Proportionality and Power Functions

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Math 122: Calculus for Business Administration and Social Sciences

Outline

1 2.1: Instantaneous Rate of Change

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2. 2.2: The Derivative Function

DEFINITION

The *instantaneous rate of change* of f at a is defined to be the limit of the average rates of change of f over successively smaller intervals around a. This is also known as the *derivative of f at a*.

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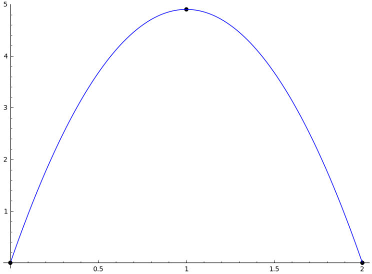

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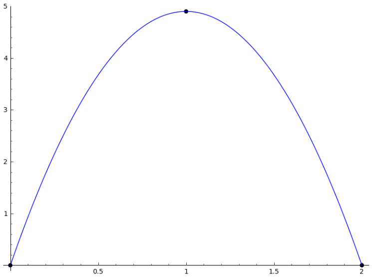

What is the instantaneous rate of change at the vertex, where $t = 1$?

Example (cont.)

Here are some values:

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So, we would guess that the instantaneous rate of change is 0 at $t = 1$.

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- In this case, we can define a function $f'(x)$ that outputs the instantaneous rate of change of f at x.
- We call $f'(x)$ the *derivative function*.

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- We call the line

$$y - f(x_0) = f'(x_0)(x - x_0)$$

the *line tangent to f at* $(x_0, f(x_0))$.

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- This means f locally looks like a line!

Non-Differentiable Function

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$$\frac{|x| - 0}{x - 0} = \frac{x}{x} = 1.$$
- So the derivative at $(0, 0)$ is **not** defined: it's -1 if we approach from left to right, and 1 if right to left.

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- $f'(x) \leq 0$, then f is decreasing on (a, b),
- $0 \leq f'(x)$, then f is increasing on (a, b),
- $f'(x) = 0$, then f is constant on (a, b).