Source code

Quicksort:

Using insertion sort for appropriate size small input:

```
def insertion_sort(arr, left=0, right=None):
    if right is None:
        right = len(arr) - 1
    for i in range(left + 1, right + 1):
        key = arr[i]
        j = i - 1
        while j >= left and arr[j] > key:
            arr[j + 1] = arr[j]
            j -= 1
        arr[j + 1] = key
```

Finding the cutoff point between pure insertion sort and pure quicksort:

```
def partition(arr, low, high):
    pivot = arr[high]
    i = low - 1
    for j in range(low, high):
        if arr[j] < pivot:
            i += 1
            arr[i], arr[j] = arr[j], arr[i]
    arr[i + 1], arr[high] = arr[high], arr[i + 1]
    return i + 1

def quicksort(arr, low=0, high=None):
    if high is None:
        high = len(arr) - 1
    if low < high:
        pi = partition(arr, low, high)
        quicksort(arr, low, pi - 1)
        quicksort(arr, pi + 1, high)

# Function to generate random arrays
def generate_random_array(n):
    return [random.randint(0, 20000) for _ in range(n)]

# Define small input sizes to test
```

```
input_sizes = list(range(5, 5000, 5))  # Small inputs from 5 to 5000
insertion_times = []
quicksort_times = []

for size in input_sizes:
    arr1 = generate_random_array(size)
    arr2 = arr1.copy()
    # Measure Insertion Sort Time
    start = time.time()
    insertion_sort(arr1)
    insertion_times.append(time.time() - start)
    # Measure Quicksort Time
    start = time.time()
    quicksort(arr2)
    quicksort_times.append(time.time() - start)

# Find the cutoff point where Insertion Sort becomes faster
cutoff_index = np.argmax(np.array(insertion_times) < np.array(quicksort_times))
best_cutoff = input_sizes[cutoff_index]
print(f"Best Cutoff Point: {best_cutoff}")
```

Implementing randomized pivoting:

```
def randomized_partition(arr, low, high):
    rand_pivot = random.randint(low, high)
    arr[high], arr[rand_pivot] = arr[rand_pivot], arr[high]
    return partition(arr, low, high)

def quicksort_random(arr, low, high, cutoff):
    if high - low + 1 <= cutoff:
        insertion_sort(arr[low:high + 1])
        return
    if low < high:
        pivot = randomized_partition(arr, low, high)
        quicksort_random(arr, low, pivot - 1, cutoff)
        quicksort_random(arr, pivot + 1, high, cutoff)
```

Implementing median-of-three pivoting:

```
def median_of_three_partition(arr, low, high):
    mid = (low + high) // 2
    pivots = [(arr[low], low), (arr[mid], mid), (arr[high], high)]
    pivots.sort()
```

```
    median_index = pivots[1][1]
    arr[high], arr[median_index] = arr[median_index], arr[high]
    return partition(arr, low, high)

def quicksort_median(arr, low, high, cutoff):
    if high - low + 1 <= cutoff:
        insertion_sort(arr[low:high + 1])
        return
    if low < high:
        pivot = median_of_three_partition(arr, low, high)
        quicksort_median(arr, low, pivot - 1, cutoff)
        quicksort_median(arr, pivot + 1, high, cutoff)
```

Radix sort

Using counting sort as the stable sort of subroutine:

```
  def counting_sort(arr, exp):
    n = len(arr)
    output = [0] * n
    count = [0] * 10

    for i in range(n):
        index = (arr[i] // exp) % 10
        count[index] += 1

    for i in range(1, 10):
        count[i] += count[i - 1]

    for i in range(n - 1, -1, -1):
        index = (arr[i] // exp) % 10
        output[count[index] - 1] = arr[i]
        count[index] -= 1

    for i in range(n):
        arr[i] = output[i]
```

