Computational Astrophysics

2019

Exercises 14. Burguer's Equation

Burguer's Equation

Burger's equation:

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} = 0 \ , \tag{1}$$

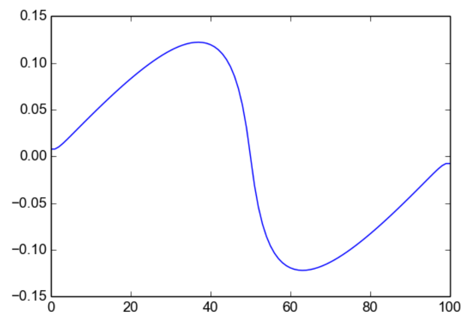
is often used as a first step into hydrodynamics. It is almost identical to the advection equation treated before, but this time the wave speed is NOT a constant v but is given by the field u itself. This fact may lead to shocks, which are typical in hydrodynamic situations.

1. Use Burger's equation to evolve a sine profile,

$$\Psi_0 = \Psi(x, t = 0) = \frac{1}{8}\sin\left(\frac{2\pi x}{L}\right) \tag{2}$$

in a $[0, L]$ domain with $L = 100$.

Use "outflow" boundary conditions (these copy the data of the last interior grid point into the boundary points).



Implement the upwind scheme and demonstrate by experiment that the solution to Burger's equation forms a shock after $t \gtrsim 140$ time has passed. Remember that upwind means "in the direction opposite of the velocity".

(2) Use Burger's equation to evolve a step profile,

$$\Phi_0 = \Phi(x, t = 0) = \begin{cases} 1 \text{ if } x < 0.5 \\ 2 \text{ if } x > 0.5 \end{cases} \tag{3}$$

in the domain $[0, 1]$.

Implement the upwind scheme and demonstrate by experiment that the solution to Burger's equation presents rarefaction.

Happy Coding :) !