MATH 111
FINAL EXAM

BLAKE FARMAN

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> Answer the questions in the spaces provided on the question sheets and turn them in at the end of the exam period. If you require extra space, use the back of the page and indicate that you have done so.
> Unless otherwise stated, all supporting work is required. Unsupported or otherwise mysterious answers will **not receive credit.**
> You may use a calculator, but you may **not** use a Computer Algebra System (CAS) or any other electronic device whatsoever, **including cell phones.**

Name: _____

Problem	Points Earned	Points Possible
1		3
2		6
3		1
4		2
5		3
6		2
7		1
8		4
9		3
10		5
11		5
12		9
13		9
14		9
15		20
16		10
17		8
Total		100

1. Definitions

1 (3 Points). *Fill in the blanks with the correct factorizations.*

(a) $A^2 - B^2 = $ _____.

(b) $A^2 + 2AB + B^2 = $ _____.

(c) $A^2 - 2AB + B^2 = $ _____.

2 (6 Points). *Let a, b be non-zero real numbers and m, n integers. Fill in the blanks*

(i) $a^0 = $ _____,

(ii) $a^{-n} = $ _____.

(iii) $a^m \cdot a^n = $ _____

(iv) $\dfrac{a^m}{a^n} = $ _____

(v) $(a \cdot b)^n = $ _____

(vi) $\left(\dfrac{a}{b}\right)^n = $ _____

3 (1 Point). *Given an equation $ax^2 + bx + c = 0$, the solutions are given by the Quadratic Formula. State the Quadratic Formula.*

4 (2 Points). *(a) State the Point-Slope form of a line passing through the point (x_1, y_1) with slope m.*

(b) State the Slope-Intercept form of a line with slope m and y-intercept b.

5 (3 Points). *(a) State the general form of an exponential function.*

(b) When does such a function model exponential growth?

(c) When does such a function model exponential decay?

6 (2 Points). *Consider the two distinct lines $f(x) = m_1 x + b_1$ and $g(x) = m_2 x + b_2$.*
(a) When are f and g parallel?

(b) When are f and g perpendicular?

7 (1 Point). *Let a be a fixed positive number. The base a logarithm of x is defined by*

$$\log_a(x) = y \ \text{ if and only if } \ \underline{\hspace{3cm}}.$$

8 (4 Points). *Let a be a positive number. Fill in the blanks.*

(a) $\log_a(1) = \underline{\hspace{3cm}}.$

(b) $\log_a(a) = \underline{\hspace{3cm}}.$

(c) $\log_a(a^x) = \underline{\hspace{3cm}}.$

(d) $a^{\log_a(x)} = \underline{\hspace{3cm}}.$

9 (3 Points). *Let $0 < a$ and C be fixed numbers. Fill in the blanks.*

(a) $\log_a(xy) = \underline{\hspace{3cm}}.$

(b) $\log_a\left(\frac{x}{y}\right) = \underline{\hspace{3cm}}.$

(c) $\log_a(x^C) = \underline{\hspace{3cm}}.$

2. PROBLEMS

10 (4 Points). *Add the following rational expressions and simplify the result,*

$$\frac{1}{x + \sqrt{3}} + \frac{1}{x - \sqrt{3}}.$$

11 (5 Points). *Consider the two lines $f(x) = 5x + 16$ and $g(x) = 8x + 7$. Find the point (that is, the (x, y) pair) where these two lines intersect.*

12 (9 Points). *Let* $f(x) = 3x^2 - 18x + 24$.

(a) Put $f(x)$ into standard form.

(b) Solve $f(x) = 0$.

(c) Use the information from parts (a) and (b) to sketch a graph of $f(x)$. To receive credit, you must label the y-intercept, any x-intercept(s), and the vertex.

13 (9 Points). *In the following problems, use the given information to find the equation of the line in slope-intercept form.*

(a) The line passing through the points $(4, 20)$ and $(1, 14)$.

(b) The line passing through the point $(6, 12)$ and parallel to the line in part (a).

(c) The line passing through $(4, 8)$ and perpendicular to the line in part (a).

14 (9 Points). *A biologist observes a population with initial size* 81. *In two years, the biologist returns to observe the population again and finds that only* 9 *remain.*

(a) Find an exponential model for the size of the population as a function of t years.

(b) Does the function from part (a) model growth or decay?

(c) Use the model from part (a) to determine how many years it will take for the size of the population to reach 1.

15 (20 Points). *(a) Simplify the expression*

$$\log_4(x+3) + \log_4(x-3).$$

(b) Solve the following equation for x

$$\log_4(x+3) + \log_4(x-3) = 2$$

16 (10 Points). *Solve the following equation for x*

$$e^{x^2} = e^{-2x-1}$$

17 (8 Points). *Let $f(x) = \sqrt{x}$ and $g(x) = x^2 - 9$.*

(a) Compute the composition $(f \circ g)(x)$. What is the domain of this function?

(b) Compute the composition $(g \circ f)(x)$. What is the domain of this function?