Parameterizing the radix sort using the base as an input parameter and finding the best base to use for fastest sort:

```
def radix_sort(arr, base=10):
    max_element = max(arr)
```

```
    exp = 1
    while max_element // exp > 0:
        counting_sort(arr, exp)
        exp *= base

# Function to generate random integers
def generate_random_array(n):
    return [random.randint(0, 20000) for _ in range(n)]

# Experimenting with different bases for Radix Sort
bases = [10, 50, 100, 256, 512, 1024]  # Different radix bases
radix_times = []

for base in bases:
    arr = generate_random_array(10000)  # Fixed input size
    start = time.time()
    radix_sort(arr, base)
    radix_times.append(time.time() - start)

# Identify the best base for Radix Sort
best_radix_base = bases[np.argmin(radix_times)]
```

Report

Experimental Setup

Processor: Intel(R) Core(TM) i5-7200U CPU @ 2.50GHz, 2712 Mhz, 2 Core(s)

Language: Python

Random Number Generator: random.randint() function using random library in Python.

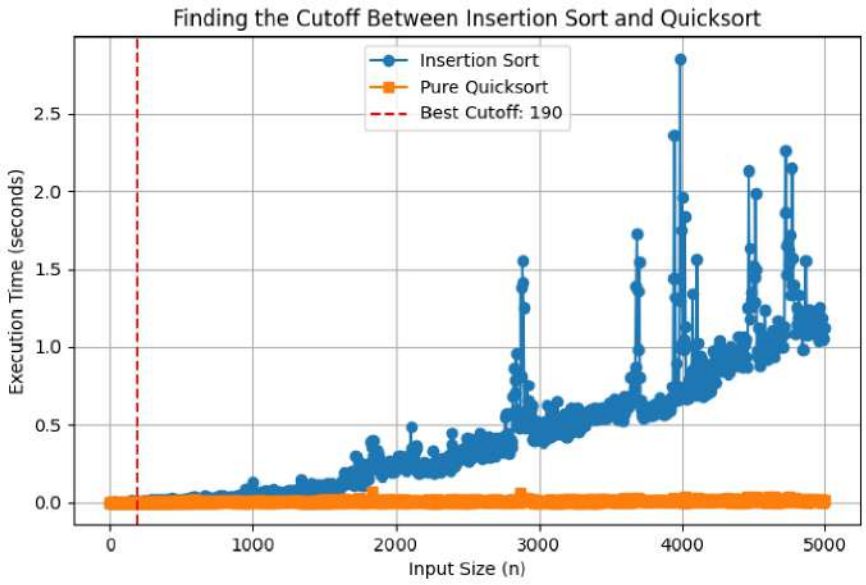
Optimizations for quicksort and for radix sort:

For Quicksort, I initially used Insertion sort using small arrays and then compared with pure quicksort to find the optimal cutoff point. After that, I used randomized pivoting and then median of three pivoting to further optimize the process.

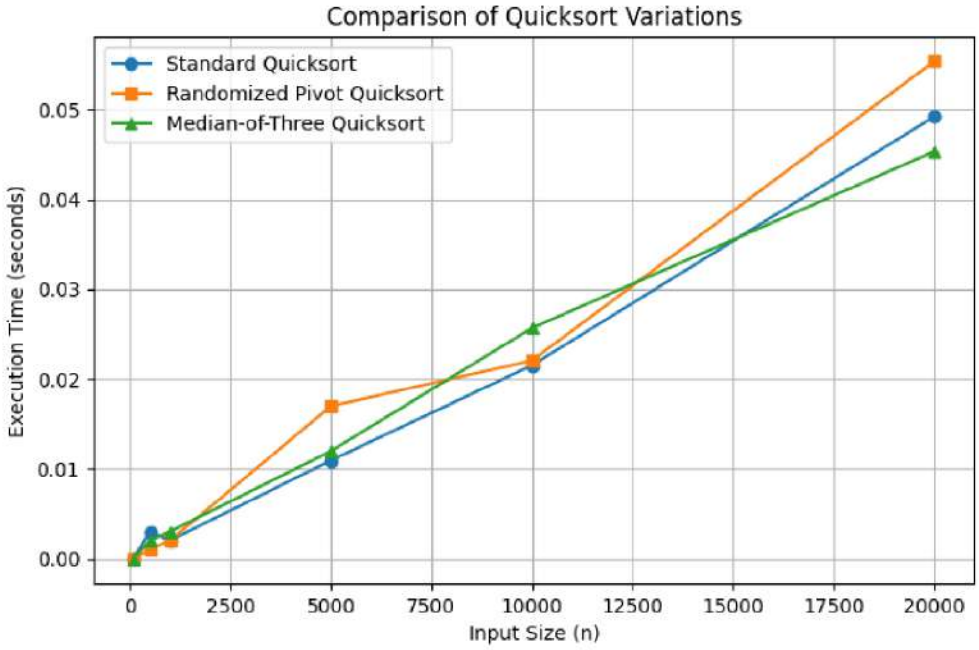
For radix sort, I started by using counting sort as a stable subroutine. Then, I iterated over all the possible options to find out the most effective base and used that best base to apply for sorting. This resulted in a more efficient response.

