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Uncertainty and Coordination in a Supply Chain

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Lack of Supply Chain Coordination and the Bullwhip Effect

- *Supply chain coordination*
  - all stages of the chain take actions that are aligned and increase total supply chain surplus
- Requires that each stage share information and take into account the effects of its actions on the other stages
- Lack of coordination results when:
  - Objectives of different stages conflict
  - Information moving between stages is delayed or distorted

Bullwhip Effect

- Fluctuations in orders increase as they move up the supply chain from retailers to wholesalers to manufacturers to suppliers
- Distorts demand information within the supply chain
- Results from a loss of supply chain coordination

Demand at Different Stages

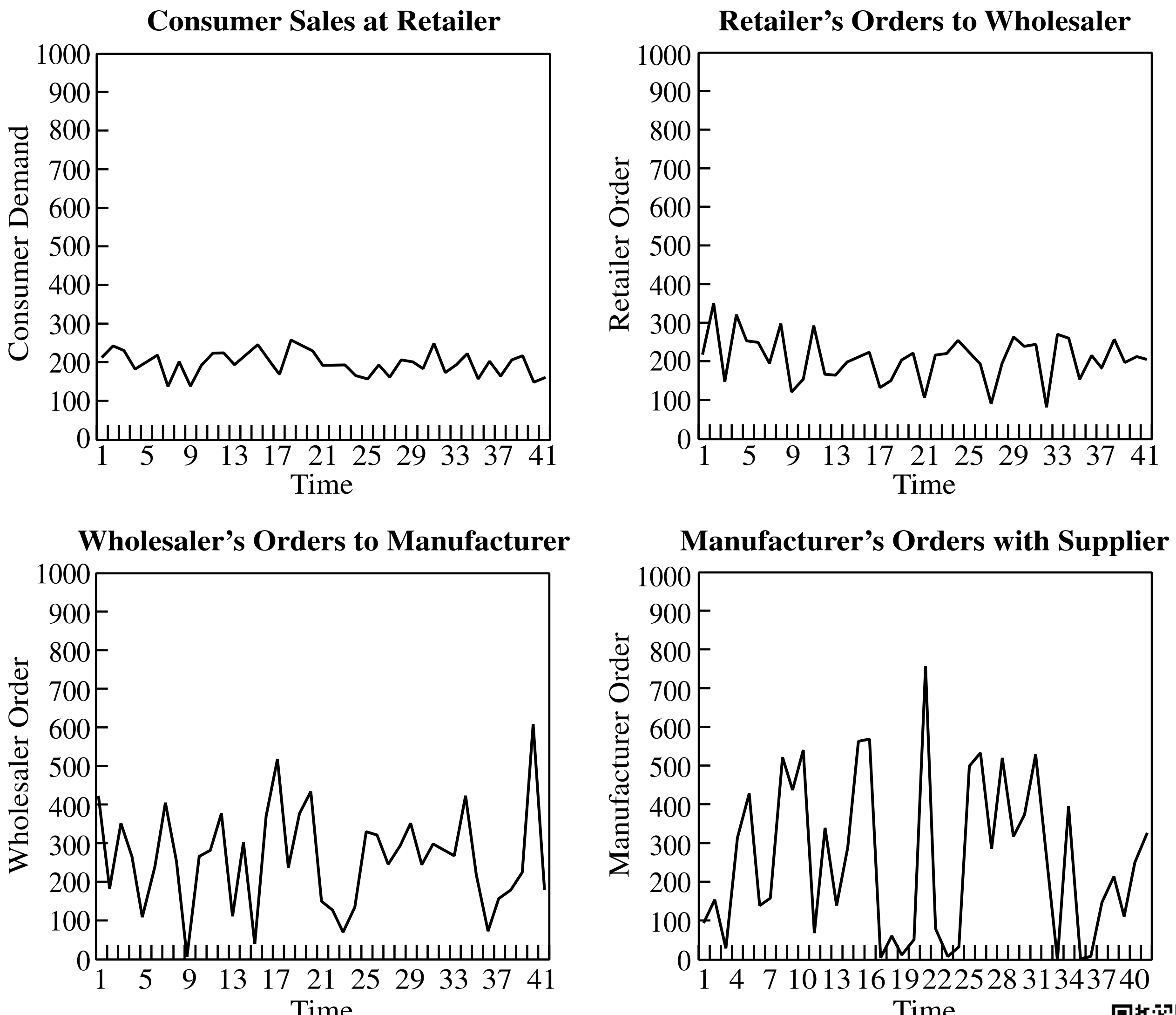


FIGURE 10-1

Key Point

The lack of coordination hurts both responsiveness and cost in a supply chain by making it more expensive to provide a given level of product availability.

