SPA: Assignment-2

Q1: Markov Chain Simulation

The first question considered a finite state Markov chain, representing the stages of writing a paper: read(r), write(w), e-mail(e), and surf(s). The transition probability matrix provided was utilized to simulate the evolution of the Markov chain and to compute various probabilities. This was the matrix:

```
(    r     w   e   s
 r 0.5, 0.3, 0, 0.2,
  w  0.2, 0.5, 0.1, 0.2,
  e 0.1, 0.3, 0.3, 0.3,
  s 0, 0.2, 0.3, 0.5
)
```

Part (a):

The Markov chain's evolution after 20 minutes was simulated as,

```
     [,1]      [,2]      [,3]      [,4]
[1,] 0.1707318 0.3360434 0.1815717 0.3116531
[2,] 0.1707318 0.3360434 0.1815718 0.3116531
[3,] 0.1707317 0.3360434 0.1815718 0.3116531
[4,] 0.1707316 0.3360433 0.1815719 0.3116532
```

and the probability $P(X_{20}=s \mid X_0=r) = 0.311653065258266.$

Part (b):

Similarly, the Markov chain was simulated to evolve for 25 minutes as,

```
     [,1]    [,2]    [,3]    [,4]
[1,] 0.18329 0.33794 0.17257 0.30620
[2,] 0.17693 0.33760 0.17692 0.30855
[3,] 0.16788 0.33559 0.18363 0.31290
[4,] 0.15883 0.33359 0.19032 0.31726
```

and the probability $P(X_{25} = s \mid X_{20} = s) = 0.31726$.

Part (c):

The stationary distribution exists and it was (0.1707317 0.3360434 0.1815718 0.3116531).

Part (d):

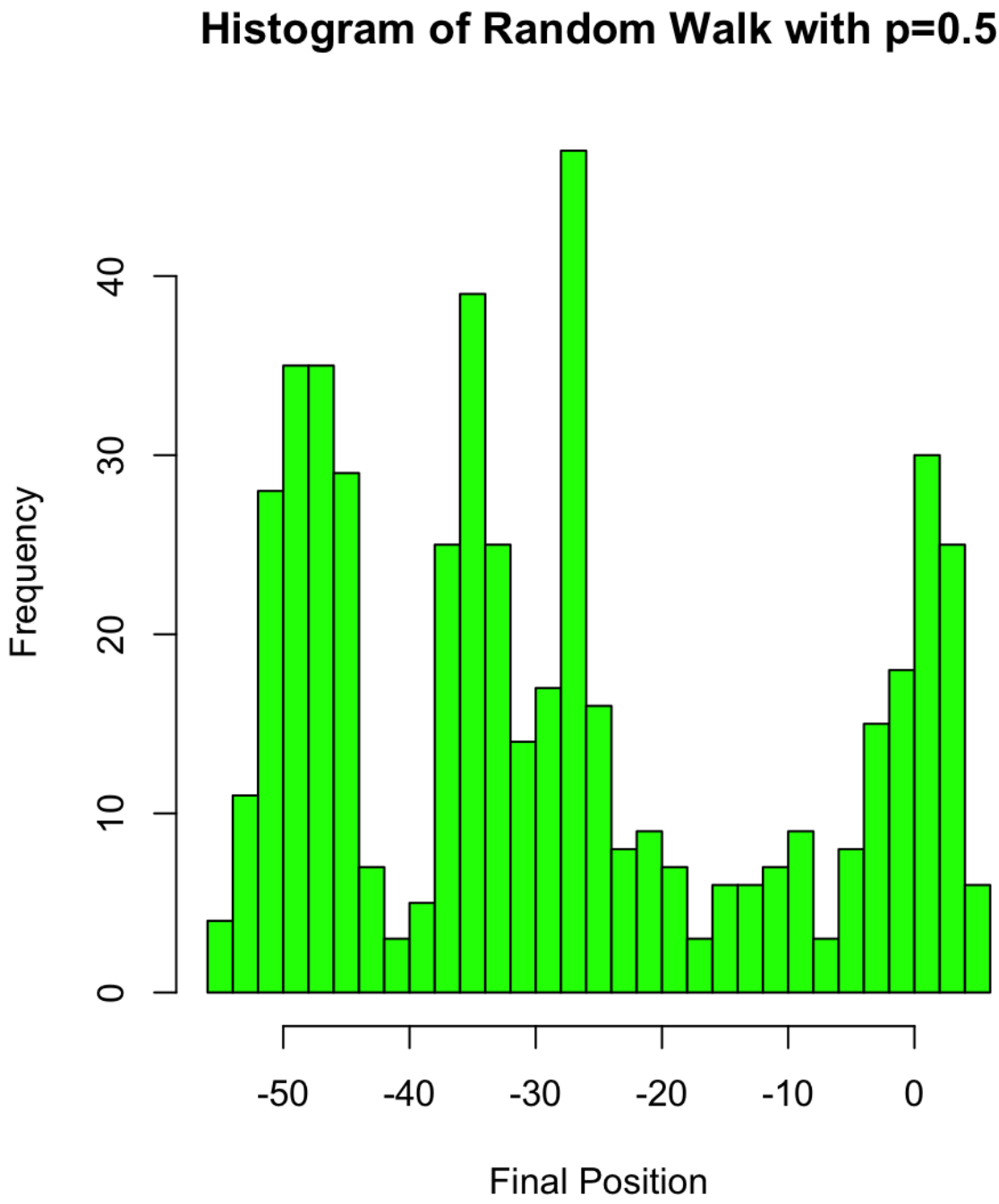
The limiting distribution also exists and it is the same as the stationary distribution.

