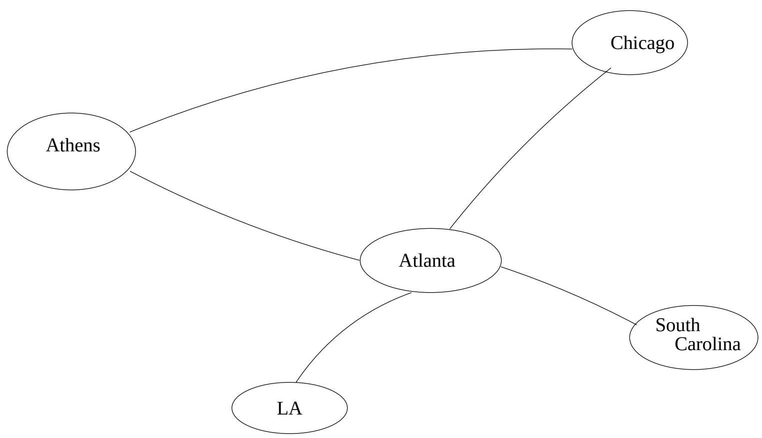
2 GRAPHS

Consists of a set of nodes or point some of which are connected by edges or lines.



A (hypothetical) graph of nonstop airline flights.

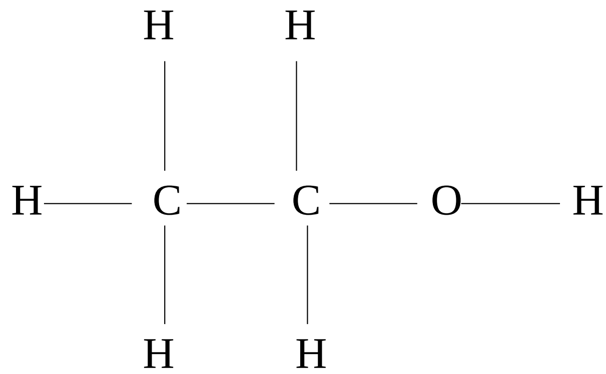
Molecule alcohol: CH_3CH_2OH

Def. A graph $G = (V, E)$ which V is the set of nodes,

$$V = \{v_1, v_2, \dots, v_n\}$$

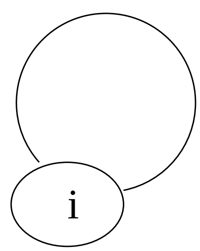
and E is the set of edges,

$$E = \{\{v_i, v_j\} | v_i \in V, v_j \in V\}$$



note:.

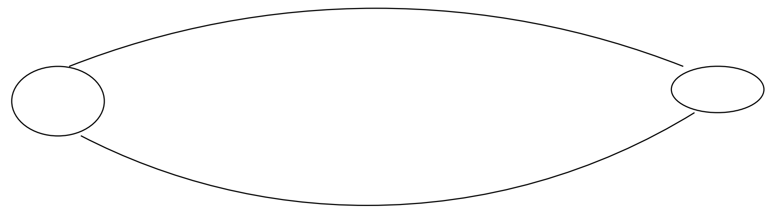
1. each edge is a set of pair of vertexes. No order implied.
2. $v\{v_i, v_j\}$, i could be equal to j.
   self-loop



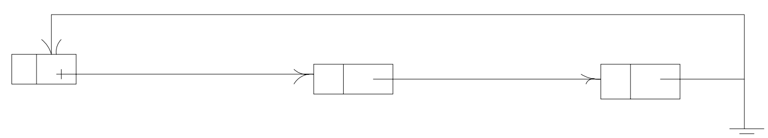
self loop

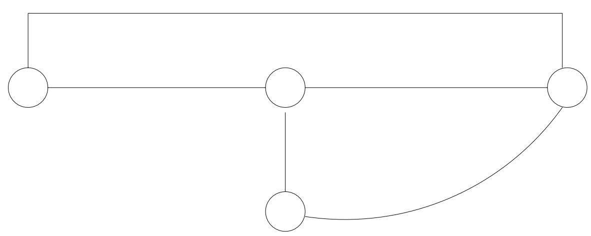
3. One also allows multiple edge between two node in a general graph

4. Cardinality of V, $|V| = n$, number of nodes, $|E| = m$, number of edges.



3 DIGRAPH





eg. A flow chart of a program

def. A graph whose edges are directed is a digraph, directed graph.
def. A digraph $G_2(V, E)$ when V is the set of vertex $V = \{v_1, v_2, \ldots, v_n\}$ and E is the set of directed edges on every $E = \{(v_i, v_j) such that v_i, v_j \in V\}$

Note

For the purpose of this chapter, we assume that V, the set of nodes, is nonempty & finite and that there no self loops or multiple edges in a graph or digraph.
Question

1. which route is cheapest?
2. which route is fastest?
3. if a node or a computer goes down in a computer network, does it get disconected?
4. is there a loop in a flowchart?

