MATH 170
FINAL EXAM

BLAKE FARMAN

UNIVERSITY OF SOUTH CAROLINA

> Answer the questions in the spaces provided on the question sheets and turn them in at the end of the class period. If you require extra space, use the back of the page and indicate that you have done so.
> Unless otherwise stated, all supporting work is required. Unsupported or otherwise mysterious answers will **not receive credit.**

Name: _____

Problem	Points Earned	Points Possible
1		10
2		10
3		10
4		10
5		10
6		15
7		15
8		10
9		10
Bonus		10
Total		100

1. PROBLEMS

1 (10 Points). *Let S be the set of all dogs. Let A be the subset of all dogs that are labradors. Let B be the subset of all dogs that are yellow.*

(a) In words, what does the set $A \cup B$ represent?

(b) In words, what does the set $A \cap B$ represent?

(c) In words, what do the sets $S \setminus A$ and $S \setminus B$ represent?

(d) In words, what does the set $(S \setminus A) \cap (S \setminus B)$ represent?

(e) In words, what does the set $(S \setminus A) \cup (S \setminus B)$ represent?

2 (10 Points). *How many three letter sequences without repetition can be made using the letters*

$$c, \ o, \ l, \ o, \ r, \ a, \ d, \ o?$$

Let $U = \{A,\ B,\ C, D,\ E,\ F,\ G\}$. Let

$$X = \{B,\ D,\ E\},$$
$$Y = \{A,\ B,\ E,\ F,\ G\}, \text{and}$$
$$Z = \{A,\ E,\ F,\}.$$

Use these sets to answer questions 3 and 4.

3 (10 Points). *Compute the following sets.*

(a) $X \cap Y$,

(b) $X \cup Z$,

(c) The complement of Z in U.

4 (10 Points). *(a) What is the cardinality of $X \times Z$?*

(b) What is the cardinality of $Y \cup Z$?

5 (10 Points). *Use a truth table to prove the following logical equivalences.*

(a)

$$q \wedge (p \Rightarrow q) \equiv q$$

(b)

$$p \wedge \neg q \equiv \neg(p \Rightarrow q)$$

6 (15 Points). *Solve the system of equations*

$$x + 3y + 5z = 7$$
$$2x + 4y + 6z = 8.$$

If there is no solution, simply write 'no solution.' If the system is dependent, express your answer in terms of z.

7 (15 Points). *Solve the system of equations*

$$
\begin{aligned}
x + 4y + 2z &= 1 \\
2y + z &= 2 \\
3x + 5y + 3z &= 3
\end{aligned}
$$

If there is no solution, simply write 'no solution.' If the system is dependent, express your answer in terms of x.

8 (10 Points). *Use* **matrix inversion** *to solve the given equation*

$$\begin{pmatrix} 2 & 1 \\ 3 & 2 \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = \begin{pmatrix} 5 \\ 7 \end{pmatrix}$$

for x and y.

9 (10 Points). *A bag contains five red marbles, two green marbles, one lavender marble, one yellow marble, and three orange marbles.*

(a) How many sets of four marbles have only green and orange marbles?

(b) How many sets of four marbles do not have any red marbles and have **at most** *two orange marbles?*

10 (Bonus - 10 Points)**.** *Let*

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

be a matrix with entries real numbers. You may assume that $a \neq 0$ and $ad - bc \neq 0$. Use Gauss-Jordan row reduction on the appropriate augmented matrix to compute the inverse of A. NOTE: I would like you to explicitly compute the inverse of A, not simply write down the matrix A^{-1}.