MATH 111:
HOMEWORK 02

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B.3

5. *Consider the expression*

$$\frac{1}{x} - \frac{2}{x+1} - \frac{x}{(x+1)^2}.$$

(a) How many terms does this expression have?
(b) Find the least common denominator of all the terms.
(c) Perform the addition and simplify.

Solution. (a) There are three terms in this expression,

$$\frac{1}{x}, \frac{2}{x+1}, \text{ and } \frac{x}{(x+1)^2}.$$

(b) The least common denominator is found by taking the least common multiple of all three of the denominators, x, $x+1$, and $(x+1)^2$. Since the last two denominators have $x+1$ is common, the least common multiple is found by combining the largest power of the $x+1$, which is $(x+1)^2$, and the first denominator x. Hence we have $x(x+1)^2$ as our least common denominator.

(c) First we manipulate the original expression so that each have a common denominator,

$$\frac{(x+1)^2}{(x+1)^2} \cdot \left(\frac{1}{x}\right) - \frac{x(x+1)}{x(x+1)} \cdot \left(\frac{2}{x+1}\right) - \frac{x}{x} \cdot \left(\frac{x}{(x+1)^2}\right) = \frac{(x+1)^2 - 2x(x+1) - x^2}{x(x+1)^2}.$$

Expanding the numerator and collecting like terms we arrive at the final answer

$$\begin{aligned} \frac{(x^2+2x+1) - (2x^2+2x) - (x^2)}{x(x+1)^2} &= (x^2 - 2x^2 - x^2) + (2x - 2x) + 1 \\ &= \frac{1-2x^2}{x(x+1)^2} \end{aligned}$$

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17. *Simplify the rational expression*

$$\frac{x^2+6x+8}{x^2+5x+4}.$$

Solution. First we start by factoring the numerator and the denominator. In the numerator, we need two numbers that add to 6 and multiply to 8. These are 2 and 4, so we have the factorization $x^2+6x+8 = (x+2)(x+4)$. In the denominator, we need two numbers that add to 5 and multiply to 4. These are 4 and 1, so we have the factorization $x^2+5x+4 = (x+1)(x+4)$. Rewriting our rational expression and simplifying we have

$$\frac{x^2+6x+8}{x^2+5x+4} = \frac{(x+2)(x+4)}{(x+1)(x+4)} = \frac{x+2}{x+1}.$$

21. *Perform the multiplication*

$$\frac{4x}{x^2-4} \cdot \frac{x+2}{16x}.$$

Solution. First we observe that we have the factorization $x^2-4 = (x+2)(x-2)$. Now, when we multiply rational expressions we multiply together the numerator and denominators, so

$$\frac{4x}{x^2-4} \cdot \frac{x+2}{16x} = \frac{4x(x+2)}{16x(x+2)(x-2)}.$$

Noting that $16 = 4^2$, we cancel common factors in the numerator and denominator to obtain

$$\frac{4x(x+2)}{16x(x+2)(x-2)} = \frac{4x(x+2)}{4^2x(x+2)(x-2)} = \frac{1}{4(x-2)}.$$

33. *Perform the addition*

$$\frac{1}{x+5} + \frac{2}{x-3}.$$

Solution. To add these rational expressions we need to first find a common denominator. This will be the product of $x+5$ and $x-3$, so

$$\begin{aligned}
\frac{1}{x+5}+\frac{2}{x-3} &= \frac{x-3}{x-3}\cdot\left(\frac{1}{x+5}\right)+\frac{x+5}{x+5}\cdot\left(\frac{2}{x-3}\right)\\
&= \frac{x-3}{(x-3)(x+5)}+\frac{2(x+5)}{(x-3)(x+5)}\\
&= \frac{(x-3)+2(x+5)}{(x-3)(x+5)}\\
&= \frac{x-3+2x+10}{(x-3)(x+5)}\\
&= \frac{3x+7}{(x-3)(x+5)}
\end{aligned}$$

43. *Rationalize the denominator*

$$\frac{2}{\sqrt{2}+\sqrt{7}}.$$

Solution. To rationalize the denominator, we need to use the conjugate of $\sqrt{2}+\sqrt{7}$, which is $\sqrt{2}-\sqrt{7}$. Recall that by the sum and difference of same terms formula,

$$(\sqrt{2}+\sqrt{7})(\sqrt{2}-\sqrt{7})=(\sqrt{2})^2-\sqrt{7}^2=2-7=-5.$$

We now multiply the numerator and denominator by the conjugate, so as to preserve its value, to obtain

$$\frac{2}{\sqrt{2}+\sqrt{7}}=\frac{\sqrt{2}-\sqrt{7}}{\sqrt{2}-\sqrt{7}}\cdot\left(\frac{2}{\sqrt{2}+\sqrt{7}}\right)=\frac{2(\sqrt{2}-\sqrt{7})}{-5}=-\frac{2(\sqrt{2}-\sqrt{7})}{5}$$

50. *Find the quotient and remainder using long division*

$$\frac{x^3+3x^2+4x+3}{3x+6}.$$

Solution.

C.1

9. *Solve the equation*

$$x-3=2x+6.$$

13. *Solve the equation*

$$2(1-x)=3(1+2x)+5.$$

23. *Solve the equation*

$$\frac{2}{t} = \frac{3}{5}.$$

27. *Solve the equation*

$$\frac{2}{t+6} = \frac{3}{t-1}.$$

35. *Find all real solutions of the equation*

$$y^2 - 24 = 0.$$

41. *Find all real solutions of the equation*

$$(x+2)^2 = 4.$$

61. *Solve the following equation for x*

$$xy = 3y - 2x$$

C.2

7. *Solve the following equation by factoring*

$$3x^2 - 5x - 2 = 0.$$

11. *Complete the square for the given expression:*

$$x^2 + 7x + \boxed{} = (x + \boxed{})^2.$$

23. *Solve the following equation by factoring or using the Quadratic Formula*

$$x^2 - 2x - 15 = 0.$$

27. *Solve the following equation by factoring or using the Quadratic Formula*

$$x^2 + 3x + 1 = 0.$$

C.3

1. *Fill in the blank with an appropriate inequality sign.*

(a) If $x < 5$, then $x - 3$ ____ 2.
(b) If $x \leq 5$, then $3x$ ____ 15.
(c) If $x \geq 2$, then $-3x$ ____ -6.
(d) If $x < -2$, then $-x$ ____ 2.

10. *Let $S = \{-2, -1, 0, \frac{1}{2}, 1, \sqrt{2}, 2, 4\}$. Determine which elements of S satisfy the inequality*

$$x^2 + 2 < 4.$$

15. *Solve the linear inequality*

$$7 - x \geq 5.$$

Express the solution using interval notation, and graph the solution set.

17. *Solve the linear inequality*

$$3x + 11 \leq 7x + 8.$$

Express the solution using interval notation, and graph the solution set.

23. *Solve the non-linear inequality*

$$(x + 2)(x - 3) < 0.$$

Express the solution using interval notation, and graph the solution set.

25. *Solve the non-linear inequality*

$$x^2 - 3x - 18 \leq 0.$$

Express the solution using interval notation, and graph the solution set.