#1. $G = (D, C, A, X)$ → Properties of the Graph − G

↳ Customer Nodes

↳ Depot Nodes

Decision variables: x_{ij}^{v}, y_d, z_c^{v}

x_{ij}^{v} → Whether vehicle v traverses (i,j) arc

y_d → Whether depot node d is open (operational)

z_c^{v} → Load of vehicle v on arrival at customer node c.

Deep-dive into constraints for z_c^{v}.

→ if $x_{ck}^{v} = 1$ i.e. if vehicle v visits node k from customer node c, then,

z_k^{v} will reduce in comparison to z_c^{v} by at least as much as the demand at ~~q~~ customer node c $\Rightarrow q_c$.

$$\therefore \quad z_k^{v} \leq z_c^{v} - q_c$$

→ if $x_{ck}^{v} = 0$ i.e. if vehicle v does not visit node k from customer node c, then,

z_k^{v} can take any value. There is no relation between z_k^{v} and z_c^{v}.

$$\therefore \quad z_k^{v} \in \mathbb{R}_+$$

We re-write this as,

$$z_k^{v} \leq M$$

where M is large enough.

Combining the two if conditions using x^v_{ck}

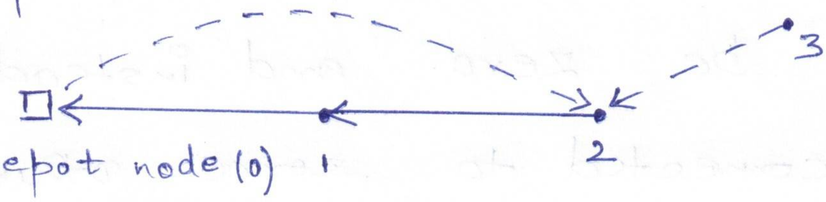
$$z^v_k \leq \underbrace{(x^v_{ck})(z^v_c - q_c)}_{\text{if } x^v_{ck}=1 \text{ this part of RHS will be active}} + \underbrace{(1-x^v_{ck})(M)}_{\text{if } x^v_{ck}=0 \text{ then this section of RHS will be active.}}$$

In addition to this constraint, we have

$z^v_c \leq q_v \longrightarrow$ Vehicle Capacity Constraint

$z^v_d = 0 \longrightarrow$ Boundary Condition.

Examining constraints on z_c^v through a small example,



$$\Rightarrow \boxed{z_0^v = 0} \quad \text{(Boundary Condition)}$$

also, for connection between 1 and 0.

- $z_0^v \leq x_{10}^v (z_1^v - q_1) + (1 - x_{10}^v) M$

Since $x_{10}^v = 1$

$$z_0^v \leq z_1^v - q_1$$

$$\Rightarrow z_1^v \geq \underbrace{\cancel{z_0^v}}_{0} + q_1 \Rightarrow \boxed{z_1^v \geq q_1}$$

also, for connection between 2 and 1.

- $z_1^v \leq x_{21}^v (z_2^v - q_2) + (1 - x_{21}^v) M$

Since $x_{21}^v = 1$

$$\underbrace{\cancel{z_1^v}}_{q_1} \leq z_2^v - q_2$$

$$\Rightarrow \boxed{z_2^v \geq q_1 + q_2}$$

Likewise, we can write,

$$z_3^v \geq q_1 + q_2 + q_3$$

if $x_{32}^v = 1$.

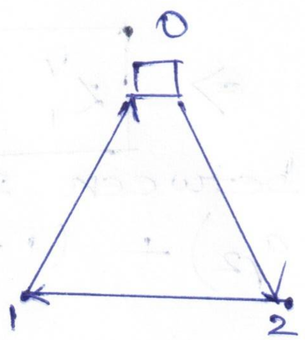
Let's now assume $q_1 + q_2 + q_3 > q_v$

vehicle capacity

#4. Because capacity constraint is violated connecting $3 \longrightarrow 2 \longrightarrow 1 \longrightarrow 0$,

x^v_{32} has to be zero and instead, node 2 should be connected to some other node

If $T(2) = \{0, 3\}$ then the only option left is to connect node 2 back to the depot node (0). Consequently, the tour should be,



Hence, the load balance, vehicle capacity, boundary condition and customer service constraints together enforce that a delivery tour starts and ends at the depot node.