

15 Trig Identities

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

$$\begin{aligned}\tan^2(\theta) + 1 &= \frac{\sin^2(\theta)}{\cos^2(\theta)} + \frac{\cos^2(\theta)}{\cos^2(\theta)} \\ &= \frac{\sin^2(\theta) + \cos^2(\theta)}{\cos^2(\theta)} \\ &= \frac{1}{\cos^2(\theta)} \\ &= \sec^2(\theta)\end{aligned}$$

$$\sec^2(\theta) - 1 = \tan^2(\theta)$$

Similarly

$$\begin{aligned}\cot^2(\theta) + 1 &= \frac{\cos^2(\theta)}{\sin^2(\theta)} + 1 = \frac{\cos^2(\theta) + \sin^2(\theta)}{\sin^2(\theta)} = \frac{1}{\sin^2(\theta)} \\ \csc^2(\theta) - 1 &= \cot^2(\theta)\end{aligned}$$

Additive Formulas

$$\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)$$

$$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$$

Q1: $\sin(2x) = \sin(x+x) = \sin(x)\cos(x) + \cos(x)\sin(x)$
 $= 2\sin(x)\cos(x)$

② $\cos(2x) = \cos(x+x) = \cos^2(x) - \sin^2(x)$

③ $\cos(2x) = \cos^2(x) + (\sin^2(x) - \sin^2(x)) - \sin^2(x)$
 $= 1 - 2\sin^2(x)$

$$\Rightarrow 2\sin^2(x) = 1 - \cos(2x)$$

$$\Rightarrow \sin^2(x) = \frac{1 - \cos(2x)}{2}$$

$$\begin{aligned}
 \cos(2x) &= \cos^2(x) + (\cos^2(x) - \sin^2(x)) - \sin^2(x) \\
 &= 2\cos^2(x) - (\cos^2(x) + \sin^2(x)) \\
 &= 2\cos^2(x) - 1 \\
 \Rightarrow \cos^2(x) &= \frac{1 + \cos(2x)}{2}
 \end{aligned}$$

$$\begin{aligned}
 (4) \sin(A-B) &= \sin(A+(-B)) \\
 &= \sin(A)\cos(-B) + \cos(A)\sin(-B) \\
 &= \sin(A)\cos(B) - \cos(A)\sin(B)
 \end{aligned}$$

$$\begin{aligned}
 (5) \cos(A-B) &= \cos(A)\cos(-B) - \sin(A)\sin(-B) \\
 &= \cos(A)\cos(B) + \sin(A)\sin(B)
 \end{aligned}$$

E.g.: ① Compute $\sin(5\pi/12)$.

$$\begin{aligned}
 5\pi/12 &= \frac{2\pi}{12} + \frac{3\pi}{12} = \pi/6 + \pi/4 \\
 \sin(5\pi/12) &= \sin(\pi/6)\cos(\pi/4) + \cos(\pi/6)\sin(\pi/4) \\
 &= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) \\
 &= \frac{\sqrt{2}(1+\sqrt{3})}{4}
 \end{aligned}$$

② Compute $\cos(\pi/12)$

$$\begin{aligned}
 \pi/12 &= \pi/12 - \frac{3\pi}{12} + \frac{3\pi}{12} = -\frac{2\pi}{12} + \pi/4 = -\pi/6 + \pi/4 \\
 \cos(\pi/12) &= \cos(-\pi/6 + \pi/4) = \cos(\pi/6)\cos(\pi/4) + \sin(\pi/6)\sin(\pi/4) \\
 &= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) \\
 &= \frac{\sqrt{2}}{4}(1+\sqrt{3})
 \end{aligned}$$

Remk: $\sin(\pi/2 - \theta) = \sin(\pi/2)\cos(\theta) - \cos(\pi/2)\sin(\theta) = \cos(\theta)$
 $\cos(\pi/2 - \theta) = \cos(\pi/2)\cos(\theta) + \sin(\pi/2)\sin(\theta) = \sin(\theta)$

$$\begin{aligned}
 \sin(A \pm B) &= \sin(A)\cos(B) \pm \cos(A)\sin(B) \\
 \cos(A \pm B) &= \cos(A)\cos(B) \mp \sin(A)\sin(B)
 \end{aligned}$$