Customer Lifetime Value (CLV or LTV)

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Marketing Analytics

- How to segment customers?
- Whom to target?
- Understand consumer preferences

- What customer is going to buy next?
- How much purchase they going to make?
- How many will stay with the firm for next period?

Prediction: Regression Type Model

- Next period predictions
  - Buy or not buy
  - Which brand
  - How much
- Using past behaviour and marketing activities
  - Supervised learning
    - Regression, Decision tree, Random forest, Xgboost, Neural networks

Beyond Next Period

- Customer life time value
  - How likely will they stay or for how long?
  - How much they will buy?
- So that we can make informed decision about
  - Is a customer worth acquiring at this cost?
  - Is a customer worth retaining at this cost?
- Broad Framework

CLV Preliminaries

- What is the value of a particular customer?
- Should I get a customer who requires $xx to acquire?
- Who are my most profitable customers?
- Should I invest (promotions etc) on a particular customer?
- Is there a way to answer the above question? Yes.

CLV Definition

- "*The net present value of the profits linked to a specific customer once the customer has been acquired, after subtracting incremental costs associated with marketing, selling, production and servicing over the customer's lifetime*"
- Observe, that firm needs to (1) forecast future sales of a customer (2) compute incremental costs per customer; and (3) determine the relevant discount rate to use in the present value calculation.

Life Time Value (LTV)

Year	Survival rate[a]	Expected profit	Discount multiplier[b]	Net discounted profit
1	1.000	$80	1.000	$80
2	0.800	$64	0.870	$56
3	0.640	$51	0.756	$39
4	0.512	$41	0.658	$27
5	0.410	$33	0.572	$19
6	0.328	$26	0.497	$13
7	0.262	$21	0.432	$9
8	0.210	$17	0.376	$6
9	0.168	$13	0.327	$4
10	0.134	$11	0.284	$3

LTV = Total net discounted profit = $256

The survival rate is the probability the customer is still a customer in a given year

Primary Models of LTV (CLV)

- Retention Model
  - Assumes if the customer has left the firm, it is not going to come back.
  - Common in industries such as financial services, B2B businesses, magazine subscriptions and pharmaceutical drugs
- Migration Model
  - Assumes customers might migrate in and out of being a customer during the normal course of their lifetimes
  - Common in industries such as retailing, catalogues and CPG.

Customer (Cohort) Life Time Value

- Subscription Renewal (insurance, magazine)
  - t1 =1000 customers acquired …
  - t2=631
  - t3=468
  - t4=382
  - t5=326
- Suppose each customer generates $100
- Assuming 10% discount rate

$$\$100 + \$100 \times \frac{0.631}{1.1} + \$100 \times \frac{0.468}{(1.1)^2} + \$100 \times \frac{0.382}{(1.1)^3} + \$100 \times \frac{0.326}{(1.1)^4}$$

CLV = Discount Sum (Probability of Survival * Value of Purchase)

Simple Retention Model

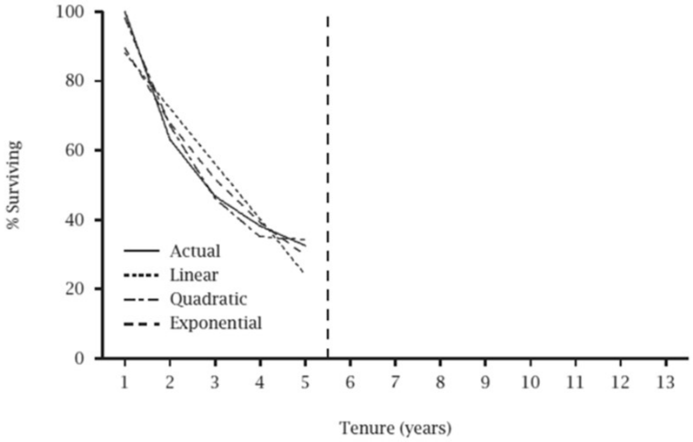
- Assumes if the customer has left the firm it is not going to come back.
- In example we saw retention rate.
- Retention rate is the probability that the customer stays with the firm given that the customer has not left the firm yet.
- How can you calculate retention rate?
  - Simplest way is to look at the past data. Using its customer base for a given year, the firm can determine what percentage of these customers remained with the company next year.

