MATH 142: EXAM 03

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Answer the questions in the spaces provided on the question sheets and turn them in at the end of the class period. Unless otherwise stated, all supporting work is required. You may **not** use any calculators.

Name: _____

Problem	Points Earned	Points Possible
1		20
2		20
3		20
4		20
5		20
Bonus		10
Total		100

Date: July 25, 2014.

1. Problems

1. *Find the area between the curves* $y = \sin\left(\frac{\pi x}{2}\right)$ *and* $y = x$.

2. *Find the volume of the solid obtained by rotating the region bounded by the curves* $y = \dfrac{1}{x}$, $x = 1$, $x = 2$, *and* $y = 0$ *about the* x*-axis.*

3. *Find a power series representation for* $\dfrac{1}{x-5}$. *Once you have found this power series, find its radius of convergence and the interval of convergence.*

4. *(a) Find the Maclaurin expansion (i.e. find the Taylor expansion about $a = 0$) for the function $f(x) = \cos(x)$. Find the radius of convergence and the interval of convergence for this series.*

(b) Use Taylor's Inequality to show that $\cos(x)$ is equal to its Maclaurin expansion on the interval of convergence you found in Part a. You will find the statement of Taylor's Inequality on the last page.

5. *Find the Maclaurin expansion of the function* $f(x) = xe^x$.

6. *Show that $f(x) = xe^x$ is equal to the power series expansion you found in Problem 5.*

7 (Taylor's Inequality)**.** *If there exists a real number, M, such that $\left|f^{(n+1)}(x)\right| \leq M$ holds whenever $|x-a| \leq d$, then the remainder, $R_n(x)$, of the Taylor series satisfies the inequality*

$$|R_n(x)| \leq \frac{M}{(n+1)!}\,|x-a|^{n+1}$$

whenever $|x-a| \leq d$.