Improving Information Visibility and Accuracy

- Sharing customer demand data
- Implementing collaborative forecasting and planning
- Designing single-stage control of replenishment
  - Continuous replenishment programs (CRP)
  - Vendor managed inventory (VMI)

Key Point

Demand planning at each stage in a supply chain based on the stream of orders received from its down- stream stage results in a magnification of fluctuation in orders as one moves up the supply chain from the retailer to the manufacturer. It is better for the entire supply chain to forecast based on end customer demand.

Continuous Replenishment and Vendor-Managed Inventories

- CRP – wholesaler or manufacturer replenishes based on POS data
- VMI – manufacturer or supplier is responsible for all decisions regarding inventory

Collaborative Planning, Forecasting, and Replenishment (CPFR)

- Sellers and buyers in a supply chain may collaborate along any or all of the following
  1. Strategy and planning
  2. Demand and supply management
  3. Execution
  4. Analysis

Achieving Coordination in Practice

- Quantify the bullwhip effect
- Get top management commitment for coordination
- Devote resources to coordination
- Focus on communication with other stages
- Try to achieve coordination in the entire supply chain network
- Use technology to improve connectivity in the supply chain
- Share the benefits of coordination equitably

The Role of Safety Inventory

- *Safety inventory* is carried to satisfy demand that exceeds the amount forecasted
  - Raising the level of safety inventory increases product availability and thus the margin captured from customer purchases
  - Raising the level of safety inventory increases inventory holding costs

The Role of Safety Inventory

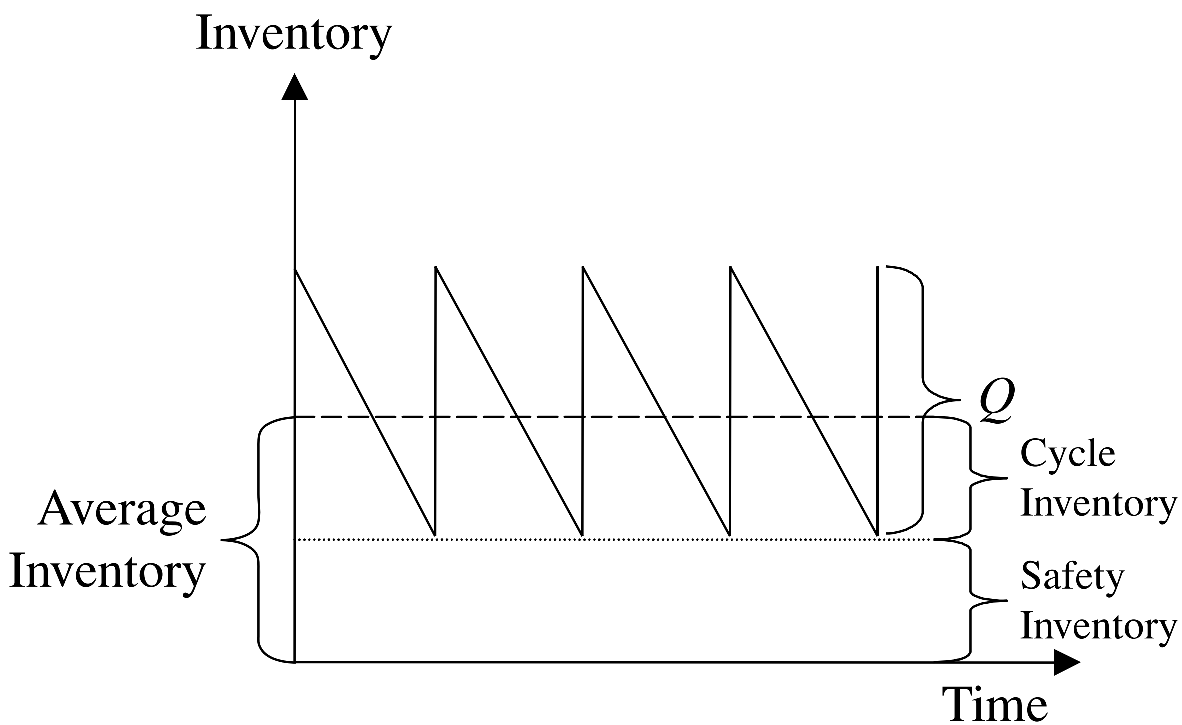


FIGURE 12-1

The Role of Safety Inventory

- Three key questions
  1. What is the appropriate level of product availability?
  2. How much safety inventory is needed for the desired level of product availability?
  3. What actions can be taken to reduce safety inventory without hurting product availability?