Simple Retention Model

- Problem with this simple approach is that
  1. it assumes retention rate to be same for all the years in future.
  2. also, we cannot calculate customer level retention rates.

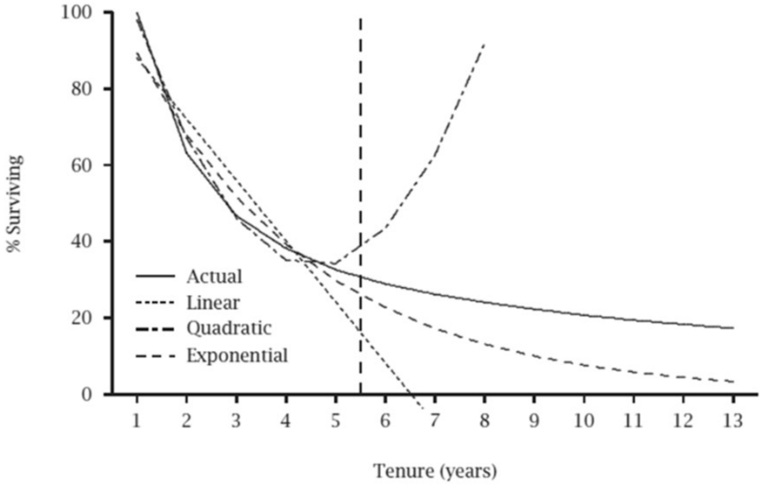
- Therefore, we need a better way to calculate retention rate.

Survival Rate



- Linear, quadratic, exponential curve fitting

Survival Curve Projection



Modeling Survival

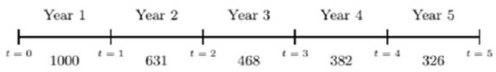
- End of each contract customer makes renewal decision
  - ☐ Attrition probability = θ
  - ☐ Renewal probability = 1- θ
- Churn probability in period T

$$P(T = t \mid \theta) = \theta(1-\theta)^{t-1}, \quad t = 1, 2, 3, \ldots$$

- Survival probability over time

$$\begin{aligned} S(t \mid \theta) &= P(T > t \mid \theta) \\ &= (1-\theta)^t, \quad t = 0, 1, 2, 3, \ldots \end{aligned}$$

Geometric Survival Model



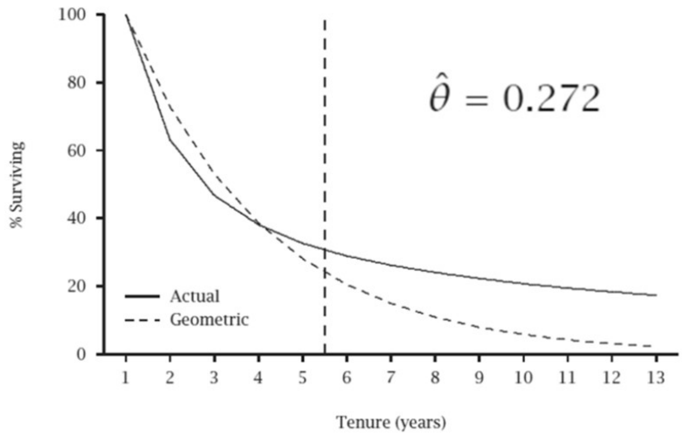
- Probability of observed renewal pattern

$$\begin{aligned}&[P(T=1\mid\theta)]^{369}[P(T=2\mid\theta)]^{163}[P(T=3\mid\theta)]^{86}\\&\qquad\times[P(T=4\mid\theta)]^{56}[S(t\mid\theta)]^{326}\\&=[\theta]^{369}[\theta(1-\theta)]^{163}[\theta(1-\theta)^2]^{86}\\&\qquad\times[\theta(1-\theta)^3]^{56}[(1-\theta)^4]^{326}\end{aligned}$$

- Maximum likelihood Estimation

(simply maximizes the log of probability by optimizing, θ)

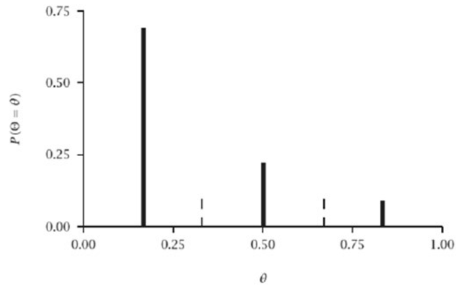
How can we improve model fit?



