AMATH 581: HW # 1

I. Consider the function

$$f(x) = x\sin(3x) - e^x$$

and solve for the x-value near $x \approx .5$ that satisfies $f(x) = 0$.

In the first part, use the Newton-Raphson method w/ initial guess $x_1 = -1.6$ to converge to the solution to 10^{-6}.

A1. Newton-Raphson method. $f(x_r) = 0$ (root finding method)

iteration (1). Guess the root, $x_0 = -1.6$ initially.

Then, $x_{n+1} = x_n - \frac{f(x_n)}{f'(x_n)}$

We may obtain the slope from the original function, $f(x) = x\sin(3x) - e^x$

$$f'(x) = \left[x\cos(3x)\cdot 3 + \sin(3x)\right] - e^x$$

See attached code for full solution. I also include the following table for convergence for reference.

iteration #(n)	x_n	$f(x_n)$
1	-1.6	-1.796
2	3.198	25.02
3	2.464	-9.549
4	1.204	-3.876
5	.6502	-1.312
6	-.1169	-.8495
7	-.6605	.08899
8	-.5219	-.07145
9	-.5666	-.005623
10	-.5707	-5.654×10^{-5}
11	-.570789	-6.0162×10^{-9} (within tolerance).

A1.

Assumptions: x_1 counts as the "first iteration" (as given, it was x_1).

In the second part, use bisection with the initial endpoints $x = -.7$ and $x = -.4$. Keep track of the midpoint values and number of iterations until an accuracy of 10^{-6} is achieved.

A2. Bisection method. Now we perform bisection with the left endpoint $-.7$ and right endpoint $-.4$.

Again I used python to compute the method, since the tolerance 10^{-6} is much higher than machine precision.

See attached code for full solution. I also include the following table for convergence for reference.

iteration #	x mid	f(x mid)
1	-.55	-.0287
2	-.625	-.061
3	-.5875	+.021
4	-.56875	-.00269
5	-.578125	.00946
6	-.5734375	.003455
7	-.5711	3.99×10^{-4}
8	-.5699	1.1×10^{-3}
9	-.5705	3.7×10^{-4}
10	-.5708	1.466×10^{-5}
11	-.57065	1.77×10^{-4}
12	-.57073	-8.156×10^{-5}
13	-.57076	-3.344×10^{-5}
14	-.570782	-9.893×10^{-6}
15	-.57079	2.63×10^{-6}
16	-.570787	-3.3×10^{-6}
17	-.570789	-3.7×10^{-7}

A2

This method, while it initially drops much faster than the Newton-Raphson method, ends up taking a lot longer to converge, ultimately.

A3. $\boxed{\begin{bmatrix} 11 \\ 17 \end{bmatrix}}$ A3.

II. let the following be defined:

$$A = \begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix}, \quad C = \begin{bmatrix} 2 & 0 & -3 \\ 0 & 0 & -1 \end{bmatrix}, \quad D = \begin{bmatrix} 1 & 2 \\ 2 & 3 \\ -1 & 0 \end{bmatrix}, \quad x = \begin{bmatrix} 1 \\ 0 \end{bmatrix}, \quad y = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, \quad z = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$$

Calculate the following:

a) $A + B$

$$\begin{bmatrix} 1 & 2 \\ -1 & 1 \end{bmatrix} + \begin{bmatrix} 2 & 0 \\ 0 & 2 \end{bmatrix} = \boxed{\begin{bmatrix} 3 & 2 \\ -1 & 3 \end{bmatrix}}$$ A4

b) $3x - 4y$

$$3\begin{bmatrix}1\\0\end{bmatrix} - 4\begin{bmatrix}0\\1\end{bmatrix} = \boxed{\begin{bmatrix}3\\-4\end{bmatrix}}$$

A5

c) Ax

$$\begin{bmatrix}1 & 2\\-1 & 1\end{bmatrix} \cdot \begin{bmatrix}1\\0\end{bmatrix} = \boxed{\begin{bmatrix}1\\-1\end{bmatrix}}$$

A6

d) $B(x-y)$

$$x-y = \begin{bmatrix}1\\0\end{bmatrix} - \begin{bmatrix}0\\1\end{bmatrix} = \begin{bmatrix}1\\-1\end{bmatrix}$$

$$A \cdot (x-y) = \begin{bmatrix}1 & 2\\-1 & 1\end{bmatrix}\begin{bmatrix}1\\-1\end{bmatrix} = \begin{bmatrix}1-2\\-1-1\end{bmatrix} = \boxed{\begin{bmatrix}-1\\-2\end{bmatrix}}$$

A7

e) Dx

$$\begin{bmatrix}1 & 2\\2 & 3\\-1 & 0\end{bmatrix}\begin{bmatrix}1\\0\end{bmatrix} = \boxed{\begin{bmatrix}1\\2\\-1\end{bmatrix}}$$

A8

3×2 $\quad 2\times1$ $\quad 3\times1$

f) $Dy + z$

$$\begin{bmatrix}1 & 2\\2 & 3\\-1 & 0\end{bmatrix}\begin{bmatrix}0\\1\end{bmatrix} + \begin{bmatrix}1\\2\\-1\end{bmatrix} = \begin{bmatrix}2\\3\\0\end{bmatrix} + \begin{bmatrix}1\\2\\-1\end{bmatrix} = \boxed{\begin{bmatrix}3\\5\\-1\end{bmatrix}}$$

A9

g) AB

$$\begin{bmatrix}1 & 2\\-1 & 1\end{bmatrix}\begin{bmatrix}2 & 0\\0 & 2\end{bmatrix} = \boxed{\begin{bmatrix}2 & 4\\-2 & 2\end{bmatrix}}$$

A10

h) BC

$$\begin{bmatrix}2 & 0\\0 & 2\end{bmatrix}\begin{bmatrix}2 & 0 & -3\\0 & 0 & -1\end{bmatrix} = \boxed{\begin{bmatrix}4 & 0 & -6\\0 & 0 & -2\end{bmatrix}}$$

A11

2×2 $\quad 2\times3$ $\quad 2\times3$

i) CD

$$\begin{bmatrix}2 & 0 & -3\\0 & 0 & -1\end{bmatrix}\begin{bmatrix}1 & 2\\2 & 3\\-1 & 0\end{bmatrix} = \boxed{\begin{bmatrix}5 & 4\\1 & 0\end{bmatrix}}$$

A12

2×3 $\quad 3\times2$ $\quad 2\times2$