MSc-12, Systems Biology Problem Set-1

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Problem-1. On a given day, 1000 birds were intorduced in an island. The birds breed at a constant continuous growth rate of 10% per year. How many brids will be there on the island after 7 years? Write a differential equation and solve it.

Problem-2 Suppose again that there are 1000 birds on an island, breeding with a constant continuous growth rate of 10% per year. But now birds migrate to the island at a constant rate of 100 new arrivals per year. How many birds are on the island after seven years?

Problem-3 Nicotine patches are used by people who wish to discontinue smoking. The patches are applied to the skin and deliver nicotine to the blood stream. The concentration A of nicotine in blood plasma (in $\mu g/L$) can be modelled by differential equations,

$$\frac{dA}{dt} = R_0 - kA \qquad \text{when patch is on}$$

$$\frac{dA}{dt} = -kA \qquad \text{when patch is off}$$

where time 't' is in hours, R_0 is the infusion rate of nicotine and k is the continuous decay rate of Nicotine in the blood stream. Assume that a particular brand of nicotine patch has an infusion rate of $R_0 = 1.0$. Let k=0.12. The Nicotin patch is to be applied for 16 hours(while awake) and then removed for 8 hours (while aslep) in a day.

1. Assume at time t = 0 there is no nicotine in the bloodstream; then the patch is applied for 16 hours. What is the concentration of nicotine at t = 16?

2. The patch is then removed for 8 hours. What is the nicotine concentration at t = 24?

Problem-4 A cell culture in a biology lab currently holds 1 million cells. The cells have a constant continuous birth rate of 1.5% and death rate of 0.5% per hour. Cells are extracted from the culture for an experiment at the rate of 5000 per hour.

How many cells will be in the culture 10 hours from now?

Problem-5 Before a patient undergoes a minor operation, a certain anaesthesia is injected in the muscle of the upper arm. From there it slowly flows into the blood where it exerts its sedating effect. From the blood it is picked up by the liver, where it is ultimately degraded. We write the following model for the amount of anaesthesia in the muscle M, blood B and liver L:

$$\frac{dM}{dt} = -eM, \quad \frac{dB}{dt} = eM - cB, \ \text{and} \ \frac{dL}{dt} = cB - \delta L$$

where the parameter e is the efflux from the muscle, c is the clearance from the blood, and δ is the degradation in the liver. All parameters are rates per hour. The degradation in the muscle and blood is assumed to be negligible.

The initial amount of anaesthesia injected is M (0): 315 mg, the amount in the muscle at time zero.

Let the parameter values of e = 0.5, c = 0.3 and $\delta = 0.4$.

(a) Sketch the amounts of anaesthesia in the muscle, M (t), and in the blood, B (t), as a function of time.

(b) How long does it take before half of the injected amount has flown from the muscle to the blood?

(c) Suppose the degradation rate is slow, i.e., if δ tends to 0, how much anaesthesia will ultimately end up in the liver?