The Level of Safety Inventory

- Determined by two factors
  - The uncertainty of both demand and supply
  - The desired level of product availability
- Measuring Demand Uncertainty

D = Average demand per period

σ_D = Standard deviation of demand (forecast error) per period

Lead time (L) is the gap between when an order is placed and when it is received

Evaluating Demand Distribution Over *L* Periods

$$D_L = \sum_{i=1}^{L} D_i \qquad \sigma_L = \sqrt{\sum_{i=1}^{L} \sigma_i^2 + 2\sum_{i>j} \rho_{ij}\sigma_i\sigma_j}$$

$$D_L = DL \qquad \sigma_L = \sqrt{L}\sigma_D$$

The *coefficient of variation*

$$cv = \sigma / \mu$$

Measuring Product Availability

1. *Product fill rate (fr)*
   – Fraction of product demand satisfied from product in inventory
2. *Order fill rate*
   – Fraction of orders filled from available inventory
3. *Cycle service level (CSL)*
   – Fraction of replenishment cycles that end with all customer demand being met

Measuring Product Availability

1. *Product fill rate (fr)*
   – Fraction of product demand satisfied from

2. *O*
   – ory

3. *C*
   – Fraction of replenishment cycles that end with all customer demand being met

replenishment cycle – the interval between two successive replenishment deliveries

Replenishment Policies

1. *Continuous review*
   – Inventory is continuously tracked
   – Order for a lot size *Q* is placed when the inventory declines to the reorder point (*ROP*)
2. *Periodic review*
   – Inventory status is checked at regular periodic intervals
   – Order is placed to raise the inventory level to a specified threshold

Determining the Appropriate Level of Safety Inventory

- Evaluating Safety Inventory Given a Replenishment Policy

Expected demand during lead time = $D \times L$

Safety inventory, $ss = \text{ROP} - D \times L$

Determining the Appropriate Level of Safety Inventory

Average demand per week, $D = 2{,}500$

Standard deviation of weekly demand, $\sigma_D = 500$

Average lead time for replenishment, $L = 2$ weeks

Reorder point, $ROP = 6{,}000$

Average lot size, $Q = 10{,}000$

Safety inventory, $ss = ROP - DL = 6{,}000 - 5{,}000 = 1{,}000$

Cycle inventory $= Q/2 = 10{,}000/2 = 5{,}000$

Determining the Appropriate Level of Safety Inventory

Average inventory	= cycle inventory + safety inventory
	= 5,000 + 1,000 = 6,000
Average flow time	= average inventory/throughput
	= 6,000/2,500 = 2.4 weeks

Determining the Appropriate Level of Safety Inventory

- Evaluating Cycle Service Level Given a Replenishment Policy

$$CSL = Prob(\text{ddlt of } L \text{ weeks} \leq ROP)$$

$$CSL = F(ROP, D_L, \sigma_L) = NORMDIST(ROP, D_L, \sigma_L, 1)$$

(ddlt = demand during lead time)

Determining the Appropriate Level of Safety Inventory

$Q = 10{,}000$, $ROP = 6{,}000$, $L = 2$ weeks

$D = 2{,}500$/week, $\sigma_D = 500$

$$D_L = D \times L = 2 \times 2{,}500 = 5{,}000$$

$$\sigma_L = \sqrt{L}\sigma_D = \sqrt{2} \times 500 = 707$$

$$CSL = F(ROP, D_L, \sigma_L) = NORMDIST(ROP, D_L, \sigma_L, 1)$$

$$= NORMDIST(6{,}000, 5{,}000, 707, 1) = 0.92$$

Determining the Appropriate Level of Safety Inventory

- Evaluating Required Safety Inventory Given a Desired Cycle Service Level

Desired cycle service level = CSL

Mean demand during lead time = D_L

Standard deviation of demand during lead time = σ_L

$$\text{Probability}(\text{demand during lead time} \leq D_L + ss) = CSL$$

- Identify safety inventory ss so that

$$F(D_L + ss, D_L, s_L) = CSL$$

Determining the Appropriate Level of Safety Inventory

$$D_L + ss = F^{-1}(CSL, D_L, \sigma_L) = NORMINV(CSL, D_L, \sigma_L)$$

or

$$ss = F^{-1}(CSL, D_L, \sigma_L) - D_L = NORMINV(CSL, D_L, \sigma_L) - D_L$$

$$ss = F_S^{-1}(CSL) \times \sigma_L = F_S^{-1}(CSL) \times \sqrt{L}\sigma_D$$

$$= NORMSINV(CSL) \times \sqrt{L}\sigma_D$$

Evaluating Fill Rate Given a Replenishment Policy

- *Expected shortage per replenishment cycle* (*ESC*) is the average units of demand that are not satisfied from inventory in stock per replenishment cycle
- Product fill rate

$$fr = 1 - ESC/Q = (Q - ESC)/Q$$

Evaluating Fill Rate Given a Replenishment Policy

$$ESC = \int_{x=ROP}^{\infty} (x - ROP) f(x)\, dx$$

$$ESC = -ss\left[1 - F_s\left(\frac{ss}{\sigma_L}\right)\right] + \sigma_L f_s\left(\frac{ss}{\sigma_L}\right)$$

$$ESC = -ss[1 - NORMDIST(ss / \sigma_L, 0, 1, 1)]$$
$$+\sigma_L NORMDIST(ss / \sigma_L, 0, 1, 0)$$

