MATH 115
FINAL EXAM

BLAKE FARMAN

UNIVERSITY OF SOUTH CAROLINA

Answer the questions in the spaces provided on the question sheets and turn
them in at the end of the class period. If you require extra space, use the
back of the page and indicate that you have done so.
Unless otherwise stated, all supporting work is required. Unsupported or
otherwise mysterious answers will **not** receive credit.
You may **not** use any calculators or other electronic devices.

Name: _____

Problem	Points Earned	Points Possible
1		10
2		10
3		10
4		10
5		10
6		10
7		10
8		10
9		10
10		10
Bonus		5
Total		100

1. Problems

1. Let $f(x) = -2x^2 + 8x - 5$.

(a) Find the solutions to $f(x) = 0$.

(b) Write $f(x)$ in vertex form.

(c) Find the coordinates of the y-intercept for $f(x)$.

(d) Use parts (a)-(c) to sketch a graph of $f(x)$.

2. *Find and simplify the difference quotient for* $f(x) = 2x^2 + x$.

3. *Graph the function*

$$f(x) = \begin{cases} -x - 3 & \text{if} & x \leq -3, \\ \sqrt{9 - x^2} & \text{if} & -3 < x < 3, \\ x - 3 & \text{if} & 3 \leq x. \end{cases}$$

4. *Solve the inequality*

$$-4x^2 + 3x + 1 \geq 0$$

for x.

5. *Solve the equation*

$$\ln(x - 2) = \ln\left(\frac{1}{x - 4}\right)$$

for x.

6. *Find the period, frequency, phase shift, and amplitude of*

$$y = 2\sin(3x + \pi) + 1,$$

then graph one period.

7. *Find the period, frequency, phase shift, and amplitude of*

$$y = -3\cos\left(x - \frac{\pi}{2}\right) - 3,$$

then graph one period.

8. *Show that*

$$\sin(\arctan(x)) = \frac{x}{1 + x^2}.$$

9. *Determine whether the following functions are even, odd, or neither.* **Justify your answers.**

(a) $f(x) = \sin(x^3)$

(b) $h(x) = \cos\left(x^3\right)$

(c) $g(x) = 1 + \csc(x)$

(d) $k(x) = \cos(x) - 1$

10. *You are given that* $\cos(\pi/3) = 1/2$, $\sin(\pi/3) = \sqrt{3}/2$, $\cos(\pi/2) = 0$, *and* $\sin(\pi/2) = 1$. *Using the list of trigonometric identities on the last page, compute the values of each of the following expressions.* **Do not** *simply write down values you have memorized from the unit circle as you will receive no credit for doing so.* *[Hint: 1/3 and 1/2 have common denominator 6]*

(a) $\sin\left(\dfrac{5\pi}{6}\right)$.

(b) $\cos\left(\dfrac{5\pi}{6}\right)$.

11 (Bonus)**.** *Solve the following system algebraically and then sketch both equations on the same coordinate system. Label the points of intersection.*

$$y = x^2 - 7x + 18$$
$$y = x + 3$$

2. Useful Formulae

- $\sin^2(\theta) + \cos^2(\theta) = 1$.
- $\tan^2(\theta) + 1 = \sec^2(\theta)$.
- $\cot^2(\theta) + 1 = \csc^2(\theta)$.
- $\cos(2\theta) = \cos^2(\theta) - \sin^2(\theta)$.
- $\cos(2\theta) = 1 - 2\sin^2(\theta)$.
- $\cos(2\theta) = 2\cos^2(\theta) - 1$.
- $\sin(2\theta) = 2\sin(\theta)\cos(\theta)$.
- $\sin(\theta + \varphi) = \sin(\theta)\cos(\varphi) + \sin(\varphi)\cos(\theta)$.
- $\cos(\theta + \varphi) = \cos(\theta)\cos(\varphi) - \sin(\theta)\sin(\varphi)$.
- $\sin\left(\dfrac{\theta}{2}\right) = \pm\sqrt{\dfrac{1 - \cos(\theta)}{2}}$
- $\cos\left(\dfrac{\theta}{2}\right) = \pm\sqrt{\dfrac{1 + \cos(\theta)}{2}}$
- $2\sin(\theta)\cos(\varphi) = \sin(\theta - \varphi) + \sin(\theta + \varphi)$.
- $2\sin(\theta)\sin(\varphi) = \cos(\theta - \varphi) - \cos(\theta + \varphi)$.
- $2\cos(\theta)\cos(\varphi) = \cos(\theta - \varphi) + \cos(\theta + \varphi)$.