Finding a sample size and confidence intervals for small sample sizes

Finding a sample size

- We are in sections 8.1, 8.2 of the textbook.
- The first topic of this section is how to find the sample size needed to achieve a certain confidence interval.
- The general formula for sample size is

$$n = \left(\frac{z_c \sigma}{E}\right)^2$$

When the formula gives a fractional number, round it up to the next whole number.

1. A population has a standard deviation of 12. How large of a sample size do we need to find a margin of error E of 3? Use a 90% confidence level. Use the formula

$$n = \left(\frac{z_c \sigma}{E}\right)^2$$

2. A population has a standard deviation of 20. How large of a sample size do we need to find a margin of error E of 1.5? Use a 95% confidence level. Use the formula

$$n = \left(\frac{z_c \sigma}{E}\right)^2$$

Confidence intervals for small sample sizes

- We are in parts 7.1 and 7.2 of the textbook.
- We see how to change the procedure to find confidence intervals when the sample size is small.
- We use the sample standard deviation s for the population standard deviation σ.
- Instead of finding a critical number z_c, we find a critical number t_c from "Student's t distribution".

3. Suppose we have a sample of size 15 with a sample standard deviation of 5 and a sample average of 60. We will use the "t distribution" to find the margin of error E and the confidence interval. We will use a 95% confidence level.

   (a) First, we find the *degrees of freedom*, which is $n-1$. In this case, the degrees of freedom is:

   (b) Next, we find the value $t_c = t_{0.95}$ from a new table. The table is on page 4 of this handout. We use the degrees of freedom that we just calculated.

   (c) We use the formula $E = t_c \frac{s}{\sqrt{n}}$ to find the margin of error.

   (d) We use the formula $(\bar{x}-E, \bar{x}+E)$ to find the confidence interval.

   (e) Repeat the steps above to find the 99% confidence interval.

4. Suppose we have a sample of size 20 with a sample standard deviation of 18 and a sample average of 125. We will use the "t distribution" to find the margin of error E and the confidence interval. We will use a 90% confidence level.

   (a) First, we find the *degrees of freedom*, which is $n-1$. In this case, the degrees of freedom is:

   (b) Next, we find the value $t_c = t_{0.90}$ from the new table. We use the degrees of freedom that we just calculated.

   (c) We use the formula $E = t_c \frac{s}{\sqrt{n}}$ to find the margin of error.

   (d) We use the formula $(\bar{x} - E, \bar{x} + E)$ to find the confidence interval.

   (e) Repeat the steps above to find the 95% confidence interval.

TABLE 6 **Critical Values for Student's *t* Distribution**

one-tail area	0.250	0.125	0.100	0.075	0.050	0.025	0.010	0.005	0.0005
two-tail area	0.500	0.250	0.200	0.150	0.100	0.050	0.020	0.010	0.0010
d.f. \ *c*	0.500	0.750	0.800	0.850	0.900	0.950	0.980	0.990	0.999
1	1.000	2.414	3.078	4.165	6.314	12.706	31.821	63.657	636.619
2	0.816	1.604	1.886	2.282	2.920	4.303	6.965	9.925	31.599
3	0.765	1.423	1.638	1.924	2.353	3.182	4.541	5.841	12.924
4	0.741	1.344	1.533	1.778	2.132	2.776	3.747	4.604	8.610
5	0.727	1.301	1.476	1.699	2.015	2.571	3.365	4.032	6.869
6	0.718	1.273	1.440	1.650	1.943	2.447	3.143	3.707	5.959
7	0.711	1.254	1.415	1.617	1.895	2.365	2.998	3.499	5.408
8	0.706	1.240	1.397	1.592	1.860	2.306	2.896	3.355	5.041
9	0.703	1.230	1.383	1.574	1.833	2.262	2.821	3.250	4.781
10	0.700	1.221	1.372	1.559	1.812	2.228	2.764	3.169	4.587
11	0.697	1.214	1.363	1.548	1.796	2.201	2.718	3.106	4.437
12	0.695	1.209	1.356	1.538	1.782	2.179	2.681	3.055	4.318
13	0.694	1.204	1.350	1.530	1.771	2.160	2.650	3.012	4.221
14	0.692	1.200	1.345	1.523	1.761	2.145	2.624	2.977	4.140
15	0.691	1.197	1.341	1.517	1.753	2.131	2.602	2.947	4.073
16	0.690	1.194	1.337	1.512	1.746	2.120	2.583	2.921	4.015
17	0.689	1.191	1.333	1.508	1.740	2.110	2.567	2.898	3.965
18	0.688	1.189	1.330	1.504	1.734	2.101	2.552	2.878	3.922
19	0.688	1.187	1.328	1.500	1.729	2.093	2.539	2.861	3.883
20	0.687	1.185	1.325	1.497	1.725	2.086	2.528	2.845	3.850
21	0.686	1.183	1.323	1.494	1.721	2.080	2.518	2.831	3.819
22	0.686	1.182	1.321	1.492	1.717	2.074	2.508	2.819	3.792
23	0.685	1.180	1.319	1.489	1.714	2.069	2.500	2.807	3.768
24	0.685	1.179	1.318	1.487	1.711	2.064	2.492	2.797	3.745
25	0.684	1.198	1.316	1.485	1.708	2.060	2.485	2.787	3.725
26	0.684	1.177	1.315	1.483	1.706	2.056	2.479	2.779	3.707
27	0.684	1.176	1.314	1.482	1.703	2.052	2.473	2.771	3.690
28	0.683	1.175	1.313	1.480	1.701	2.048	2.467	2.763	3.674
29	0.683	1.174	1.311	1.479	1.699	2.045	2.462	2.756	3.659
30	0.683	1.173	1.310	1.477	1.697	2.042	2.457	2.750	3.646
35	0.682	1.170	1.306	1.472	1.690	2.030	2.438	2.724	3.591
40	0.681	1.167	1.303	1.468	1.684	2.021	2.423	2.704	3.551
45	0.680	1.165	1.301	1.465	1.679	2.014	2.412	2.690	3.520
50	0.679	1.164	1.299	1.462	1.676	2.009	2.403	2.678	3.496
60	0.679	1.162	1.296	1.458	1.671	2.000	2.390	2.660	3.460
70	0.678	1.160	1.294	1.456	1.667	1.994	2.381	2.648	3.435
80	0.678	1.159	1.292	1.453	1.664	1.990	2.374	2.639	3.416
100	0.677	1.157	1.290	1.451	1.660	1.984	2.364	2.626	3.390
500	0.675	1.152	1.283	1.442	1.648	1.965	2.334	2.586	3.310
1000	0.675	1.151	1.282	1.441	1.646	1.962	2.330	2.581	3.300
∞	0.674	1.150	1.282	1.440	1.645	1.960	2.326	2.576	3.291

For degrees of freedom *d.f.* not in the table, use the closest *d.f.* that is *smaller*.