Problem Set 1

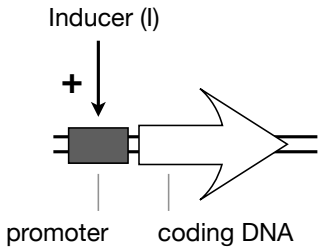

Figure 1: Induction of a protein P by a single small molecule inducer I.

1. The expression of protein P (μmol/gDW) from gene $\mathcal{G}_P$ ($\mathcal{L}$ = 3075 nt) is induced by an extracellular inducer I (mM) in a well-mixed population of growing *E. coli* cells (doubling time of 30 min) from a plasmid present at 2500 copies per cell (average) (Fig. 1). Treat the plasmid and inducer concentrations as constant. Assume transcription follows the elementary reaction scheme:

$$\begin{aligned}
\mathcal{G}_P + R_X &\rightleftharpoons (\mathcal{G}_P : \mathrm{R_X})_C && (1)\\
(\mathcal{G}_P : \mathrm{R_X})_C &\longrightarrow (\mathcal{G}_P : \mathrm{R_T})_O && (2)\\
(\mathcal{G}_P : \mathrm{R_X})_O &\longrightarrow R_X + \mathcal{G}_P && (3)\\
(\mathcal{G}_P : \mathrm{R_X})_O &\longrightarrow m_P + R_X + \mathcal{G}_P && (4)
\end{aligned}$$

where $\mathcal{G}_P$, R_X denote the gene and RNAP concentration, and $(\mathcal{G}_P : \mathrm{R_X})_O$, $(\mathcal{G}_P : \mathrm{R_X})_C$ denote the open and closed complex concentrations, respectively. Let the *overall* specific rate of transcription, $\hat{r}_{X,P} = r_{X,P}u(I)$, be the product of a kinetic term $r_{X,P}$ and a control term $u(I)$, where $u(I)$ is a Voigt type model (Moon *et al Nature* **491**:249, 2012). Assume $r_{X,P}$ is first order with respect to the open complex.

   a) Derive an expression for $\hat{r}_{X,P}$ and estimate unknown parameter values appearing in $r_{X,P}$ using `BioNumbers`, literature or the CHEME-7770 GitHub parameter page.

   b) Is transcription initiation of elongation limited?

   c) Plot (on a semi-log scale) the steady-state mRNA concentration m_P (μmol/gDW) as a function inducer I for I = 0.0001 mM to I = 10.0 mM. Let W_1 = 0.26, W_2 = 300.0, K = 0.30 mM and n = 1.5.