Q2: 1-D Random Walk Simulation

The second question addressed the simulation of a 1-D random walk under different step-forward probabilities.

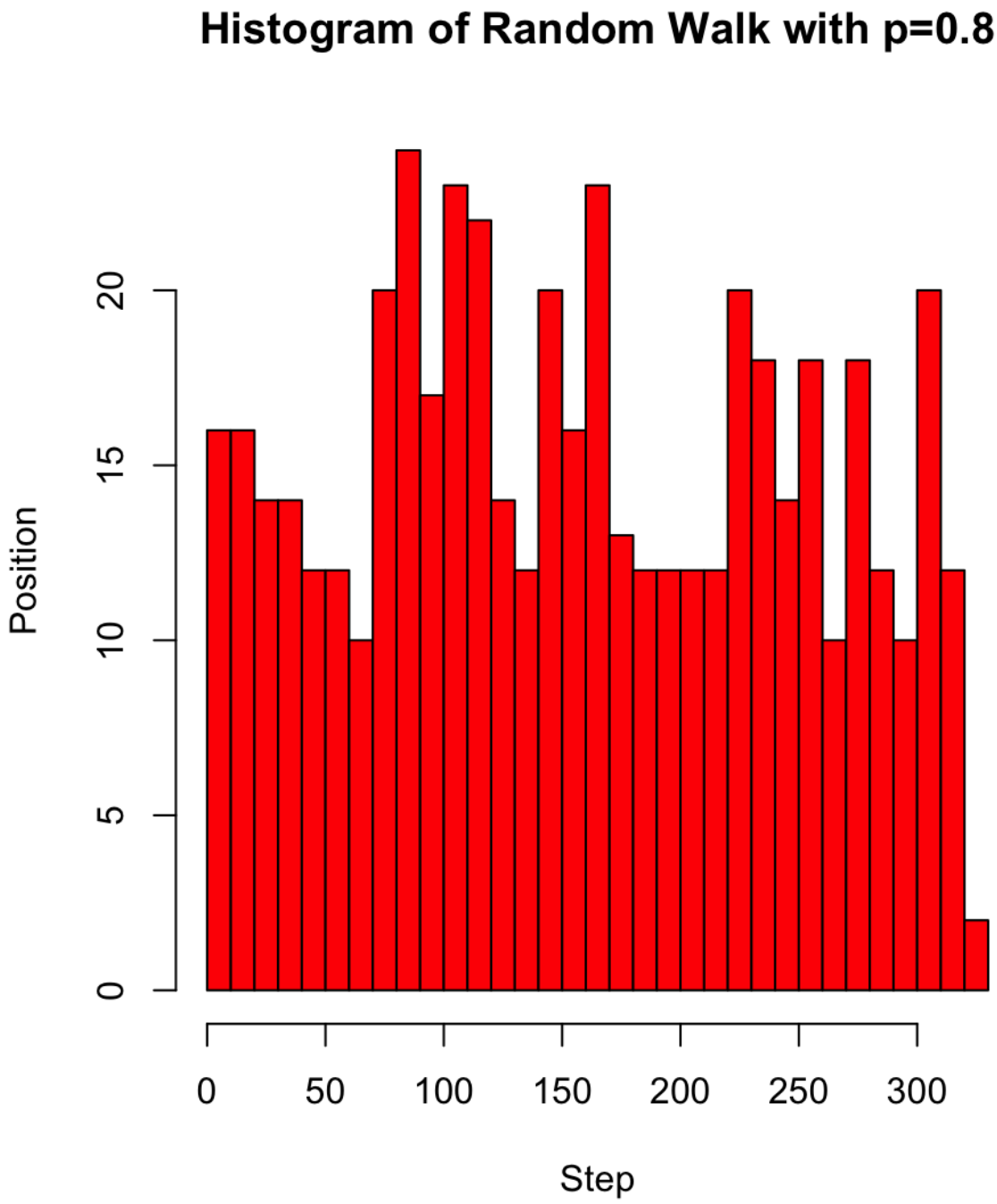
Part (a):