Evaluating Fill Rate Given a Replenishment Policy

Lot size, Q = 10,000

Average demand during lead time, D_L = 5,000

Standard deviation of demand during lead time, σ_L = 707

Safety inventory, $ss = ROP - DL$ = 6,000 – 5,000 = 1,000

$$ESC = -1{,}000[1 - NORMDIST(1{,}000/707,0,1,1)] \\ +707\,NORMDIST(1{,}000/707,0,1,0) = 25$$

$fr = (Q - ESC)/Q$ = 110,000 – 252/10,000 = 0.9975

Evaluating Fill Rate Given a Replenishment Policy

	A	B	C	D	E
1	*Inputs*				
2	Q	D	σ_D	L	ss
3	10,000	2,500	500	2	1,000
4	*Distribution of demand during lead time*				
5	D_L	σ_L			
6	5,000	707			
7	*Cycle Service Level and Fill Rate*				
8	CSL	ESC	fr		
9	0.92	25.13	0.9975		

Cell	Cell Formula	Equation
A6	=B3*D3	12.2
B6	=SQRT(D3)*C3	12.2
A9	=NORMDIST(A6+E3, A6, B6, 1)	12.4
B9	=-E3*(1-NORMDIST(E3/B6, 0, 1, 1)) + B6*NORMDIST(E3/B6, 0, 1, 0)	12.8
C9	=(A3-B9)/A3	12.5

FIGURE 12-2

Key Point

Both fill rate and cycle service level increase as the safety inventory is increased. For the same safety inventory, an increase in lot size increases the fill rate but not the cycle service level.

Evaluating Safety Inventory Given Desired Fill Rate

- Expected shortage per replenishment cycle is

$$ESC = (1 - fr)Q$$

- No equation for *ss*
- Try values or use *GOALSEEK* in Excel

Evaluating Safety Inventory Given Desired Fill Rate

Desired fill rate, $fr = 0.975$

Lot size, $Q = 10{,}000$ boxes

Standard deviation of ddlt, $\sigma_L = \sqrt{2} \times 500 = 707$

$$ESC = (1 - fr)Q = (1 - 0.975)10{,}000 = 250$$

Evaluating Safety Inventory Given Desired Fill Rate

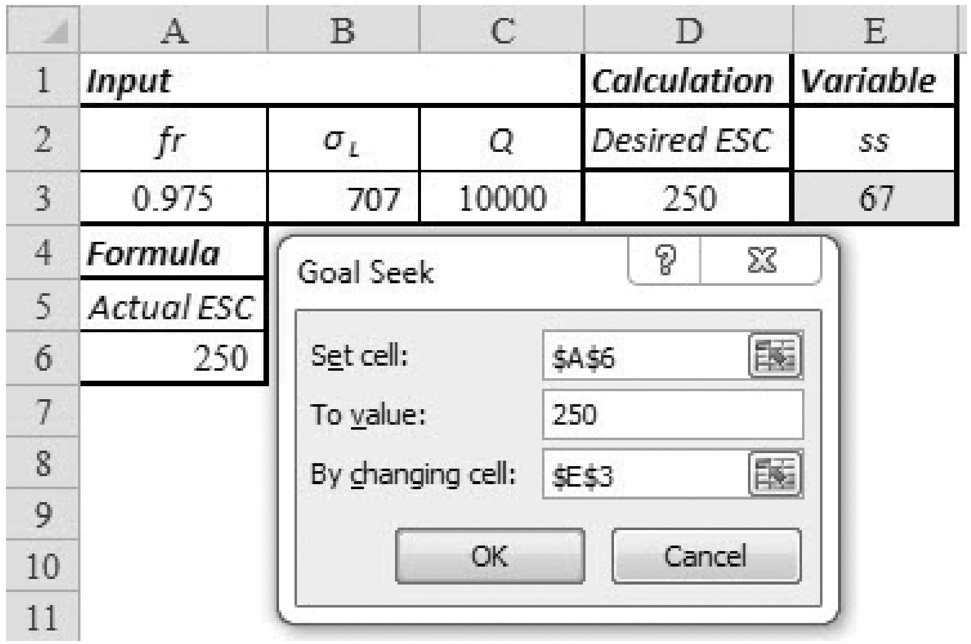
$$ESC = 250 = -ss\left[1 - F_s\left(\frac{ss}{\sigma_L}\right)\right] + \sigma_L f_s\left(\frac{ss}{\sigma_L}\right)$$