R library(survival) will help you estimate more advance models
e.g. non parametric Kapla-Meier or Cox's PH Model to estimate effects of covariates

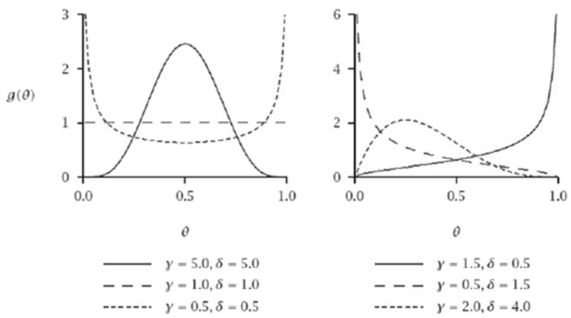
Survival Heterogeneity $\hat{\theta} = 0.272$

- Suppose there are three different types of customers



What if the types of customers are not identified in the data?

Heterogeneity in Churn Probability



BG: Beta-Geometric Model

$$
\begin{aligned}
P(T=t \mid \gamma,\delta) &= \int_0^1 P(T=t \mid \theta)\, g(\theta \mid \gamma,\delta)\, d\theta \\
&= \int_0^1 \theta(1-\theta)^{t-1} \frac{\theta^{\gamma-1}(1-\theta)^{\delta-1}}{B(\gamma,\delta)}\, d\theta \\
&= \frac{1}{B(\gamma,\delta)} \int_0^1 \theta^{\gamma}(1-\theta)^{\delta+t-2}\, d\theta \\
&= \frac{B(\gamma+1,\delta+t-1)}{B(\gamma,\delta)}.
\end{aligned}
$$

every customer has its unique attrition probability, θ

$$B(\gamma,\delta) = \frac{\Gamma(\gamma)\Gamma(\delta)}{\Gamma(\gamma+\delta)}$$

$$\Gamma(n) = (n-1)!$$

$$
\begin{aligned}
S(t \mid \gamma,\delta) &= \int_0^1 S(t \mid \theta)\, g(\theta \mid \gamma,\delta)\, d\theta \\
&= \int_0^1 (1-\theta)^{t} \frac{\theta^{\gamma-1}(1-\theta)^{\delta-1}}{B(\gamma,\delta)}\, d\theta \\
&= \frac{1}{B(\gamma,\delta)} \int_0^1 \theta^{\gamma-1}(1-\theta)^{\delta+t-1}\, d\theta \\
&= \frac{B(\gamma,\delta+t)}{B(\gamma,\delta)}.
\end{aligned}
$$

Churn Probabilities

$$
\begin{aligned}
&[P(T=1 \mid \theta)]^{369}[P(T=2 \mid \theta)]^{163}[P(T=3 \mid \theta)]^{86} \\
&\quad \times [P(T=4 \mid \theta)]^{56}[S(t \mid \theta)]^{326} \\
&= [\theta]^{369}[\theta(1-\theta)]^{163}[\theta(1-\theta)^2]^{86} \\
&\quad \times [\theta(1-\theta)^3]^{56}[(1-\theta)^4]^{326}
\end{aligned}
$$

