Session 10

Analysis of Variance (ANOVA)

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Learning Objectives

- Why is analysis of variance (ANOVA) required to compare means of populations?
- What is the principal of sum of squares?
- How to conduct the ANOVA test?
- What follow-up analysis should be done if ANOVA test is significant?

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Example: More weight reduction programs

- Suppose the nutrition expert would like to do a comparative evaluation of three diet programs (Atkins, South Beach, GM)
- She randomly assigns equal number of participants to each of these programs from a common pool of volunteers
- Suppose the average weight losses in each of the groups (arms) of the experiments are 4.5 kg, 7 kg, 5.3 kg
- What can she conclude?

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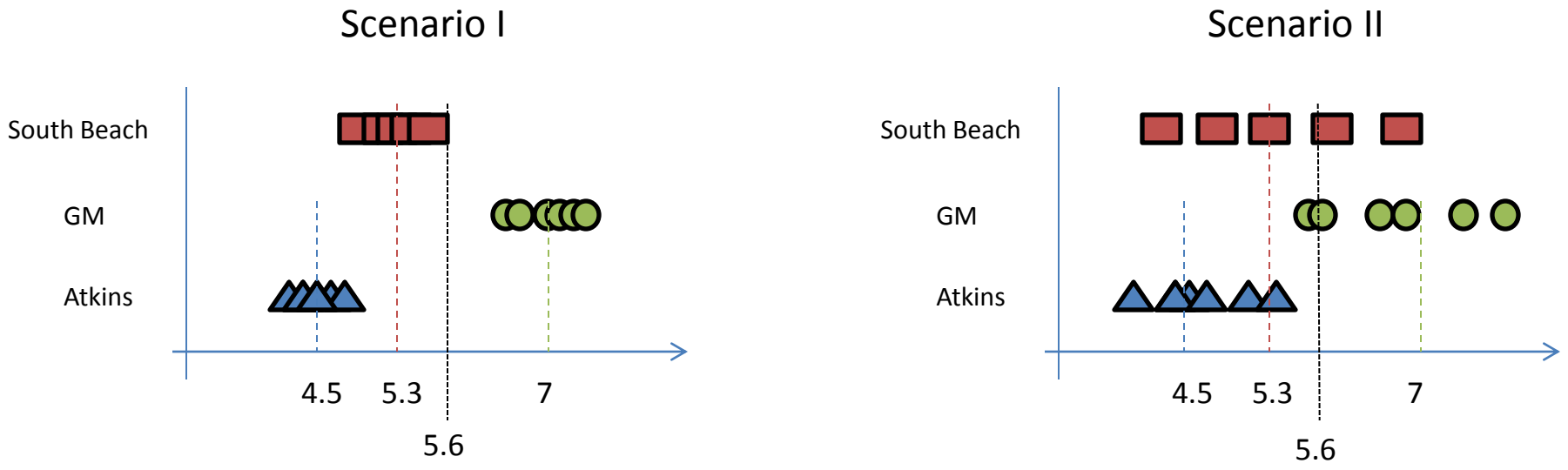
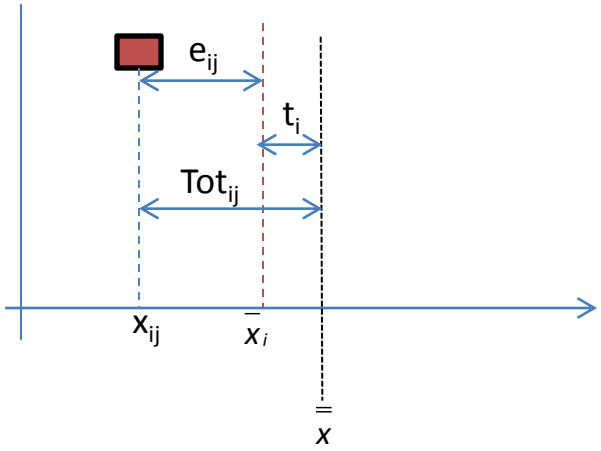
Two kinds of variation matter



- Not every individual in each program will respond identically to the diet program
- Easier to identify variations across programs if variations within programs are smaller
- Hence the method is called Analysis of Variance (ANOVA)

Formalizing the intuition behind variations



- It should be obvious that for every observation: $Tot_{ij} = t_i + e_{ij}$

- What is more surprising and useful is:

$$\sum_{i=1}^{r}\sum_{j=1}^{n_i} Tot_{ij}^2 = \sum_{i=1}^{r} n_i t_i^2 + \sum_{i=1}^{r}\sum_{j=1}^{n_i} e_{ij}^2$$

Sum of Squares Total (SST)	Sum of Squares Treatment (SSTR)	Sum of Squares Error (SSE)

Statistical test for equality of means

- n subjects equally divided into r groups

- Hypotheses
  - H0: $\mu_1 = \mu_2 = \mu_3 = \ldots = \mu_r$
  - Not all μ_i are equal

- Calculate
  - Mean Square Treatment **MSTR** = SSTR / (r-1)
  - Mean Square Error **MSE** = SSE / (n-r)
  - The ratio of two squares **f** = MSTR/MSE
  - Strength of this evidence **p-value** = $\Pr(F_{(r-1,n-r)} \geq f)$

- Reject the null hypothesis if p-value < α

Example: Weight reduction programs

- Suppose 12 participants are allocated to each of the three diet programs
  - r = 3
  - n = 36

- ANOVA Table

	DF	Sum Sq.	Mean Sq.	F value	Pr(>F)
Diet	2	38.89	19.444	3.571	0.0394
Residuals (Error)	33	179.67	5.444		

- Can we conclude that GM diet is more effective than Atkins diet?

Further analysis: Pairwise differences

- To test whether any two means are different
  - Construct the test statistic $$q = \frac{\left|\bar{x}_i - \bar{x}_j\right|}{\sqrt{\frac{MSE}{n}}}$$
  - Calculate the p-value associated with this test statistic: ptukey(q,r,n-r)
  - Reject the null hypothesis that the two means are equal if p-value < α

Summary of Session X

- The extent of variation between and within groups determines the strength of evidence against the null hypothesis that means of all groups are equal

- The sum of squared deviations total (around the grand mean) is equal to the sum of squared deviations errors (around respective group means) plus the sum of squared deviations treatment (group means around grand mean)

- ANOVA test compares mean squared treatment with mean squared errors. If this ratio is "significantly" greater than 1, we can reject the null hypothesis that the means are equal

- We can conduct a series of Tukey tests for pair-wise comparisons of group means

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