A random walk starting from state `i` with `p = 0.5` was simulated over 500 steps. The simulated probability of stepping right was approximately 0.446.

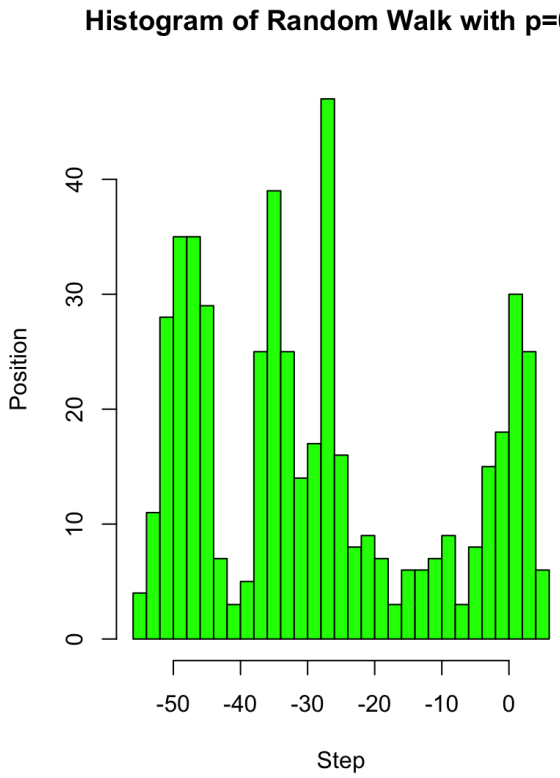


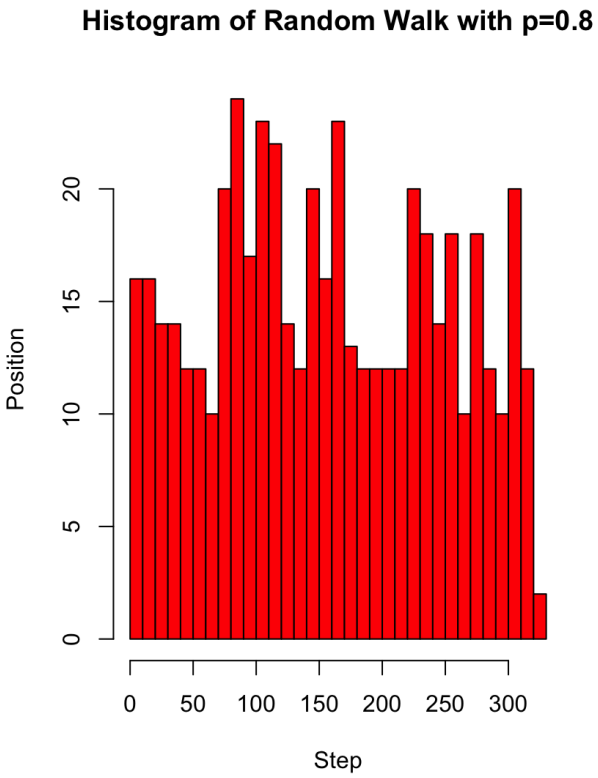
Part (b):

