Theory Exercise 1

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1 Assignment 1 - Planes and cylinders

The first thing to do is to derive an implicit parametrization for the infinite cylinder. To do so we assume we are given with the center $\boldsymbol{c}$, the radius r and the direction $\boldsymbol{a}$ of the axis of the cylinder. Then we impose that a point $\boldsymbol{x}$ is on the surface of the cylinder if it satisfies

$$||(\boldsymbol{x}-\boldsymbol{c}) - \langle \boldsymbol{x}-\boldsymbol{c}, \boldsymbol{a}\rangle \boldsymbol{a}||^2 = r^2 \tag{1}$$

This means that we are imposing that the distance of the point from the axis is equal to the radius of the cylinder.

Now, we consider a ray $\boldsymbol{x}(t) = \boldsymbol{o} + t\boldsymbol{d}$ and we intersect it with the cylinder obtaining

$$||(\boldsymbol{o}+t\boldsymbol{d}-\boldsymbol{c}) - \langle (\boldsymbol{o}+t\boldsymbol{d}-\boldsymbol{c}), \boldsymbol{a}\rangle \boldsymbol{a}||^2 = r^2 \tag{2}$$

Now we expand the equation:

$$\begin{aligned} 0 &= ||(\boldsymbol{o}+t\boldsymbol{d}-\boldsymbol{c}) - \langle (\boldsymbol{o}+t\boldsymbol{d}-\boldsymbol{c}), \boldsymbol{a}\rangle \boldsymbol{a}||^2 - r^2 \\ &= (||d||^2 - \langle \boldsymbol{a}, \boldsymbol{d}\rangle^2)t^2 + (2\langle \boldsymbol{d}, \boldsymbol{o}-\boldsymbol{c}\rangle - 2\langle \boldsymbol{a}, \boldsymbol{d}\rangle\langle \boldsymbol{a}, \boldsymbol{o}-\boldsymbol{c}\rangle)t + ||\boldsymbol{o}-\boldsymbol{c}||^2 - \langle \boldsymbol{a}, \boldsymbol{o}-\boldsymbol{c}\rangle^2 - r^2 \end{aligned} \tag{3}$$

So, we have a quadratic equation $at^2 + bt + c = 0$ with the following coefficients:

$$a = ||d||^2 - \langle \boldsymbol{a}, \boldsymbol{d}\rangle^2$$
$$b = 2\langle \boldsymbol{d}, \boldsymbol{o}-\boldsymbol{c}\rangle - 2\langle \boldsymbol{a}, \boldsymbol{d}\rangle\langle \boldsymbol{a}, \boldsymbol{o}-\boldsymbol{c}\rangle$$
$$c = ||\boldsymbol{o}-\boldsymbol{c}||^2 - \langle \boldsymbol{a}, \boldsymbol{o}-\boldsymbol{c}\rangle^2 - r^2$$

Regarding the normals, we can compute them in the following way. Assuming we have an intersection point $\overline{\boldsymbol{x}}$ we can compute its projection on the axis of the cylinder as $\boldsymbol{y} = \langle \overline{\boldsymbol{x}} - \boldsymbol{c}, \boldsymbol{a}\rangle \boldsymbol{a}$, so that the normal vector is given by the difference $(\overline{\boldsymbol{x}} - \boldsymbol{c}) - \boldsymbol{y}$ which has only to be normalized.