

- * Merge Sort
 - ↳ Divide And Conquer Algorithm
 - ↳ Stable Algorithm
 - ↳ $O(n \log n)$ time and $O(n)$ Aux Space
 - ↳ Best for linked lists. Works even $O(1)$ Auxiliary Space.
 - ↳ Used in External Sorting.
 - ↳ For general arrays, Quick Sort outperforms Quick Sort.

Java 8 → uses Merge + Quick Sort

Python → uses Merge + Insertion Sort = Tim Sort

$$\text{Impl Time complexity} = \Theta(\log_2 n) * \Theta(n)$$

$$= \Theta(n \log_2 n)$$

Generally, $\lceil \log_2 n \rceil + 1$ total no. of steps.

At each step merging takes n iterations
therefore $\Theta(\log_2 n * n)$ time complexity.

- QUICK SORT (Only Naive Partition is Stable partition)
 - Lomuto Partition → Unstable. ↳ $O(n)$ extra space.
 - Not Inplace
 - ↳ Inplace

We work on the approach that elements till $0 \rightarrow i$ are less than pivot and elements from $i+1$ to $j-1$ are greater than pivot

pivot is always last element then :-
Code:

```

int pivot = a[h];           // h → last index
int i = l-1;                // l → lowest index
for (int j=l; j<=h-1; j++) {
    if (a[j] < pivot) {
        i++;
        Swap(a[i], a[j]);
    }
}
Swap(a[i+1], a[h]);
return i+1;
    
```

↳ position of pivot.

Corner Cases :-

① When pivot is smallest

$$\{70, 60, 80, 40, 30\}$$

i=1

→ No swapping will be done so i will remain -1 and at the end.

$$i+ \Rightarrow i=0 \{30, 60, 80, 40, 70\}$$

② When pivot is largest

$$\{30, 40, 20, 50, 80\}$$

here at each iteration ~~is~~ pivot

each element will be swapped by itself until it reaches end of array.

Time Complexity :- $O(N)$

Space " :- $O(1)$

What if we want pivot to be any element then just add the code given below to the beginning of source code :-

Swap($a[p], a[h]$);

$p = h$;

→ Hoare's Partition → Better than Lamutu Partition.

↪ Inplace but Not Stable.

→ It takes $O(1)$ extra space and $O(n)$ time. Also 1 traversal of Input array.

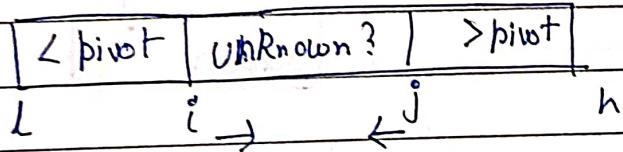
→ We consider low always as the pivot element.

→ We move i from starting of array till when we get the element that is greater than pivot.

Similarly we move j from end of array to the beginning until we get to the starting of array or get an element less than pivot.

→ Then we swap $j \leftrightarrow i$.

→ If $i \geq j$ we break from the loop and return j^{th} index that will be the index $(j+1)$ from which all elements left of it are less than that and right of that are greater.



Corner Case

* We do not fix pivot to its correct position rather, we return an index from which all to the left of it are lesser and right of it are greater.

Corner Cases

① Pivot is smallest $\{5, 10, 9, 12\}$ $i = -1, j = h \rightarrow \text{Pivot}$

initially $i = -1, j = 4$
 Now j will increment only once i.e. $i=0$
 after that j will decrement till it reaches
 \downarrow i^{th} element.
 finally $i=0, j=0$ return j .

② Pivot is largest

$\{12, 10, 5, 9\}$
 \uparrow
 $i=0, j=3$ $\{9, 10, 5, 12\}$
 $i=3, j=2$ $\{9, 10, 5, 12\}$
 \uparrow
 return j

③ All elements are same \rightarrow this proves however is
 not stable partition due to
 which Quicksort will also be unstable.

$\{5, 5, 5, 5\}$
 \uparrow \uparrow
 $i=0$ $j=3$

Swap 1st 5 and last 5
 hence prove Unstable.