3.1 Subgraph

$V' \subseteq V, E' \subseteq E$
Induced Subgraph by V'
$G' = (V', E'), E' = \{u, v | u, v \in V'\}$

Complete graph
$E = \{\{v_i, v_j | 1 \leq j \leq n\}$ eg. if (v, w) is an edge then v & w are adjacent to each other and v and w are said to be incident with the edge (v, w).

def A path from v to w is a sequencs of vertex $v_0, V_1, v_2, \ldots,$ such that $v_0 = v, v_k = w\&$ $\{V_i, v_j\} \in E$.
(Books defenition is not good)
If $v_0 = v, v_k = w\&$ all $v_0, v_1, \ldots, v_k$ are distinct, then the length of the path is k. v_0 is a path of length 0.

Connected Graph:

Cycle is path $v_0 v_1 v_2 \ldots v_k$ such that $v_0 = v_k$
A graph without any cycle is called acylic.

Tree: Acyclic connected graph

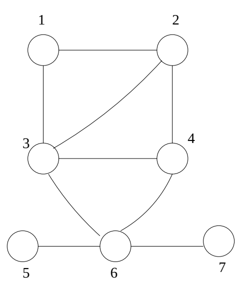
Rooted tree has a designated root vertex establishing parent & child relationship
number of edge in a tree is $n - 1$
Could be proved by induction
A connected component of a graph G is a maximal connected subgraph of G

Weighted Graph
$G = (V, E, w)$
$W : E \leftarrow 2+$
$w(e)$ weight of e (Capacity or distance)

3.2 Representation of a graph

3.2.1 Adjacency Matrix

$A = (a_{ij})_{n*n}$
$a_{ij} = 1$ if $v_i, v_j \in E$
0 else
for $1 \leq i, j \leq n$

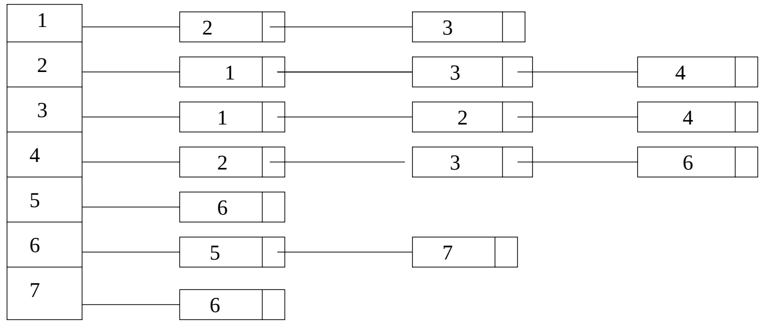


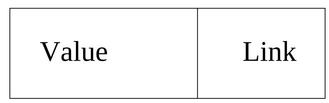
$$
\begin{array}{c c}
 & \begin{array}{ccccccc} 1 & 2 & 3 & 4 & 5 & 6 & 7 \end{array} \\
\begin{array}{c} 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \end{array} &
\left[\begin{array}{ccccccc}
0 & 1 & 0 & 1 & 0 & 0 & 0 \\
1 & 0 & 1 & 1 & 0 & 0 & 0 \\
1 & 1 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 \\
0 & 0 & 1 & 1 & 0 & 1 & 1 \\
0 & 0 & 0 & 0 & 0 & 1 & 0
\end{array}\right]
\end{array}
$$

Space usage $\Leftarrow O(n^2)$

3.2.2 Adjacency list

On any $A[1 \ldots n]$ of linked list, $A[j]$ is the linked list of all the vertices adjacent to v_j.