The process was repeated with `p = 0.8`. The final position and the simulated probability (approximately 0.822) indicated a significant drift.



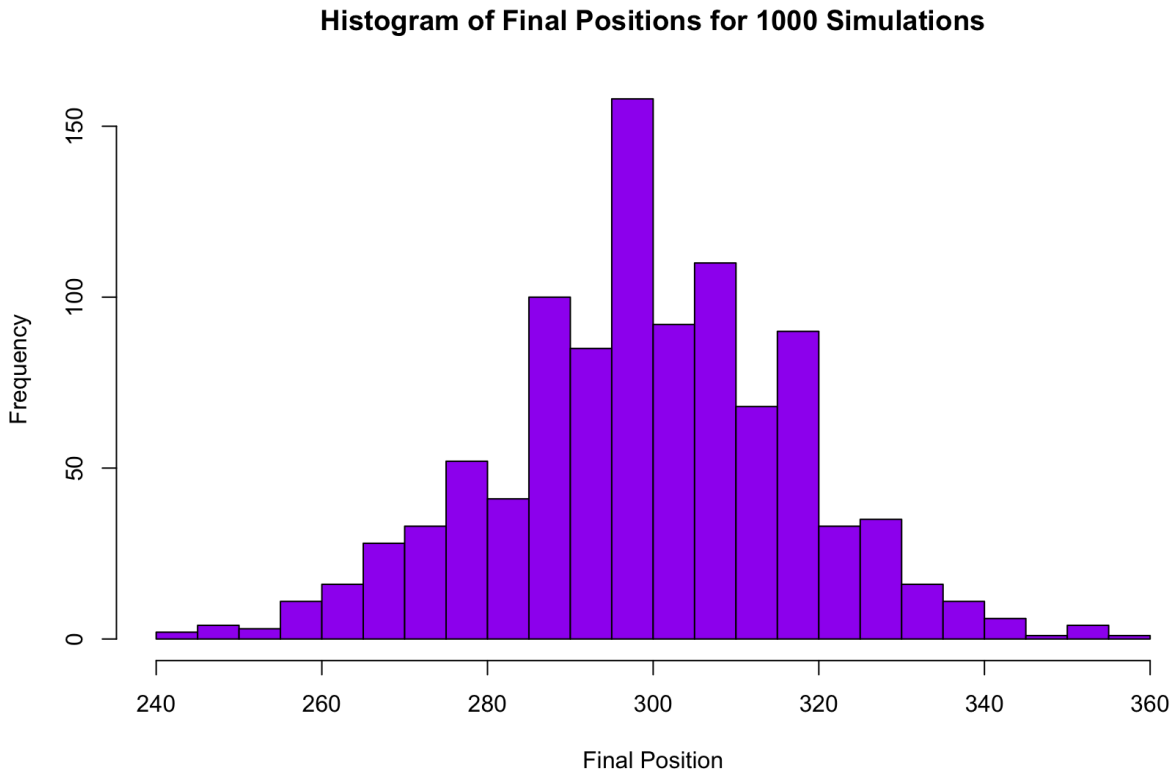
Comparison of Part(a) and Part(b) demonstrates the time-dependent evolution of the MC.



