &\quad - (rate\ at\ which\ prey\ eaten\ by\ predator)(prey)(predator) \\ \frac{d[predator]}{dt} &= (growth\ rate\ for\ predator)(predator)(prey) \\ &\quad - (death\ rate\ of\ predator)(predator)\end{aligned}$$

Let the initial number of prey is 15 and predator is 7.

Write an R script to solve this system of equations.

(a) Sketch the number of predator and prey as function of time.

(b) Explain the results.

(c) Explain what happens when the initial number of either prey or predator is 0.