Estimating a proportion

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- We are in section 8.3 of the textbook
- Suppose we want to estimate the population proportion using the sample proportion.
- The sample proportion
$$\hat{p} = \frac{r}{n}$$
is the point estimate for the population proportion.
- **Rule of thumb:** this method is reliable as long as n is large enough so that $np > 5$ and $n(1-p) > 5$.
- First, we choose a confidence level, usually 95% or 99%. We find the corresponding **critical value** using the a z-table or the chart below.

Confidence Interval Critical Values z_c

Level of Critical Confidence c	Value of z_c
0.70, or 70%	1.04
0.75, or 75%	1.15
0.80, or 80%	1.28
0.85, or 85%	1.44
0.90, or 90%	1.645
0.95, or 95%	1.96
0.98, or 98%	2.33
0.99, or 99%	2.58

- We find the **margin of error**, E, using the formula
$$E = z_c \left(\frac{\sqrt{p(1-p)}}{\sqrt{n}} \right)$$
- If the sample proportion is $\hat{p}$, the confidence interval is $(\hat{p} - E, \hat{p} + E)$.
- The meaning of the confidence interval is: if we repeatedly took random samples of size n, we would get a list of different confidence intervals. Every sample would have a different confidence interval with the same width. The confidence level tells us how often the confidence interval we get would contain the actual population proportion.

1. Suppose that we know in 500 trials there were 421 successes. What can we say about the actual proportion (percentage) of successes, which we don't know?

   (a) What is the sample proportion of successes?

   (b) What is the standard deviation of our sampling distribution? Use the formula $\frac{\sqrt{p(1-p)}}{\sqrt{n}}$.

   (c) What is the critical value z_c for 95% confidence? Use the table on page 1.

   (d) What is the margin of error E of our 95% confidence interval, E? Use the formula

   $$E = z_c \left(\frac{\sqrt{p(1-p)}}{\sqrt{n}} \right)$$

   (e) What is our confidence interval for 95% confidence? Use the formula $(\hat{p} - E, \hat{p} + E)$.

2. Suppose that we know in $1,050$ trials there were 160 successes. What can we say about the actual proportion (percentage) of successes, which we don't know?

   (a) What is the sample proportion of successes?

   (b) What is the standard deviation of our sampling distribution? Use the formula $\frac{\sqrt{p(1-p)}}{\sqrt{n}}$.

   (c) What is the critical value z_c for 90% confidence? Use the table on page 1.

   (d) What is the margin of error E of our 90% confidence interval, E? Use the formula

   $$E = z_c \left(\frac{\sqrt{p(1-p)}}{\sqrt{n}} \right)$$

   (e) What is our confidence interval for 90% confidence? Use the formula $(\hat{p} - E, \hat{p} + E)$.

3. Suppose that we know in 80 trials there were 64 successes. What can we say about the actual proportion (percentage) of successes, which we don't know?

   (a) What is the sample proportion of successes?

   (b) What is the standard deviation of our sampling distribution? Use the formula $\frac{\sqrt{p(1-p)}}{\sqrt{n}}$.

   (c) What is the critical value z_c for 99% confidence? Use the table on page 1.

   (d) What is the margin of error E of our 99% confidence interval, E? Use the formula

   $$E = z_c \left(\frac{\sqrt{p(1-p)}}{\sqrt{n}} \right)$$

   (e) What is our confidence interval for 99% confidence? Use the formula $(\hat{p} - E, \hat{p} + E)$.