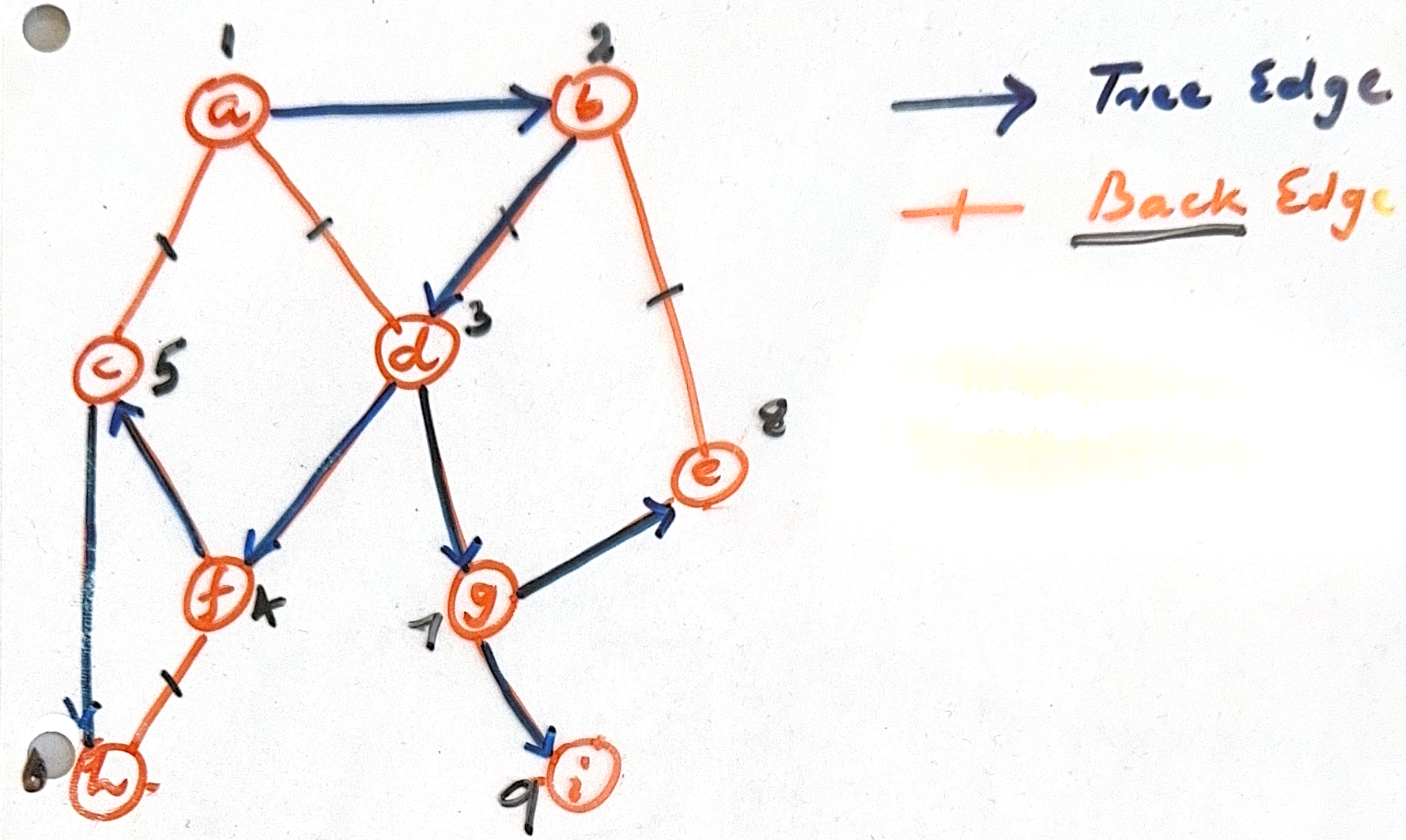


Space usage $\Leftarrow O(n) + O(m) = O(m+n)$
$= O(n^2)$ if m is $O(n^2)$

3.3 Weighted Graph Represatation

Adjacency Matrix $a_{ij} = W(v_i, v_j)$ if $\{v_i v_j\} \in E$
$= 0$ otherwise
Adjacent link

