15 Trig Identities

$$\sin^2(\theta) + \cos^2(\theta) = 1$$

$$\tan^2(\theta) + 1 = \frac{\sin^2(\theta)}{\cos^2(\theta)} + \frac{\cos^2(\theta)}{\cos^2(\theta)}$$

$$= \frac{\sin^2(\theta) + \cos^2(\theta)}{\cos^2(\theta)}$$

$$= \frac{1}{\cos^2(\theta)}$$

$$= \sec^2(\theta)$$

$$\sec^2(\theta) - 1 = \tan^2(\theta)$$

Similarly

$$\cot^2(\theta) + 1 = \frac{\cos^2(\theta)}{\sin^2(\theta)} + 1 = \frac{\cos^2(\theta) + \sin^2(\theta)}{\sin^2(\theta)} = \csc^2(\theta)$$

$$\csc^2(\theta) - 1 = \cot^2(\theta)$$

Additive Formulas

$$\sin(A+B) = \sin(A)\cos(B) + \cos(A)\sin(B)$$
$$\cos(A+B) = \cos(A)\cos(B) - \sin(A)\sin(B)$$

Cor: ① $\sin(2x) = \sin(x+x) = \sin(x)\cos(x) + \cos(x)\sin(x)$
$$= 2\sin(x)\cos(x)$$

② $\cos(2x) = \cos(x+x) = \cos^2(x) - \sin^2(x)$

③ $\cos(2x) = \cos^2(x) + (\sin^2(x) - \sin^2(x)) - \sin^2(x)$
$$= 1 - 2\sin^2(x)$$

$$\Rightarrow 2\sin^2(x) = 1 - \cos(2x)$$

$$\Rightarrow \sin^2(x) = \frac{1 - \cos(2x)}{2}$$

$$\cos(2x) = \cos^2(x) + (\cos^2(x) - \cos^2(x)) - \sin^2(x)$$
$$= 2\cos^2(x) - (\cos^2(x) + \sin^2(x))$$
$$= 2\cos^2(x) - 1$$
$$\Rightarrow \cos^2(x) = \frac{1 + \cos(2x)}{2}$$

④ $\sin(A-B) = \sin(A+(-B))$
$$= \sin(A)\cos(-B) + \cos(A)\sin(-B)$$
$$= \sin(A)\cos(B) - \cos(A)\sin(B)$$

⑤ $\cos(A-B) = \cos(A)\cos(-B) - \sin(A)\sin(-B)$
$$= \cos(A)\cos(B) + \sin(A)\sin(B)$$

E.g.: ① Compute $\sin(5\pi/12)$.

$$\frac{5\pi}{12} = \frac{2\pi}{12} + \frac{3\pi}{12} = \frac{\pi}{6} + \frac{\pi}{4}$$

$$\sin(5\pi/12) = \sin(\pi/6)\cos(\pi/4) + \cos(\pi/6)\sin(\pi/4)$$
$$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$
$$= \frac{\sqrt{2}(1+\sqrt{3})}{4}$$

② Compute $\cos(\pi/12)$

$$\frac{\pi}{12} = \frac{\pi}{12} - \frac{8\pi}{12} + \frac{3\pi}{12} = \frac{-2\pi}{12} + \frac{\pi}{4} = \frac{-\pi}{6} + \frac{\pi}{4}$$

$$\cos(\pi/12) = \cos(\pi/6 + \pi/4) = \cos(\pi/6)\cos(\pi/4) + \sin(\pi/6)\sin(\pi/4)$$
$$= \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$
$$= \frac{\sqrt{2}}{4}(1 + \sqrt{3})$$

Rmk: $\sin(\pi/2 - \theta) = \sin(\pi/2)\cos(\theta) - \cos(\pi/2)\sin(\theta) = \cos(\theta)$
$\cos(\pi/2 - \theta) = \cos(\pi/2)\cos(\theta) + \sin(\pi/2)\sin(\theta) = \sin(\theta)$

$$\sin(A \pm B) = \sin(A)\cos(B) \pm \cos(A)\sin(B)$$
$$\cos(A \pm B) = \cos(A)\cos(B) \mp \sin(A)\sin(B)$$