Computational Astrophysics
2019
Exercises 11. ODE. Boundary values.

A simple Boundary-Value Problem

Consider the Boundary-Value Problem (BVP)

$$\frac{d^2y}{dx^2} = 12x - 4\,, \quad y(0) = 0\,, \quad y(1) = 0.1\ . \tag{1}$$

(1) Solve the BVP ODE with the shooting method.

First, we need to rewrite the second-order ODE as two first-order ODEs:

$$\begin{aligned} \frac{dy}{dx} &= u(x)\ , \\ \frac{du}{dx} &= 12x - 4\ . \end{aligned} \tag{2}$$

This is a really simple BVP and therefore it will require only very few shooting iterations for reasonable initial guesses. Hence use the initial values $u(0) = z_0 = -1100000.0$ and $u(0) = z_1 = -10000000.0$ as initial guesses so that you get to do at least two iterations of shooting.

In the code directory you will find a skeleton that you may use. A forward Euler integrator is already implemented. Solve the problem by implementing an RK2 (or higher) integrator. Demonstrate convergence. Make a plot of your solution for $y(x)$ and compare it with the true solution is $y(x) = 2x^3 - 2x^2 + 0.1x$.

(2) Solve the BVP ODE with the finite-difference method.

Happy Coding :) !