Plots

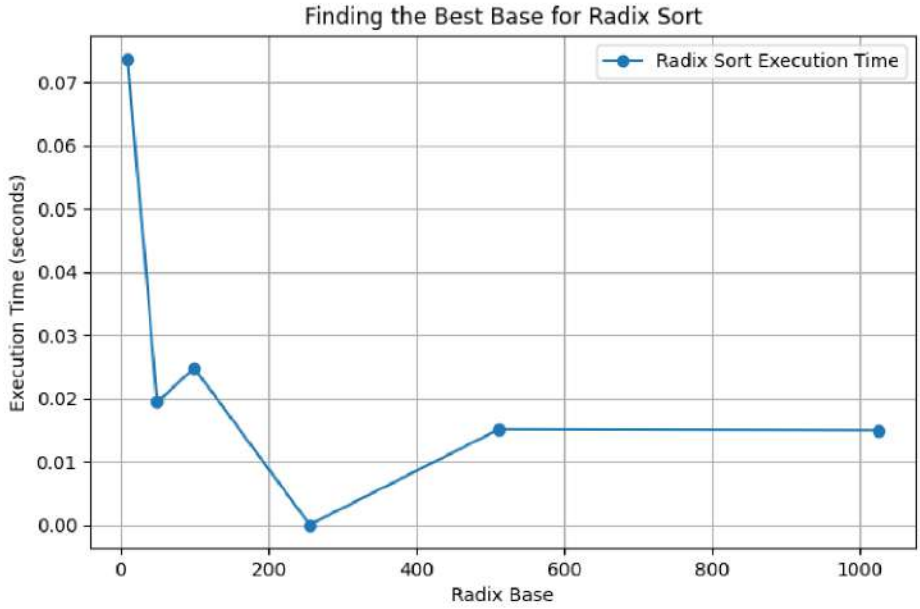
1. **Plot to find cut off point of insertion sort and quicksort**:



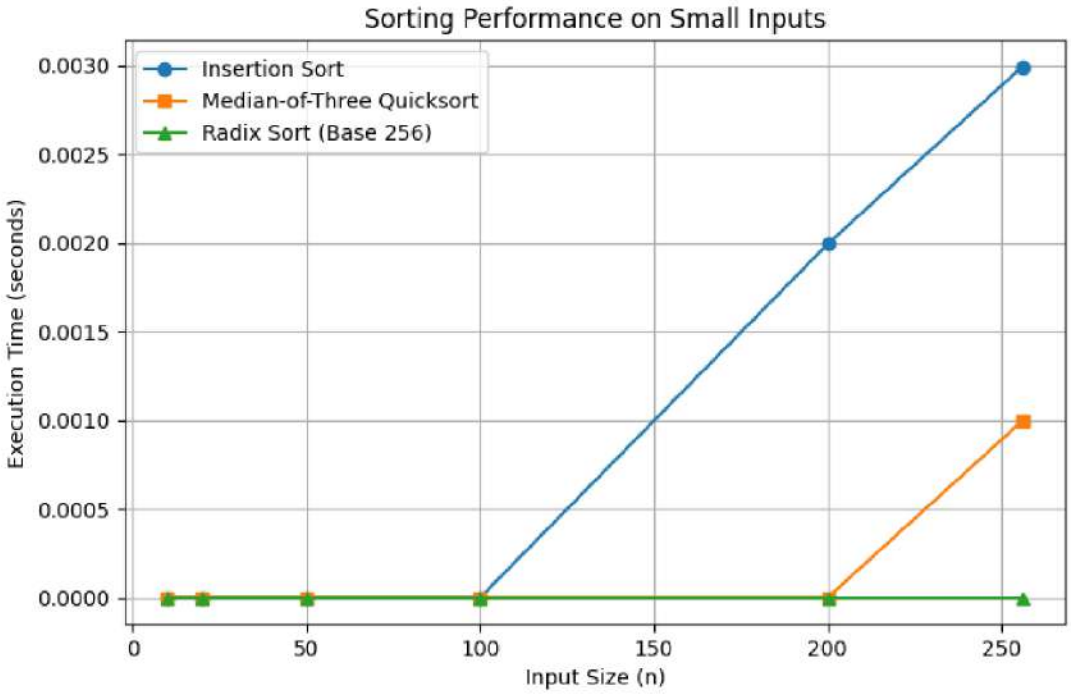
2. **Plot to find the best quicksort among its all variations:**