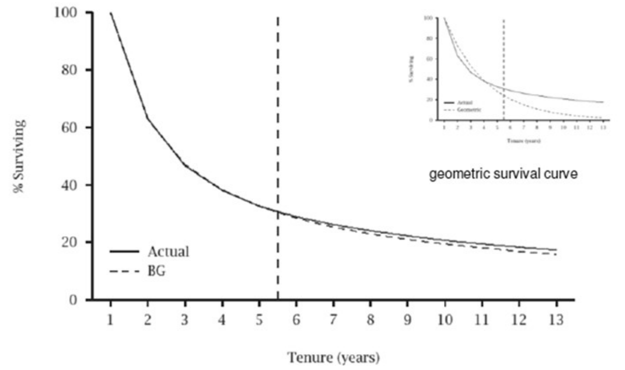
- Churn function

$$
P(T=t \mid \gamma,\delta) = \begin{cases} \dfrac{\gamma}{\gamma+\delta} & t=1 \\[2ex] \dfrac{\delta+t-2}{\gamma+\delta+t-1} \times P(T=t-1) & t=2,3,\ldots \end{cases}
$$

- Survival function

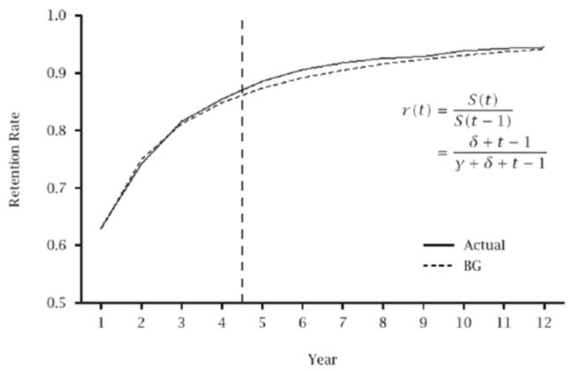
$$
S(t \mid \gamma,\delta) = \begin{cases} 1 & \text{if } t=0 \\ \dfrac{\delta+t-1}{\gamma+\delta+t-1} \times S(t-1) & \text{if } t=1,2,3,\ldots \end{cases}
$$

Beta-Geometric Survival Function



easy to estimate, can be done even in excel by minimizing predicted vs actual retention rates by choosing appropriate parameter values, γ, δ

Projected Retention Rate



easy to estimate, can be done even in excel by minimizing predicted vs actual retention rates by choosing appropriate parameter values, γ, δ

Expected Customer Life Time Value

- Total value of customer cohort

$$E(CLV) = \sum_{t=0}^{\infty} \frac{v(t)\, S(t)}{(1+d)^t}$$

Monetary value

Survival probability

- Residual value of customer cohort

$$= \sum_{t=n}^{\infty} \frac{v(t)\, S(t \mid T > n-1)}{(1+d)^{t-n}}$$

$$S(t \mid T > n-1) = S(t)/S(n-1)$$

Frequency Modeling

Purchase Frequency (aggregate)

# Bottles	0	1	2	3	4	5	6	7	8+
Frequency	400	60	30	20	8	8	9	6	27

- How many transactions
- How many visits
- How many actions

Purchase Frequency (aggregate)

# Bottles	0	1	2	3	4	5	6	7	8+
Frequency	400	60	30	20	8	8	9	6	27

- How many transactions
- How many visits
- How many actions

Purchase Frequency Model

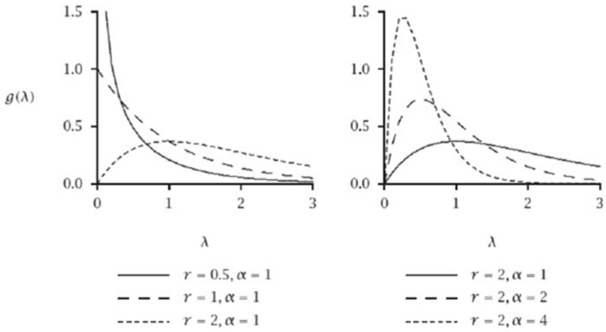
- Count Data
- Number of people purchasing, X = 0,1,2,3…
- Assume X is Poisson distributed with mean λ

$$P(X = x \mid \lambda) = \frac{\lambda^x e^{-\lambda}}{x!}$$

- Mean purchases λ differ across individuals follow gamma distribution

$$g(\lambda \mid r, \alpha) = \frac{\alpha^r \lambda^{r-1} e^{-\alpha\lambda}}{\Gamma(r)}$$