$$= -ss\left[1 - F_s\left(\frac{ss}{707}\right)\right] + 707 f_s\left(\frac{ss}{707}\right)$$

$$250 = -ss[1 - NORMDIST(ss/707,0,1,1)]$$
$$+707\,NORMDIST(ss/707,0,1,0)$$

- Use *GOALSEEK* to find safety inventory ss = 67 boxes

Evaluating Safety Inventory Given Desired Fill Rate



Cell	Cell Formula	Equation
A6	-E3*(1-NORMSDIST(E3/B3, 0, 1,1)) + B3*NORMDIST(E3/B3, 0, 1, 0)	12.10

FIGURE 12-3

Impact of Desired Product Availability and Uncertainty

- As desired product availability goes up the required safety inventory increases

Fill Rate	Safety Inventory
97.5%	67
98.0%	183
98.5%	321
99.0%	499
99.5%	767

TABLE 12-1

Key Point

The required safety inventory grows rapidly with an increase in the desired product availability.

Impact of Desired Product Availability and Uncertainty

- Goal is to reduce the level of safety inventory required in a way that does not adversely affect product availability
  1. Reduce the supplier lead time L
  2. Reduce the underlying uncertainty of demand (represented by σ_D)

Benefits of Reducing Lead Time

D = 2,500/week, σ_D = 800, CSL = 0.95

$$ss = NORMSINV(CSL) \times \sqrt{L}\sigma_D$$
$$= NORMSINV(.95) \times \sqrt{9} \times 800 = 3{,}948$$

- If lead time is reduced to one week

$$ss = NORMSINV(.95) \times \sqrt{1} \times 800 = 1{,}316$$

- If standard deviation is reduced to 400

$$ss = NORMSINV(.95) \times \sqrt{9} \times 400 = 1{,}974$$

Impact of Supply Uncertainty on Safety Inventory

- We incorporate supply uncertainty by assuming that lead time is uncertain

D: Average demand per period

σ_D: Standard deviation of demand per period

L: Average lead time for replenishment

s_L: Standard deviation of lead time

$$D_L = DL \qquad \sigma_L = \sqrt{L\sigma_D^2 + D^2 s_L^2}$$

Impact of Lead Time Uncertainty on Safety Inventory

Average demand per period, D = 2,500

Standard deviation of demand per period, $\sigma_D = 500$

Average lead time for replenishment, L = 7 days

Standard deviation of lead time, s_L = 7 days

Mean ddlt, $D_L = DL = 2{,}500 \times 7 = 17{,}500$

Standard deviation of ddlt $\sigma_L = \sqrt{L\sigma_D^2 + D^2 s_L^2}$

$$= \sqrt{7 \times 500^2 + 2{,}500^2 \times 7^2}$$

$$= 17{,}500$$

Impact of Lead Time Uncertainty on Safety Inventory

- Required safety inventory

$$ss = F_S^{-1}(CSL) \times \sigma_L = NORMSINV(CSL) \times \sigma_L$$
$$= NORMSINV(0.90) \times 17{,}500$$
$$= 22{,}491 \text{ tablets}$$

TABLE 12-2

s_L	σ_L	*ss* (units)	*ss* (days)
6	15,058	19,298	7.72
5	12,570	16,109	6.44
4	10,087	12,927	5.17
3	7,616	9,760	3.90
2	5,172	6,628	2.65
1	2,828	3,625	1.45
0	1,323	1,695	0.68

Key Point

A reduction in supply uncertainty can help to dramatically reduce the required safety inventory without hurting product availability.

Impact of Aggregation on Safety Inventory

- How does aggregation affect forecast accuracy and safety inventories

D_i: Mean weekly demand in region i, $i = 1,\ldots, k$

σ_i: Standard deviation of weekly demand in region i, $i = 1,\ldots, k$

ρ_{ij}: Correlation of weekly demand for regions i, j, $1 \leq i \neq j \leq k$

Impact of Aggregation on Safety Inventory

Total safety inventory in decentralized option $= \sum_{i=1}^{k} F_S^{-1}(CSL) \times \sqrt{L} \times \sigma_i$

$$D^C = \sum_{i=1}^{k} D_i; \qquad \mathrm{var}\left(D^C\right) = \sum_{i=1}^{k} \sigma_i^2 + 2\sum_{i>j} \rho_{ij}\sigma_i\sigma_j;$$

$$\sigma_D^C = \sqrt{\mathrm{var}\left(D^C\right)}$$

$$D^C = kD \qquad \sigma_D^C = \sqrt{k\sigma^2 + k\left(k-1\right)\rho\sigma^2}$$

