MATH 116
FINAL EXAM

BLAKE FARMAN
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Answer the questions in the spaces provided on the question sheets and turn them in at the end of the class period. If you require extra space, use the back of the page and indicate that you have done so.
Unless otherwise stated, all supporting work is required. Unsupported or otherwise mysterious answers will **not receive credit.** You may *not* use any calculators.

Name: _____

Definition	Points Earned	Points Possible	Problem	Points Earned	Points Possible
1		6	1		8
2		1	2		4
3		1	3		2
4		2	4		10
5		2	5		10
6		7	6		10
7		6	7		4
8		1	8		5
9		1	9		5
Subtotal		27	10		15
Total					100

Date: December 10, 2016.

1. Definitions

1 (6 Points)**.** Let a, b be non-zero real numbers and m, n rational numbers. Fill in the blanks

(i) $a^0 =$ _____,

(ii) $\left(\frac{a}{b}\right)^{-n} =$ _____.

(iii) $a^m \cdot a^n =$ _____

(iv) $\frac{a^m}{a^n} =$ _____

(v) $(a \cdot b)^n =$ _____

(vi) $\left(\frac{a}{b}\right)^{n} =$ _____

2 (1 Points)**.** State the Quadratic Formula. That is, state the values of x that satisfy the equation $ax^2 + bx + c = 0$.

3 (1 Points)**.** Fill in the blanks:

To make $x^2 + bx$ a perfect square, add and subtract $\square$. This gives

$$x^2 + bx + \square - \square = \left(x + \square\right)^2 - \square.$$

4 (2 Points)**.** (i) State the Point-Slope form of a line passing through the point (x_0, y_0) with slope m.

(ii) State the Slope-Intercept form of a line with slope m and y-intercept b.

5 (2 Points)**.** Let $f(x)$ be a function with composition inverse $f^{-1}(x)$. Fill in the blanks:

(i) $f \circ f^{-1}(x) \ =$ _____, (ii) $f^{-1} \circ f(x) \ =$ _____.

6 (7 Points)**.** Let $0 < a \neq 1$, $0 < b \neq 1$, and let $0 < x, 0 < y$ be given. Fill in the blanks:

(i) $\log_a(1) =$ _____,

(ii) $\log_a(xy) =$ _____,

(iii) $\log_a\left(\frac{x}{y}\right) =$ _____,

(iv) $\log_a(x^r) =$ _____,

(v) $\log_a(x) = \frac{\log_b(\_\_\_)}{\log_b(\_\_\_)}$,

(vi) $\log_a(a^x) =$ _____,

(vii) $a^{\log_a(x)} =$ _____.

7 (6 Points)**.** Fill in the blanks:

(i) $\arcsin \circ \sin(x) =$ _____, (iv) $\tan \circ \arctan(x) =$ _____,

(ii) $\sin \circ \arcsin(x) =$ _____, (v) $\operatorname{arcsec} \circ \sec(x) =$ _____,

(iii) $\arctan \circ \tan(x) =$ _____, (vi) $\sec \circ \operatorname{arcsec}(x) =$ _____.

8 (1 Point)**.** Fill in the blanks:

The conjugate of the expression $x + \sqrt{a}$ is given by

_____.

The product of $x + \sqrt{a}$ and its conjugate is

$$\left(x + \sqrt{a}\right) \cdot (\_\_\_\_\_) \ = \_\_\_\_\_ .$$

9 (1 Point)**.** Write the difference quotient for the function $f(x)$.

2. Problems

1 (8 Points)**.** Find the period, frequency, and amplitude of $y = 2\cos(2x) + 2$, then graph one period.

2 (4 Points)**.** Graph the function $f(x) = x^2 + 4x + 3$. Label the y-intercept, the vertex, and any x-intercepts.

3 (2 Points)**.** Let $f(x) = x^2 - 4$ and $g(x) = \sqrt{x} + 2$.

(a) Compute $(f \circ g)(x)$.

(b) Compute $(g \circ f)(x)$.

4 (10 Points)**.** Find all the solutions (real and complex) to the equation

$$x^4 - 27x = 0.$$

Hint: $x^3 - y^3 = (x - y)(x^2 + xy + y^2)$.

5 (10 Points)**.** Show that

$$\sin \circ \operatorname{arcsec}(x) = \frac{\sqrt{x^2 - 1}}{x}.$$

6 (10 Points)**.** Consider the polynomial $p(x) = x^3 - 5x^2 - 4x + 20$. Observe that

$$p(5) = (5)^3 - 5(5)^2 - 4(5) + 20 = 5^3 - 5^3 - 20 + 20 = 0.$$

Use this information to **factor** $p(x)$ completely.

7 (4 Points)**.** Use the identity

$$\sin(\theta + \phi) = \sin(\theta)\cos(\phi) + \sin(\phi)\cos(\theta)$$

to show that

$$\sin\left(\frac{2\pi}{3}\right) = \cos\left(\frac{\pi}{6}\right).$$

Hint: $\frac{2}{3} = \frac{1}{6} + \frac{1}{2}$.

8 (5 Points)**.** Simplify

$$\frac{x+\sqrt{2}}{x^2-2}$$

by rationalizing the numerator.

9 (5 Points)**.** Determine whether $f(x) = \mathrm{e}^{2x-1}$ is invertible. If it is, then compute the inverse. Otherwise, explain why it does not have an inverse.

10 (15 Points)**.** Solve the following equations for x.

(a)

$$\log_2(x^2 - 9) - \log_2(x - 3) = 2$$

(b)

$$2^{2x^2} = 4^{2x-1}$$