Gamma Distribution



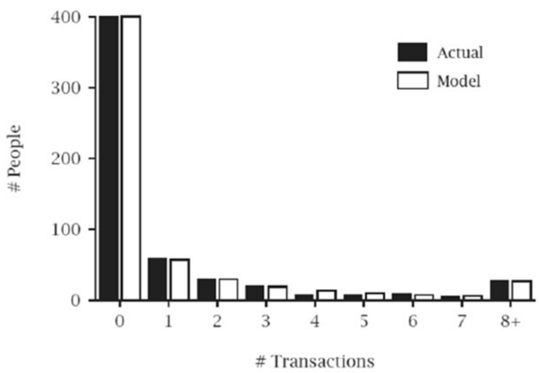
NBD Model: Gamma-Poisson

- Gamma mixture of poissons

$$\begin{aligned} P(X = x \mid r, \alpha) &= \int_0^\infty P(X = x|\lambda)\, g(\lambda \mid r, \alpha)\, d\lambda \\ &= \frac{\Gamma(r+x)}{\Gamma(r)x!} \left(\frac{\alpha}{\alpha+1}\right)^r \left(\frac{1}{\alpha+1}\right)^x \end{aligned}$$

$$P(X = x \mid r, \alpha) = \begin{cases} \left(\dfrac{\alpha}{\alpha+1}\right)^r & x = 0 \\ \dfrac{r+x-1}{x(\alpha+1)} \times P(X = x-1) & x \ge 1 \end{cases}$$

# Bottles	0	1	2	3	4	5	6	7	8+
Frequency	400	60	30	20	8	8	9	6	27



Purchase Incidences / Visits to store

- So far we assumed contractual setting
  - Subscription paid per period
  - We observe when customer leaves or stop paying
- Non contractual setting
  - We do not observe customer leaving
  - BG/BB and Pareto/NBD

CLV = Discount Sum (Probability of Survival * Probability of Purchase *Value of Purchase)

Customer Level Data
(Non-Contractual Settings)

ID	1995	1996	1997	1998	1999	2000	2001	2002	2003	2004	2005	2006
100001	1	0	0	0	0	0	0	?	?	?	?	?
100002	1	0	0	0	0	0	0	?	?	?	?	?
100003	1	0	0	0	0	0	0	?	?	?	?	?
100004	1	0	1	0	1	1	1	?	?	?	?	?
100005	1	0	1	1	1	0	1	?	?	?	?	?
100006	1	1	1	1	0	1	0	?	?	?	?	?
100007	1	1	0	1	0	1	0	?	?	?	?	?
100008	1	1	1	1	1	1	1	?	?	?	?	?
100009	1	1	1	1	1	1	0	?	?	?	?	?
100010	1	0	0	0	0	0	0	?	?	?	?	?
⋮			⋮			⋮			⋮			⋮
111102	1	1	1	1	1	1	1	?	?	?	?	?
111103	1	0	1	1	0	1	1	?	?	?	?	?
111104	1	0	0	0	0	0	0	?	?	?	?	?

Probability of Observed Pattern
(Individual Customer Level Data)

$$
\begin{aligned}
f(100100 \mid p, \theta) &= p(1-p)(1-p)p \underbrace{(1-\theta)^4\theta}_{P(\mathrm{AAAADD})} \\
&+ p(1-p)(1-p)p(1-p) \underbrace{(1-\theta)^5\theta}_{P(\mathrm{AAAAAD})} \\
&+ \underbrace{p(1-p)(1-p)p}_{P(Y_1=1,Y_2=0,Y_3=0,Y_4=1)}(1-p)(1-p) \underbrace{(1-\theta)^6}_{P(\mathrm{AAAAAA})}
\end{aligned}
$$

Probability of Survival * Probability of Purchase

Model Development (homogeneous)

$$L(p,\theta \mid x,t_x,n) = p^x(1-p)^{n-x}(1-\theta)^n + \sum_{i=0}^{n-t_x-1} p^x(1-p)^{t_x-x+i}\theta(1-\theta)^{t_x+i}$$

x is number of transactions
n is number of time periods
tx is last purchase period
p purchase probability
θ is death probability

