Runtime Analysis

a)

```
void f1(int n)
{
    int i=2;
    while (i<n){
        i = i*i;
    }
}
```

$$\sum_i^n \theta\left(\frac{1}{i}\right) = \theta(\log n)$$

b)

```
void f2(int n)
{
    for i=1 until $i \leq n$
    {
        if ( i % (int) sqrt(n)) == 0 )
        {
            for k=0 until $k < i^3$
                O(1)
        }
    }
}
```

ex n = 9

{if} triggers for i = 3, 6, 9

n = 16

{if} triggers for i = 4, 8, 12, 16

triggers $\sqrt{n}$ times

$$\sum_{i=1}^{n}\left[\theta(1) + 0\left(\sum_{j=0}^{n^3-1}\theta(1)\right)\right]$$

$$T(n) = \cancel{\sum_{i=1}^{n}\theta}(1) + \sum_i^{\sqrt{n}}\sum_{j=0}^{n^3}\theta(1) \Rightarrow n^{\frac{1}{2}+3} = n^{7/2}$$

$$= \theta(n^{7/2})$$

c)

```
for int i until $i \leq n$
{
    for int k until $k \leq n$
    {
        if (A[k]==i)
        {
            for int m until $m \leq n$, m doubles
            {
                O(1) action
            }
        }
    }
}
```

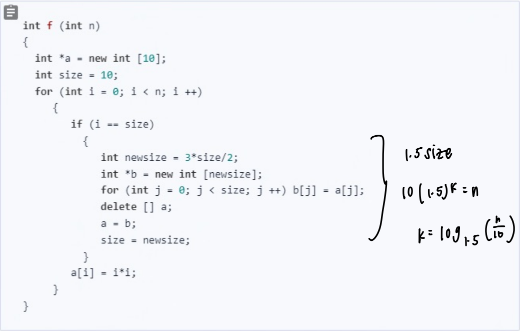
$$m = 2^x \rightarrow 1, 2, 4, 8, 16, 32 \ldots$$

$$\sum_{i=1}^{n}\sum_{j=1}^{n}\theta(1) + \sum_{i=1}^{n}\sum_{k=1}^{\log_2 n}\theta(1)$$

$$= \theta(n^2) + \cancel{\theta(n} \log n)$$

$$= \theta(n^2)$$

d)



```
int f (int n)
{
  int *a = new int [10];
  int size = 10;
  for (int i = 0; i < n; i ++)
    {
      if (i == size)
        {
          int newsize = 3*size/2;
          int *b = new int [newsize];
          for (int j = 0; j < size; j ++) b[j] = a[j];
          delete [] a;
          a = b;
          size = newsize;
        }
      a[i] = i*i;
    }
}
```

1.5 size

$10(1.5)^k = n$

$k = \log_{1.5}\left(\frac{n}{10}\right)$

$$\sum_i^n\left[\theta(1) + \sum \theta\,\text{size}\right]$$

$$\downarrow$$

$$\sum_i^n[\theta(1)] + \sum_k^{\log_{1.5}\left(\frac{n}{10}\right)} \theta\left(10(1.5)^k\right)$$

$$\downarrow$$

$$\theta(n) + \cancel{\theta\left(10(1.5)^k\right)}$$

$$= \theta(n)$$