Simplified to $\sigma_D^C = \sqrt{k}\sigma_D$

Impact of Aggregation on Safety Inventory

Require safety inventory on aggregation

$$= \sum_{i=1}^{k} F_S^{-1}(CSL) \times \sqrt{L} \times \sigma_D^C$$

Holding – cost savings on aggregation per unit sold

$$= \frac{F_S^{-1}(CSL) \times \sqrt{L} \times H}{D^C} \times \left(\sum_{i=1}^{k} \sigma_i - \sigma_D^C \right)$$

Impact of Aggregation on Safety Inventory

- The safety inventory savings on aggregation increase with the desired cycle service level *CSL*
- The safety inventory savings on aggregation increase with the replenishment lead time *L*
- The safety inventory savings on aggregation increase with the holding cost *H*
- The safety inventory savings on aggregation increase with the coefficient of variation of demand (σ_D/D)
- The safety inventory savings on aggregation decrease as the correlation coefficients increase

Trade-offs of Physical Centralization

- Use four regional or one national distribution center

$$D = 1{,}000/\text{week},\ \sigma_D = 300,\ L = 4 \text{ weeks},\ CSL = 0.95$$

- Four regional centers

Total required
safety inventory, $ss = 4 \times F_s^{-1}(CSL) \times \sqrt{L} \times \sigma_D$

$$= 4 \times NORMSINV(0.95) \times \sqrt{4} \times 300 = 3{,}948$$

Trade-offs of Physical Centralization

- One national distribution center, $\rho = 0$

Standard deviation
of weekly demand, $\sigma_D^C = \sqrt{4} \times 300 = 600$

$$ss = F_s^{-1}(0.95) \times \sqrt{L} \times \sigma_D^C$$
$$= NORMSINV(0.95) \times \sqrt{4} \times 600 = 1{,}974$$

Decrease in holding costs = (3,948 – 1,974) $1,000 x 0.2
= $394,765

Decrease in facility costs = $150,000

Increase in transportation = 52 x 1,000 x (13 – 10)
= $624,000

Product Substitution

- The use of one product to satisfy demand for a different product
  1. *Manufacturer-driven substitution*
     - Allows aggregation of demand
     - Reduce safety inventories
     - Influenced by the cost differential, correlation of demand
  2. *Customer-driven substitution*
     - Allows aggregation of safety inventory

Component Commonality

- Without common components
  - Uncertainty of demand for a component is the same as for the finished product
  - Results in high levels of safety inventory
- With common components
  - Demand for a component is an aggregation of the demand for the finished products
  - Component demand is more predictable
  - Component inventories are reduced

Value of Component Commonality

27 servers, 3 components, 3 x 27 = 81 distinct components

Monthly demand = 5,000

Standard deviation = 3,000

Replenishment lead time = 1 month

CSL = 0.95

Total safety inventory required

$$= 81 \times NORMSINV(0.95) \times \sqrt{1} \times 3{,}000$$
$$= 399{,}699 \text{ units}$$

Safety inventory per common component

$$= NORMSINV(0.95) \times \sqrt{1} \times \sqrt{9} \times 3{,}000$$
$$= 14{,}804 \text{ units}$$