$\{5, 5, 5, 5\}$
 \uparrow \uparrow
 $i=1$ $j=2$

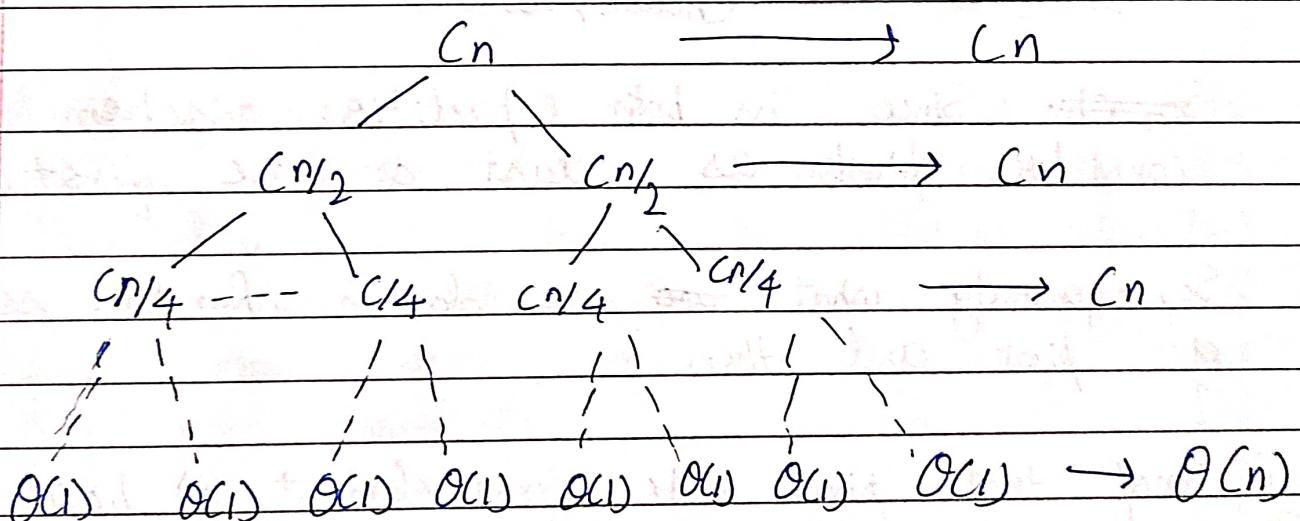
Swap 2nd 5 and 3rd 5.
 Unstable.

Quick Sort Algo

- ① Divide And Conquer Algorithm.
- ② The key part is partitioning (Hoare, Lomuto, Naiive)
- ③ Worst case time complexity $O(n^2)$
- ④ Despite having higher worst case time, it is preferred over other algorithm many time due to :-

- ① Tail Recursive
- ② In-Place
- ③ Cache friendly
- ④ Average case $(n \log n)$

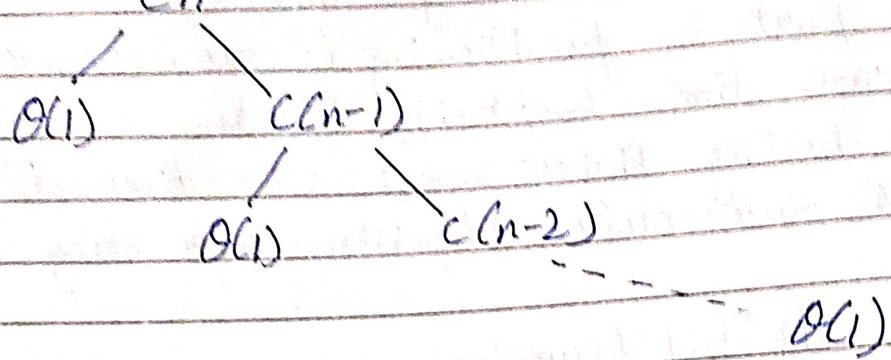
Best Case $\Theta(n \log n)$



$$(C_n + C_n + \dots + C_n) \underbrace{\quad}_{\log_2 n \text{ times}} \rightarrow \Theta(n \log n)$$

$$T(n) = 2T(n/2) + cn$$

- WORST CASE $O(n^2)$ → whenever partition divides $(n-1)$ elements on one side and 1 element on the other side.



$$T(n) = T(n-1) + \Theta(n)$$

Impt. If array is sorted ascending or descending order, both partition scheme goes into worst case. i.e. $O(n^2)$ (Hoare, Lamuto)

So, since in both partition scheme we consider pivot as last or the first element.

So, generally what we do is, Random selection of pivot. And then

- ① Swap that pivot with first element → hoare's
- ② " " " " " last " → lamuto

* Best way to choose pivot is choosing middle element.