MATH 122
FINAL EXAM

BLAKE FARMAN
UNIVERSITY OF SOUTH CAROLINA

Answer the questions in the spaces provided on the question sheets and turn them in at the end of the class period. If you require extra space, use the back of the page and indicate that you have done so.
Unless otherwise stated, all supporting work is required. Unsupported or otherwise mysterious answers will **not receive credit.** You may use a calculator **without a CAS** if you like, but a calculator is not necessary. By writing your name on the line below, you acknowledge that you have read and understand these directions.

Name: _____

Definitions	Points Earned	Points Possible	Problems	Points Earned	Points Possible
1		3	1		16
2		3	2		16
3		3	3		16
4		6	4		16
5		2	5		16
6		3	Subtotal		80
Subtotal		20	Bonus		5
			Total		100

Date: April 27, 2017.

1. Definitions

Throughout this section, let f and g be differentiable functions. Fill in the blanks.

1 (3 Points)**.** Let a be a constant.

(a)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(af(x)\right) = \underline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}$$

(b)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(f(x)+g(x)\right) = \underline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}$$

(c)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(f(x)-g(x)\right) = \underline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}$$

2 (3 Points)**.** (a) For n a number,

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(x^n\right) = \underline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}$$

(b)

$$\frac{\mathrm{d}}{\mathrm{d}x}\ln(x) = \underline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}$$

(c)

$$\frac{\mathrm{d}}{\mathrm{d}x}e^x = \underline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}$$

3 (3 Points)**.** Write the formula for each of the following derivatives.

(a)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(f(x)g(x)\right)$$

(b)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(\frac{f(x)}{g(x)}\right)$$

(c)

$$\frac{\mathrm{d}}{\mathrm{d}x}\left(f \circ g(x)\right)$$

4 (6 Points). If $F'(t)$ is a continuous function on the interval $[a, b]$, then

$$\int_a^b F'(t)\,\mathrm{d}t \;=\; \underline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}$$

5 (4 Points). Assume that $\int f(x)\,\mathrm{d}x$ and $\int g(x)\,\mathrm{d}x$ exist.

(a)

$$\int f(x) \pm g(x)\,\mathrm{d}x \;=\; \underline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}$$

(b) Let a be a number.

$$\int af(x)\,\mathrm{d}x \;=\; \underline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}$$

6 (5 Points). Let $n \neq -1$ be a fixed number.

(a)

$$\int x^n\,\mathrm{d}x \;=\; \underline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}$$

(b)

$$\int e^x\,\mathrm{d}x \;=\; \underline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}$$

(c)

$$\int \frac{1}{x}\,\mathrm{d}x \;=\; \underline{\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad}$$

2. Problems

1 (16 Points)**.** Compute the derivative of the following functions.

(a) $f(x) = \frac{1}{3}x^3 + 2\sqrt{x} + e^x + 5\ln(x)$

(b) $f(x) = \ln((x+1)^2) + e^{x^2} + \sqrt{x^2+35}$

(c) $f(x) = x^2\ln(x) + (x+5)e^{5x}$

(d) $f(x) = \frac{x^2-1}{x^2+1}$

2 (16 Points)**.** Let $f(x) = \frac{2}{3}x^3 - x^2 - 12x$.

(a) Find the derivative of f.

(b) Find the critical points of f.
[Hint: Factoring after taking the derivative will make this task much easier.]

(c) Find any local maxima and local minima of f. Clearly indicate whether a point is a maximum or a minimum.

(d) Find any inflection points of f.
[Hint: Same as for part (b).]

3 (16 Points)**.** A company sells a product for \$30 each and the manufacturing costs can be modeled by the function

$$C(q) = q^3 - 9q^2 + 45q + 15$$

of q units produced.

(a) What is the revenue as a function of q units?

(b) What is the profit as a function of q units?

(c) What is the marginal profit function (i.e. the derivative)?

(d) For each of the quantities below, determine whether the company should increase, decrease, or not change the production levels in order to maximize profit. Justify your answers using calculus. **You will not receive credit for guess and check solutions.**

(a) $q = 2$ (b) $q = 6$ (c) $q = 7$

4 (16 Points)**.** Compute the following indefinite integrals.

(a) $\displaystyle\int \left(\frac{2}{x} + 4x^3 + 4e^x\right) \mathrm{d}x$

(b) $\displaystyle\int (2x+3)e^{x^2+3x+7}\,\mathrm{d}x$

(c) $\displaystyle\int \frac{x^2+2}{x^3+6x}\,\mathrm{d}x$

(d) $\displaystyle\int \frac{x}{\sqrt{x^2+3}}\,\mathrm{d}x$

5 (16 Points)**.** Use the Fundamental Theorem of Calculus to compute the following definite integrals.

(a) $\displaystyle\int_3^5 (x+2)\,\mathrm{d}x$

(b) $\displaystyle\int_1^3 \left(3x^2+2x+7\right)\mathrm{d}x$

(c) $\displaystyle\int_0^{\ln(3)} e^x\,\mathrm{d}x$

(d) $\displaystyle\int_1^e \frac{2\ln(x)}{x}\,\mathrm{d}x$

6 (Bonus)**.** Write down a function that has a local minimum at $x = -1$, a local maximum at $x = 0$, and a local minimum at $x = 1$. Justify why the function you wrote down satisfies this criteria.
[Hint: You don't need to get too fancy here. This can be done with a polynomial with integer coefficients. The easiest way to do this is to start by defining what the derivative *should* be, then use an integral to get the function you want.]