SA1 Tutorial II

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Agenda

Review of concepts and practice on:

- Hypothesis Testing (Identifying type of test and specifying right alternate hypotheses)
- P- Value (What does it tell you and how to calculate the right value)
- ANOVA
- Pairwise testing
- t, Chi-Squared and F statistics

Practice Problem-1 (Conceptual)

For each of the following statements, indicate whether it is True/False. If false, explain why.

The average purchase amount at a retailer's online site is \$80. The retailer is evaluating a new design for its website that, it hopes, would encourage shoppers to spend more. Let μ represent the average amount spent per customer at its redesigned website.

- The appropriate null hypothesis for testing the profitability of the new design is $H_0 : \mu \leq 80$.
- If the p-value of the test is less than α, then we will always commit a Type II error.
- If the p-value of the test is less than α, then we will always commit a Type I error.

Practice Problem-1: Solution

- True. $H_0 : \mu \leq 80$, $H_A : \mu > 80$. If the test rejects H_0, then the data would have provided enough evidence in favor of the new design in terms of higher spending amounts than the current design.
- False. The test rejects H_0. This cannot be a Type II error since a Type II error results from failing to reject.
- False. The test rejects H_0. If null hypothesis is false then we would not have committed an error. However, if null hypothesis is true then we would have committed a Type I error (the probability of which is the p-value).

Practice Problem-2 (Conceptual, Easy)

It is found that Web surfers will lose interest in a Web page if downloading takes more than 12 seconds at 28K baud rate. If you wish to test the effectiveness of a newly designed Web page in regard to its download time, how will you set up the null and alternative hypotheses?

Practice Problem-2: Solution

$$H_0 : \mu \leq 12$$

$$vs.$$

$$H_A : \mu > 12$$

Practice Problem-3 (Challenge)

Banks frequently compete by adding special services that distinguish them from rivals. These services can be expensive to provide. A bank hopes to retain customers that keep high balances in accounts with low interest rates. Typical customers at this bank keep an average balance of $\$3,500$ in savings accounts that pay 2% interest annually. The bank loans this money to other customers at an average rate of 6%, thereby earning a 4% profit on these balances. A random sample of 65 customers was offered a special "personalized" account. After 3 months, the average balance in the 2% savings accounts for these customers was $\$5,000$ ($s = \$3,000$). The personalized service costs the bank \$50 extra per customer per year over the costs of a normal savings account. Is this personalized account offering going to be more profitable than the normal savings account?

- State the null and alternative hypotheses. Describe the parameters.
- Find the p-value of the test. Do the data reject the null hypothesis at $\alpha = 0.05$. (Assume that the data meet the sample size condition.)

Practice Problem-3: Solution

- Suppose μ denotes the average increase in interest profit on a savings account when offered this personalized service (4% of the average balance in the account). The cost of this service is \$50. So, $H_0 : \mu \leq \$50$ (personalized service is not profitable) and $H_A : \mu > 50$ (personalized service is profitable).
- The average increase in the balances is \$1500, earning an additional $0.04 \times \$1500 = \$60$ profit. The SD of this gain is $0.04 \times \$3000 = \$120$. The test statistic is $t = \frac{60-50}{120/\sqrt{65}} = 0.6719$ with 64 degrees of freedom. The p-value is larger than 0.05. (Will use R to show how to calculate p-values). Hence we do not reject H_0 since the p-value is larger than α. Therefore, although the sample indicates that there might be an improvement in the profitability, there is not enough evidence to indicate that this improvement that the sample shows is a feature of the population and is not due to sampling variation alone.

Practice Problem-3 (Concept, R practice)

A nationwide retailer wants to test whether new product shelf facings are effective in increasing sales volume. New shelf facings for the soft drink Country Time are tested at a random sample of 15 stores throughout the country. Data on total sales of Country Time for each store, for the week before and the week after the new facings are installed, are given below:

- Store : 1 2 3 4 5 6 7 8 9 10 11 12 13 14 15
- Before: 57 61 12 38 12 69 5 39 88 9 92 26 14 70 22
- After : 60 54 20 35 21 70 1 65 79 10 90 32 19 77 29

Using the 0.05 level of significance, do you believe that the new shelf facings increase sales of Country Time?