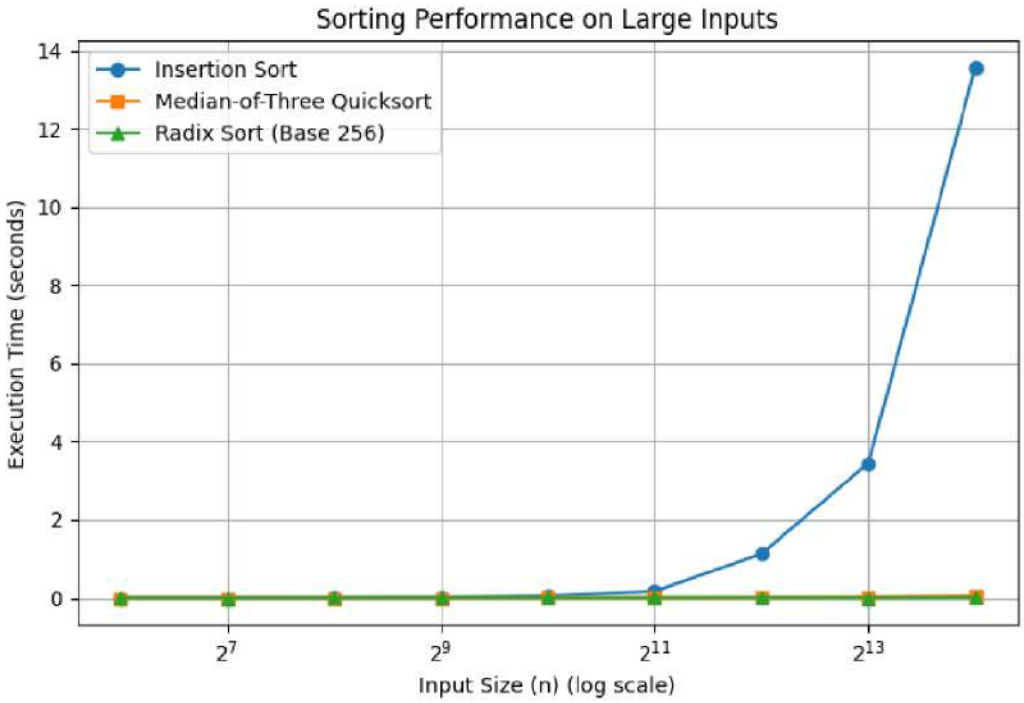
3. Plot to find the best base to be employed for Radix sort:



4(a). Plot to compare best quicksort, insertion sort, and radix sort (input lengths up to 256):



4(b). Plot to compare best quicksort, insertion sort, and radix sort (with input lengths ranging from 64 up to 2^{14}):



The average execution times with the plot are given below:

Average Quicksort Time: 0.007853s
Average Radix Sort Time: 0.005267s