## CS 5633: Analysis of Algorithms

## Homework 3

- 1. For this problem, use of the proper notation is worth half the points. Clearly define your random variables.
  - (a) What is the expected value of a single roll of a fair, 6-face die.
  - (b) Use indicator random variables to compute the expected value of the sum of n dice.
- 2. Suppose you have the option of playing a game in which two fair, 6-face dice are rolled. If you roll two 1's, you win \$10. If you roll one 1, you win \$1. If you do not roll a 1, you pay 50 cents. What is the expected value of the game from your perspective?
- 3. When Randomized Quicksort runs, how many calls are made to the random number generator in the
  - (a) Best Case?
  - (b) Worst Case?
  - (c) Average Case?
- 4. Consider the quicksort algorithm on an input which may contain the same value multiple times.
  - (a) How does deterministic quicksort behave on an array with n equal keys? What is its runtime? What is the behavior and the runtime of randomized quicksort in this case? Justify your answer.
  - (b) If you change  $A[j] \leq x$  to A[j] < x in the pseudocode for partition, how does quicksort behave on an array with n equal keys? What is its runtime?
  - (c) How does deterministic quicksort behave on an array with just two distinct keys (the total number of keys is still n)?
  - (d) Give pseudo-code for a 3-way partition that partitions the array into three parts: keys less than the pivot, keys equal to the pivot, keys greater than the pivot. Your code should be in-place (so it should use at most constant extra storage) and it should run in linear time.
  - (e) Consider an implementation of quicksort which uses 3-way partition and only recurses on the portions of the array with keys less than the pivot and with keys greater than the pivot. If the array of n keys contains only 2 different values, what is the worst-case runtime?