Practice Problem-3: Solution

- Type of test: paired t-test (samples are dependent)

```
Before <- c(57, 61, 12, 38, 12, 69, 5, 39, 88, 9, 92, 26, 1
After <- c(60, 54, 20, 35, 21, 70, 1, 65, 79, 10, 90, 32, 1
Diff <- Before-After
t.test(Diff)
```

```
    One Sample t-test

data:  Diff
t = -1.4691, df = 14, p-value = 0.1639
alternative hypothesis: true mean is not equal to 0
95 percent confidence interval:
 -7.871876  1.471876
sample estimates:
mean of x
     -3.2
```

Practice Problem-3: Alternative Solution

```
t.test(Before,After, paired = TRUE)
```

```
    Paired t-test

data:  Before and After
t = -1.4691, df = 14, p-value = 0.1639
alternative hypothesis: true difference in means is not equ
95 percent confidence interval:
 -7.871876  1.471876
sample estimates:
mean of the differences
                  -3.2
```

Practice Problem-4 (Concept)

Marcus Robert Real Estate Company wants to test whether the average sale price of residential properties in a certain size range in Bel Air, California, is approximately equal to the average sale price of residential properties of the same size range in Marin County, California. The company gathers data on a random sample of 32 properties in Bel Air and finds $\bar{x} = \$2.5$ million and $s = \$0.41$ million. A random sample of 35 properties in Marin County gives $\bar{x} = \$4.32$ million and $s = \$0.87$ million. Is the average sale price of all properties in both locations approximately equal or not? Explain.

Practice Problem-4: Solution

- Two diffferent populations, independent
- two-sample t test
- t = - 11.101
- reject the null

Practice Problem-5 (Concept, R practice)

In the theory of finance, a market for any asset or commodity is said to be efficient if items of identical quality and other attributes (such as risk, in the case of stocks) are sold at the same price. A Geneva-based oil industry analyst wants to test the hypothesis that the spot market for crude oil is efficient. The analyst chooses the Rotterdam oil market, and he selects Arabian Light as the type of oil to be studied.(Differences in location may cause price differences because of transportation costs, and differences in the type of oil—hence, in the quality of oil—also affect the price. Therefore, both the type and the location must be fixed.) A random sample of eight observations from each of four sources of the spot price of a barrel of oil during February 2007 is collected. Data, in U.S. dollars per barrel, are as follows. Perform an Anova and comment on the result.

Practice Problem-5: Data

U.K.	Mexico	U.A.E.	Oman
$62.10	$56.30	$55.60	$53.11
63.20	59.45	54.22	52.90
55.80	60.02	53.18	53.75
56.90	60.00	56.12	54.10
61.20	58.75	60.01	59.03
60.18	59.13	53.20	52.35
60.90	53.30	54.00	52.80
61.12	60.17	55.19	54.95

Figure 1: Data

Practice Problem-5: Solution

Null hypothesis:

$H_0 : \mu_1 = \mu_2 = \mu_3$

I.e. the average prices of all the four locations are same.

Alternative hypothesis:

H_A:Not H_0 i.e. not all μ's are equal

Practice Problem-5: Solution

- Need to input data in a particular format to use easily in R

```
   Location Price
1        UK 62.10
2        UK 63.20
3        UK 55.80
4        UK 56.90
5        UK 61.20
6        UK 60.18
7        UK 60.90
8        UK 61.12
9    Mexico 56.30
10   Mexico 59.45
```

Practice Problem-5: Solution

```
fit <- aov(Price ~ factor(Location), data=q3)
summary(fit)
```

```
                  Df Sum Sq Mean Sq F value   Pr(>F)
factor(Location)   3  188.8   62.94   11.55 4.19e-05 ***
Residuals         28  152.6    5.45
---
Signif. codes:  0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 '
```

Practice Problem-3: Solution

- We can see that that the p-value is 34.27e-05 is less than $\alpha = 0.05$.
- Therefore we can reject the null hypothesis at 5% level of significance and conclude that the average of all the four locations differ significantly.
- Now, we must do further tests to determine which of the locations have prices that are different from the others

Practice Problem-5: Solution

We perform the "Tukey's test for pairwise comparison":

```
TukeyHSD(fit)
```

```
  Tukey multiple comparisons of means
    95% family-wise confidence level

Fit: aov(formula = Price ~ factor(Location), data = q3)

$`factor(Location)`
                  diff       lwr          upr     p adj
Oman-Mexico -4.26625 -7.452753 -1.07974693 0.0054664
UAE-Mexico  -3.21250 -6.399003 -0.02599693 0.0476033
UK-Mexico    1.78500 -1.401503  4.97150307 0.4340886
UAE-Oman     1.05375 -2.132753  4.24025307 0.8033837
UK-Oman      6.05125  2.864747  9.23775307 0.0000942
UK-UAE       4.99750  1.810997  8.18400307 0.0010676
```

Practice Problem-5: Solution

Therefore we conclude at 5% level of significance, that:

- U.A.E has significantly lesser average price than U.K.
- OMAN has significantly lesser average price than U.K.
- U.A.E has significantly lesser average price than MEXICO.
- OMAN has significantly lesser average price than MEXICO.