Probability Rules

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- We are in section 4.2 of the textbook.
- **Basic facts about probability:** The total probability of the entire sample space is 1. For any event A, $0 \leq P(A) \leq 1$.
- Two events are *independent* if the probability of each one occurring does not depend on whether the other one occurred.
  - Say we roll a die and also draw a card from a deck. The results of each part are independent of the result of the other part.
  - Say we draw a card from a deck, put it back, shuffle, and draw a second card. These are independent, because the result of the first draw does not affect the result of the second draw.
  - Say we draw a card and then draw a second card without putting the first one back. These are not independent, because when we draw the second card there are only 51 possibilities, which depend on the first card.
- **Multiplication rule for independent events:** If A and B are independent events then $P(A \text{ and } B) = P(A) \times P(B)$.
- Two events are *mutually exclusive* if they cannot both occur at the same time. For example, rolling a 5 and rolling an even number are mutually exclusive events.
- **Addition rule for mutually exclusive events:** If A and B are mutually exclusive events, $P(A \text{ or } B) = P(A) + P(B)$.
- **General addition rule:** For any events A and B, $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$.
- The *complement* of an event is the collection of outcomes that are *not* in the original event. The complement of an event A is written A^c.
- **Complement rule**. For any event A, $P(A^c) = 1 - P(A)$. This means $P(A) + P(A^c) = 1$.

1. Suppose that I select one student at random from Marshall's undergraduate students. Here are two events:

   F = person is a freshman
   B = person's major is in the College of Business

   Write a symbolic form for each of these:

   (a) The probability the person is a freshman.

   (b) The probability the person is not a freshman.

   (c) The probability the person a freshman with a major in the College of Business.

   (d) The probability the person a freshman or has a major in the College of Business.

   (e) The probability the person is not a freshman, but does have a major in the College of Buisness.

   (f) Do we expect the events F and B to be independent? Explain.

   (g) Do we expect the events F and B to be mutually exclusive? Explain.

2. Suppose that I have two events A such that $P(A) = 0.4$, $P(B) = 0.6$, and $P(A \text{ and } B) = 0.1$. What is $P(A \text{ or } B)$?

3. Suppose that I have two events A such that $P(A) = 0.6$, $P(B) = 0.8$, and $P(A \text{ or } B) = 0.9$. What is $P(A \text{ and } B)$?

4. Suppose that I have two events A such that $P(A) = 0.7$ and $P(B) = 0.5$.

   (a) What is the smallest possible value for $P(A \text{ and } B)$?

   (b) How large could $P(A \text{ and } B)$ possibly be?

5. I draw a card from a deck of 52 cards. What is the probability that I get a red card or a Jack? The "or" here allows for both to happen. Solve the problem using the general addition rule.

6. I draw a card from a fair deck, put it back and reshuffle, and then draw a card again.

   (a) What is the probability that I draw a Jack the first time, and a Queen the second time?

   (b) What is the probability that I draw a Jack the first time or a Queen the second time?

   (c) What is the probability that I draw the same card both times?