BG/BB Model (heterogeneous)

$$L(p,\theta \mid x,t_x,n) = p^x(1-p)^{n-x}(1-\theta)^n + \sum_{i=0}^{n-t_x-1} p^x(1-p)^{t_x-x+i}\theta(1-\theta)^{t_x+i}$$

$$g(p \mid \alpha,\beta) = \frac{p^{\alpha-1}(1-p)^{\beta-1}}{B(\alpha,\beta)}$$

$$g(\theta \mid \gamma,\delta) = \frac{\theta^{\gamma-1}(1-\theta)^{\delta-1}}{B(\gamma,\delta)}$$

Beta-geometric * Beta-binomial

$$\begin{aligned}
L(\alpha,\beta,\gamma,\delta \mid x,t_x,n) &= \int_0^1\int_0^1 L(p,\theta \mid x,t_x,n)g(p \mid \alpha,\beta)g(\theta \mid \gamma,\delta)\,dp\,d\theta \\
&= \frac{B(\alpha+x,\beta+n-x)}{B(\alpha,\beta)}\frac{B(\gamma,\delta+n)}{B(\gamma,\delta)} \\
&\quad + \sum_{i=0}^{n-t_x-1}\frac{B(\alpha+x,\beta+t_x-x+i)}{B(\alpha,\beta)}\frac{B(\gamma+1,\delta+t_x+i)}{B(\gamma,\delta)}
\end{aligned}$$

Pareto/NBD: Non-Contractual Setting
(recency/frequency, continuous time model)

$$L(r, \alpha, s, \beta \mid x, t_x, T) = \frac{\Gamma(r+x)\alpha^r \beta^s}{\Gamma(r)} \left\{ \left(\frac{s}{r+s+x}\right) \frac{{}_2F_1(r+s+x, s+1; r+s+x+1; \frac{\alpha-\beta}{\alpha+t_x})}{(\alpha+t_x)^{r+s+x}} + \left(\frac{r+x}{r+s+x}\right) \frac{{}_2F_1(r+s+x, s; r+s+x+1; \frac{\alpha-\beta}{\alpha+T})}{(\alpha+T)^{r+s+x}} \right\}, \text{ if } \alpha \geq \beta$$

$$L(r, \alpha, s, \beta \mid x, t_x, T) = \frac{\Gamma(r+x)\alpha^r \beta^s}{\Gamma(r)} \left\{ \left(\frac{s}{r+s+x}\right) \frac{{}_2F_1(r+s+x, r+x; r+s+x+1; \frac{\beta-\alpha}{\beta+t_x})}{(\beta+t_x)^{r+s+x}} + \left(\frac{r+x}{r+s+x}\right) \frac{{}_2F_1(r+s+x, r+x+1; r+s+x+1; \frac{\beta-\alpha}{\beta+T})}{(\beta+T)^{r+s+x}} \right\}, \text{ if } \alpha \leq \beta$$

BTYD and BTYDplus Package in R

- Buy Til You Die

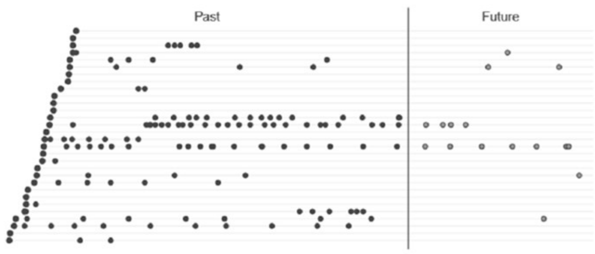


Figure 1: Timing patterns for sampled grocery customers

Broad Framework CLV Models

- Recency Model (contractual, aggregate retention, single purchase)
  - **Geometric,** exponential (homogeneous customers)
  - **Beta-Geometric,** pareto (heterogeneous customers)
- Purchase Frequency (aggregate, non-contractual, multiple purchases)
  - **Poisson,** binomial (homogeneous customers)
  - **NBD,** beta-binomial (heterogeneous customers)
- Frequency and Recency (individual, non-contractual, single purchase)
  - **BG/BB**, Pareto/NBD and BG/NBD (heterogenous customers)