Computational Astrophysics

2019

Exercises 12. Parabolic PDEs

Advection Equation

Advection is an important process in many aspects of astrophysics. For example, some transport models consider shock-accelerated particle distributions in the heliosphere [**?**] that are described, in a first approximation, by a one-dimensional advection-difussion equation for the particle density. In the weak diffusion approximation the equation to solve is

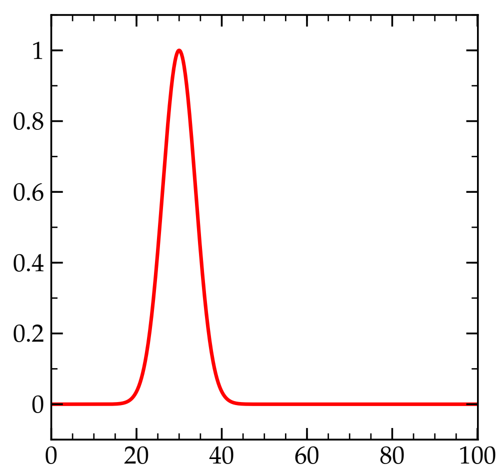
$$\frac{\partial u}{\partial t} + v\frac{\partial u}{\partial x} = 0 \,, \tag{1}$$

where v is a constant advection speed taht can be interpreted as the background solar wind speed [**?**]. As a first example in solving partial differential equations, we will use this equation to advect a Gaussian profile

$$\Psi_0 = \Psi(x, t = 0) = e^{-\frac{(x-x0)^2}{(2\sigma^2)}} \,, \tag{2}$$

with $x0 = 30$, $\sigma = \sqrt{15}$, with positive velocity $v = 0.1$ in a $[0, 100]$ domain.

In order to handle the boundaries, we will choose "outflow" boundary conditions, that simply copy the data of the last interior grid point into the boundary points.



(2) Implement the upwind scheme and demonstrate by experiment that it is stable for $0 \leq \alpha = v\Delta t/\Delta x \leq 1$. Implement an error measure and make a plot of the error as a function of time. Now try a Gaussian with a 5 times smaller σ. What do you observe regarding the error and the visual comparison with the analytic result?

(2) Implement the unstable FTCS scheme and observe the development of instability. Make a few plots and describe what you observe.

(4) Implement the Lax-Friedrich Method and compare it (visually) to upwinding. Make a few plots. Describe what you find.

(5) Implement the Leapfrog scheme and Lax-Wendroff. Compare the two results.

Happy Coding :) !

References

[1] Y. E. Litvinenko and F. Effenberger. The Astrophysical Journal, **796**, 125 (2014)