Value of Component Commonality

- With component commonality
- Nine distinct components

Total safety inventory required $= 9 \times 14,804 = 133,236$

Value of Component Commonality

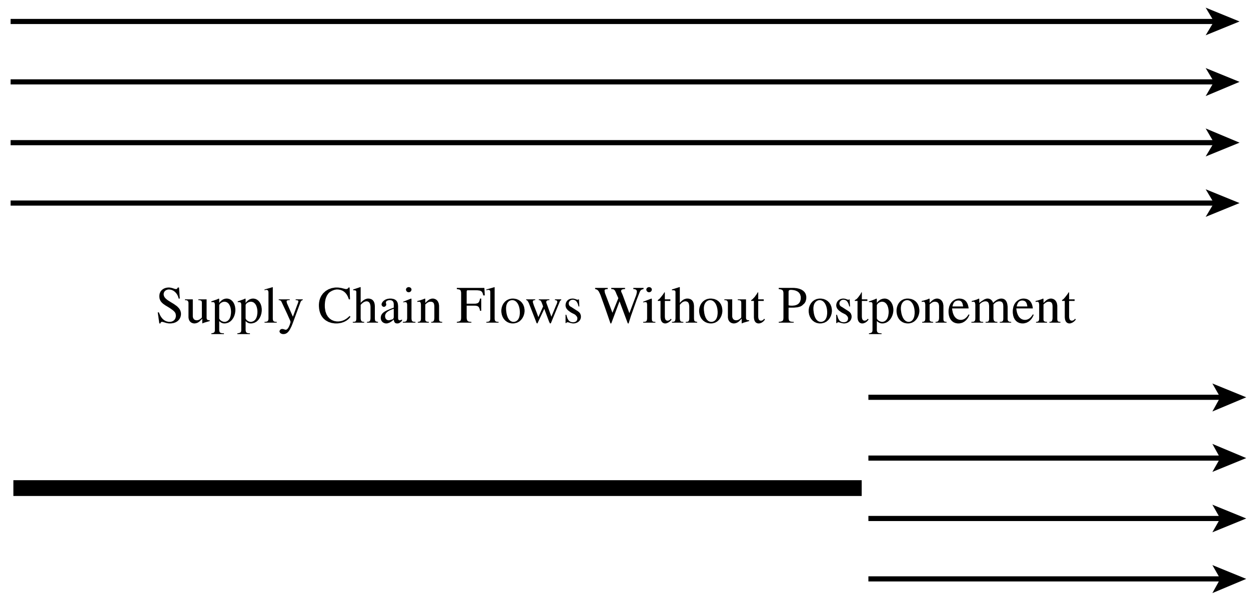
Number of Finished Products per Component	Safety Inventory	Marginal Reduction in Safety Inventory	Total Reduction in Safety Inventory
1	399,699		
2	282,630	117,069	117,069
3	230,766	51,864	168,933
4	199,849	30,917	199,850
5	178,751	21,098	220,948
6	163,176	15,575	236,523
7	151,072	12,104	248,627
8	141,315	9,757	258,384
9	133,233	8,082	266,466

TABLE 12-5

Postponement

- Delay product differentiation or customization until closer to the time the product is sold
  - Have common components in the supply chain for most of the push phase
  - Move product differentiation as close to the pull phase of the supply chain as possible
  - Inventories in the supply chain are mostly aggregate

Postponement



Supply Chain Flows Without Postponement

Supply Chain Flows with Component Commonality and Postponement

FIGURE 12-5

Value of Postponement

100 different paint colors, $D = 30/\text{week}$, $\sigma_D = 10$,

$L = 2$ weeks, $CSL = 0.95$

Total required
safety inventory, $ss = 100 \times F_s^{-1}(CSL) \times \sqrt{L} \times \sigma_D$

$$= 100 \times NORMSINV(0.95) \times \sqrt{2} \times 10 = 2{,}326$$

Standard deviation of
base paint weekly demand, $\sigma_D^C = \sqrt{100} \times 10 = 100$

$$ss = F_s^{-1}(CSL) \times \sqrt{L} \times \sigma_D^C = NORMSINV(0.95) \times \sqrt{2} \times 100 = 233$$

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Impact of Replenishment Policies on Safety Inventory

- Continuous Review Policies

D: Average demand per period

σ_D: Standard deviation of demand per period

L: Average lead time for replenishment

Mean demand during lead time, $D_L = D \times L$

Standard deviation of demand during lead time, $\sigma_L = \sqrt{L}\sigma_D$

$$ss = F_S^{-1}(CSL) \times \sigma_L = NORMSINV(CSL) \times \sqrt{L}\sigma_D, ROP = D_L + ss$$

Impact of Replenishment Policies on Safety Inventory

- Periodic Review Policies
  - Lot size determined by prespecified *order-up-to level* (*OUL*)

D: Average demand per period

σ_D: Standard deviation of demand per period

L: Average lead time for replenishment

T: Review interval

CSL: Desired cycle service level

Impact of Replenishment Policies on Safety Inventory

Probability(demand during $L + T \leq OUL$) = CSL

$$\text{Mean demand during } T+L \text{ periods, } D_{T+L} = (T+L)D$$

$$\text{Std dev demand during } T+L \text{ periods, } \sigma_{T+L} = \sqrt{T+L}\,\sigma_D$$