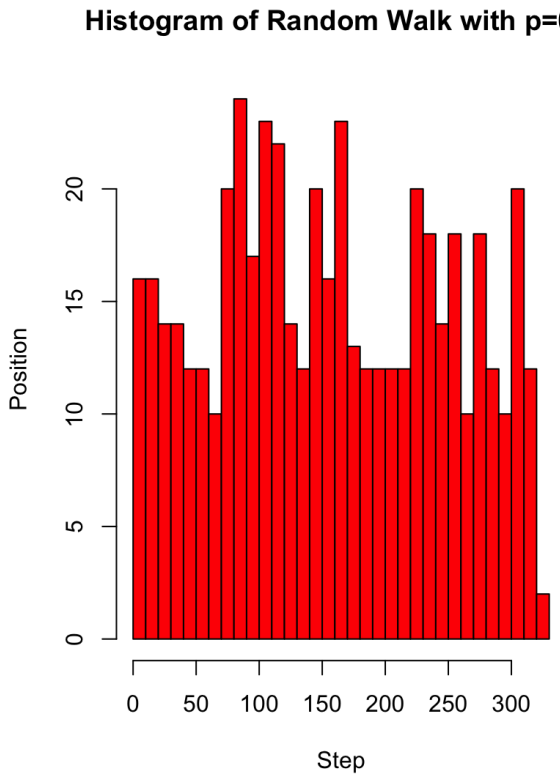


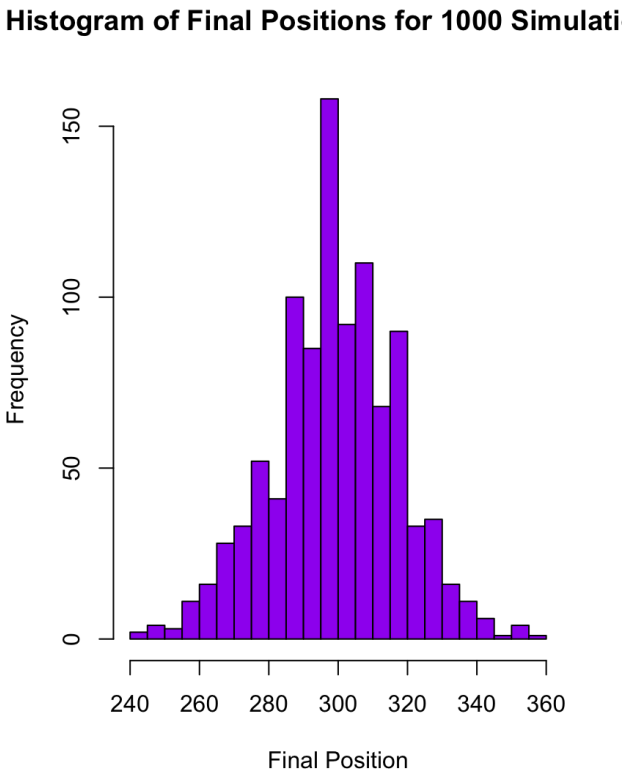
Part (c):

The simulation was done 1000 times. The histogram showed a narrow distribution around the mean final position.



In comparison with Part (b), the expected drift was seen through multiple simulations.


Histogram of Random Walk with p=0.8
Position
Step
0
5
10
15
20
0
50
100
150
200
250
300


Histogram of Final Positions for 1000 Simulati
Frequency
Final Position
0
50
100
150
240
260
280
300
320
340
360