Assignment - 7

Answer to the Question No. – 1

(a) The given example is,

i	1	2	3	4	5	6	7	8	9	10	11
S_i	1	3	0	5	3	5	6	8	8	2	12
F_i	4	5	6	7	9	9	10	11	12	14	16
Active time	3	2	6	2	6	4	4	3	4	12	4

Now, if we choose the activity that will be active the least amount of time, then the solution for this greedy algorithm will be $\{A_2, A_8, A_{11}\}$. But this is not the optimal solution. If we choose the optimal solution, then the result will be $\{A_1, A_4, A_8, A_{11}\}$.

(b) If we sort the activities in decreasing order of start time then it will give another optimal solution.

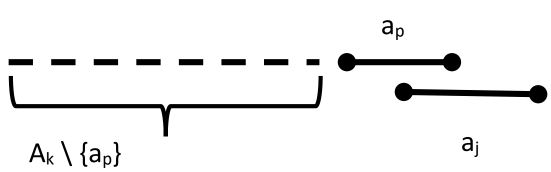
i	1	2	3	4	5	6	7	8	9	10	11
S_i	12	2	8	8	6	5	3	5	0	3	1
F_i	16	14	12	11	10	9	9	7	6	5	4

The optimal solution will be $\{A_1, A_3, A_8, A_{11}\}$.

```
OptimalGreedySolution (S, f){
        n = length(S);
        A = {a_i};
        k = 1;
        for j = 2 to n {
                if f(j) <= S(k) { A = A U {a_j}; k = j; }
        }
        return A;
}
```

Theorem: Consider any non-empty sub-problem S_k and let a_j denote the activity in S_k with the last activity to start. Then a_j is included in some maximum size subset of compatible activities of S_k.
To prove this algorithm is correct we need to prove the above theorem.

Proof: let A_k be an optimal solution for S_k and let a_p be the activity in A_k with last start time. If $a_p = a_j$ we are done.
So, assume $a_p \ != a_j$. Now, consider the set $A_k' = A_k \setminus \{a_p\} \cup \{a_j\}$



Here all the activity in A_k has finish time earlier than the start time of a_p and the start time of a_j is later than the start time of a_p. so, A_k' is feasible and $A_k' = A_k$.
So, A_k' is optimal and the algorithm is correct.

Answer to the Question No. – 2

Initially the number of in-degrees of each node: a = 0, b = 2, c =1, d = 1, e =1, f = 2.

So the valid topological sorts are –

1. a -> b -> c -> d -> e -> f
2. a -> b -> d -> c -> e -> f
3. a -> b -> d -> e -> c -> f
4. a -> d -> e -> b -> c -> f
5. a -> d -> b -> c -> e -> f
6. a -> d -> b -> e -> c -> f

Answer to the Question No. – 3

```
bool dfs(int node, int parent) {
  visited[node] = 1;
  for (int i = 0; i < n; i++) {
    if (graph[node][i]) {
      if (!visited[i]) {
        if (dfs(i, node)) return true;
        }
       else if (i != parent) return true;
  }}
  return false;
}
bool hasCycle() {
  for (int i = 0; i < n; i++) visited[i] = 0;
  for (int i = 0; i < n; i++) {
    if (!visited[i]) {
      if (dfs(i, -1)) return true;
  }}
return false;
}
```

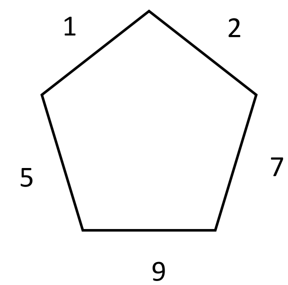
The DFS explores each edge once. If it encounters a visited node that is not the direct parent, a cycle is detected. Here the time complexity is O(n + m) where n is the number of nodes and m is the number of edges.

Answer to the Question No. – 4

Proof: suppose, our connected undirected graph G, with edges of unique weights, has two distinct and acceptable minimum spanning tree (MST) T and T' and let t = W(T) = W(T').

Next consider the overlap of T and T'. In this overlap we will see some cycle c with k edges, where k-1 edges are from T and k-1 edges from T'. Consider the heaviest edge e on this cycle and assume without loss of generality that e is definitely in T. Because all edge weights are unique for any given cycle from our original graph, an MST will not use the heaviest weighted edge in this cycle. Hence e does not belong to any MST, contradicting T is an MST.

Example:



From the above graph we can see that every edge has distinct weight. The MST will have the edges with weight {1, 2, 5, 7}, as 9 is the largest weight on the other edge we cannot take this edge in MST. So there cannot be any other MST with the edge weight 9 and only one unique MST is possible whether we use Prim's or Krushkal's algorithm to find the MST.