$$OUL = D_{T+L} + ss$$

$$ss = F_S^{-1}(CSL) \times \sigma_{D+L} = NORMSINV(CSL) \times \sigma_{T+L}$$

$$\text{Average lot size, } Q = D_T = D \times T$$

Impact of Replenishment Policies on Safety Inventory

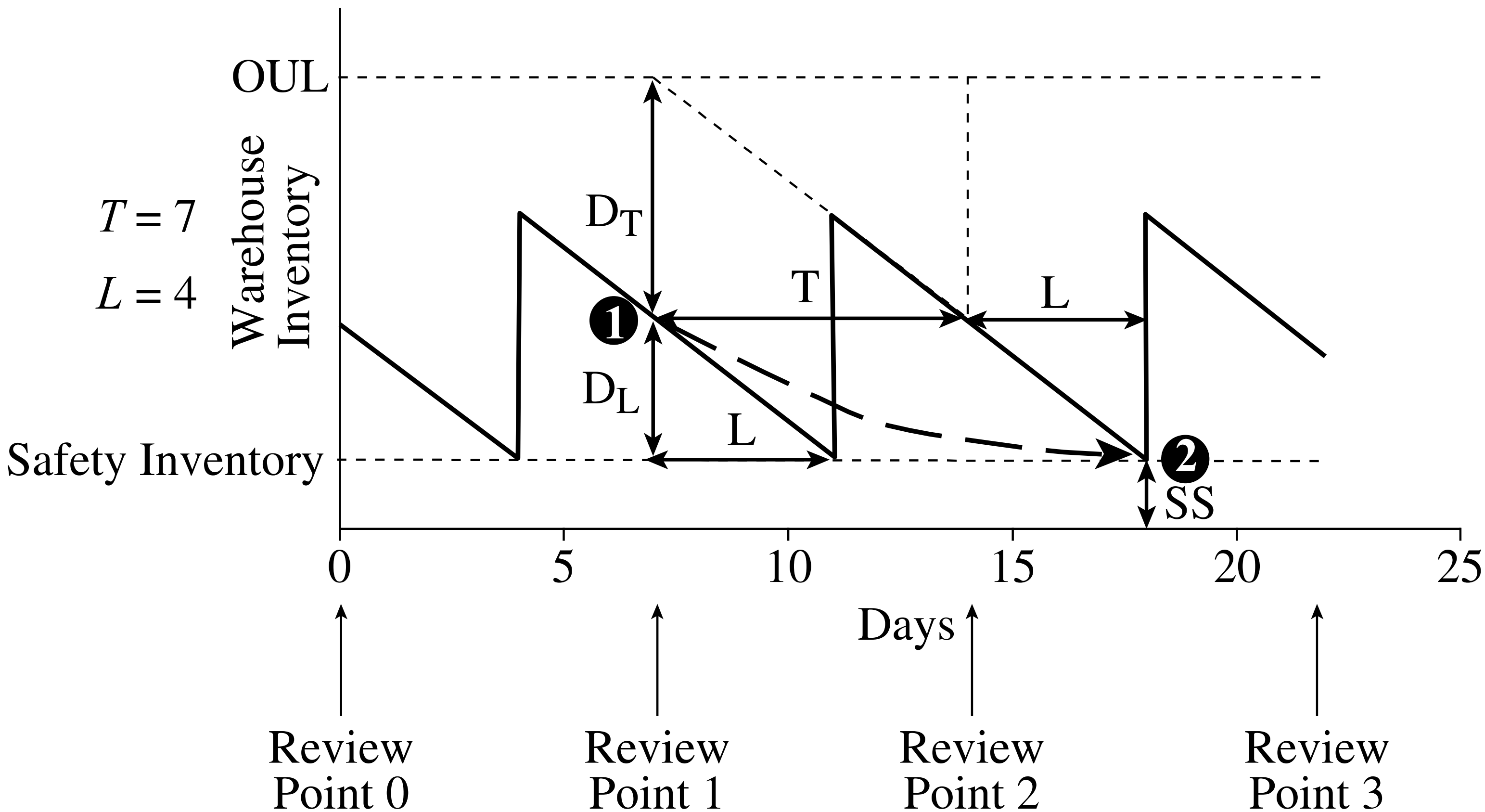

FIGURE 12-6

Evaluation Safety Inventory for a Periodic Review Policy

D = 2,500, σ_D = 500, L = 2 weeks, T = 4 weeks

Mean demand during $T+L$ periods, $D_{T+L} = (T+L)D$

$$= (2+4)2{,}500 = 15{,}000$$

Std dev demand during $T+L$ periods, $\sigma_{T+L} = \sqrt{T+L}\sigma_D$

$$= \left(\sqrt{4+2}\right)500 = 1{,}225$$

$$ss = F_S^{-1}(CSL) \times \sigma_{D+L} = NORMSINV(CSL) \times \sigma_{T+L}$$

$$= NORMSINV(0.90) \times 1{,}225 = 1{,}570 \text{ boxes}$$

$$OUL = D_{T+L} + ss = 15{,}000 + 1{,}570 = 16{,}570$$

Key Point

Periodic review replenishment policies require more safety inventory than continuous review policies for the same lead time and level of product availability.

The Role of IT in Inventory Management

- IT systems can help
  - Improve inventory visibility
  - Coordination in the supply chain
  - Track inventory (RFID)
- Value tightly linked to the accuracy of the inventory information

Summary of Learning Objectives

1. Describe different measures of product availability
2. Understand the role of safety inventory in a supply chain
3. Identify factors that influence the required level of safety inventory
4. Utilize managerial levers available to lower safety inventory and improve product availability