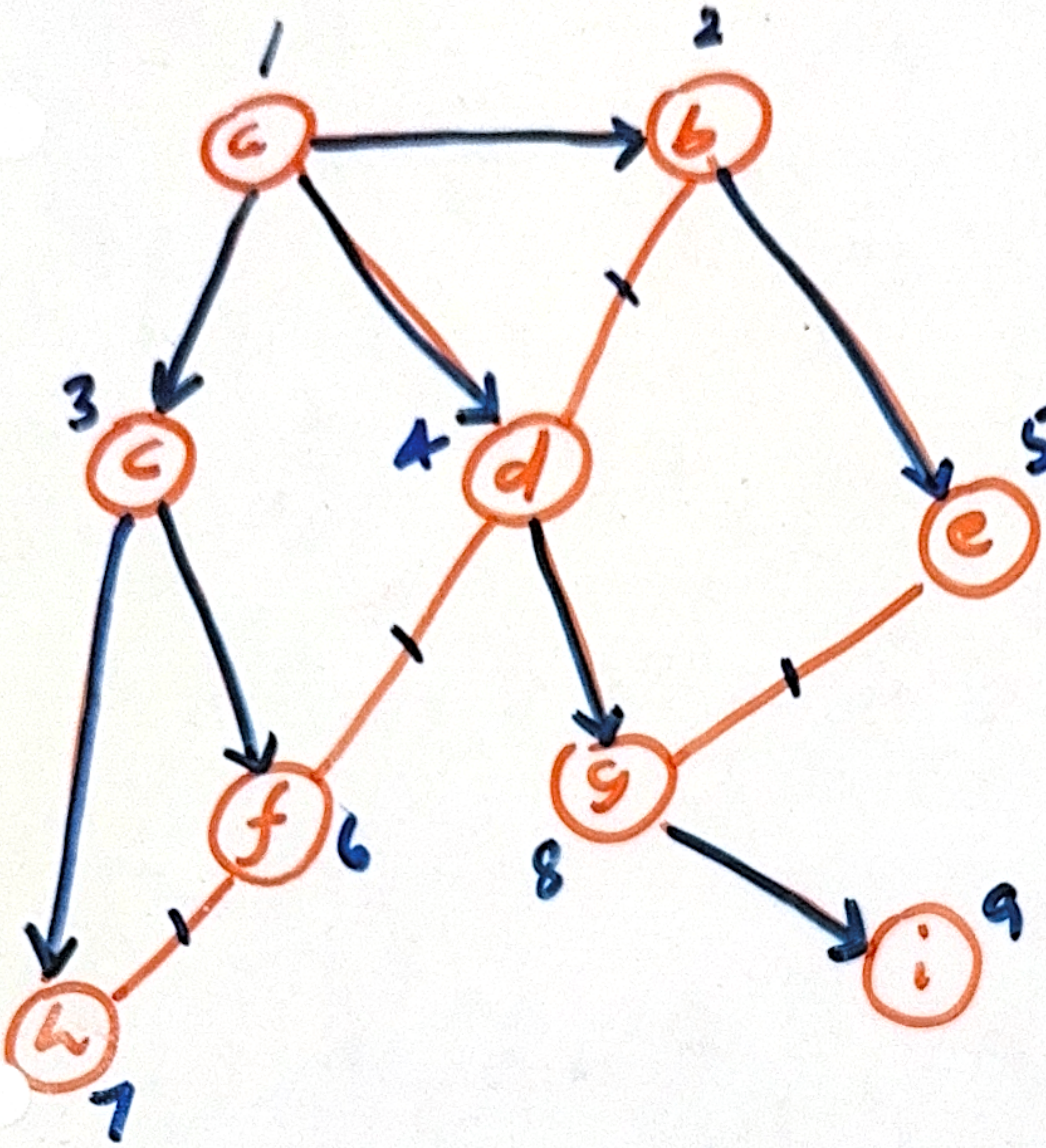
Traversing Graphs



DFS (v) : Depth-First Search

```
mark v
while there is an unmarked node
w adjacent to node v
        DFS(w)
end
```

[Connected Components]



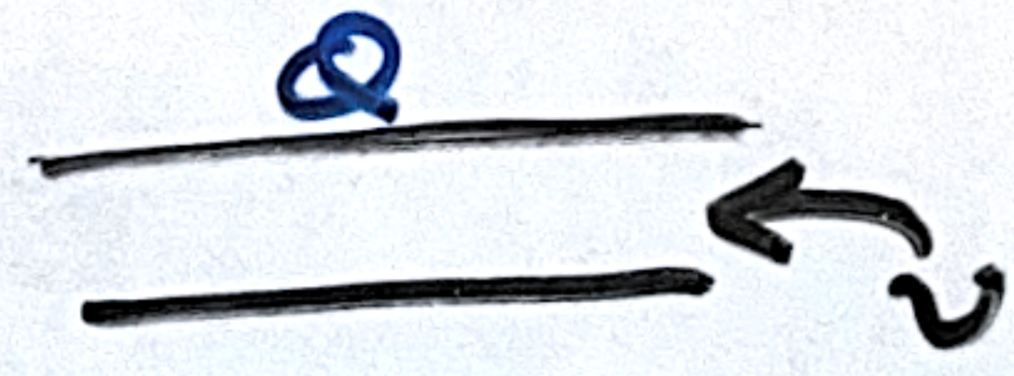
$O(m+n)$

cross edges

BFS (v) : Breadth-First SEARCH

mark v

Q = Qv



While Q is nonempty do

x = deleteFront (Q)

for each unmarked node w adjacent to x do

mark w

Q = Qw

end

4 TRAVERSING GRAPHS

DFS(v): Depth-First Search
mark v
while there is an unmarked node w adjacent to node v DFS(w)
end

connected components
$O(m+n)$

BFS(v): Breadth-First Search
mark v
Q=Qv
while Q is nonempty do

$x =$ delete Front(Q)

for each unmarked node w adjacent to x do mark w
    Q=Qw