Expected Value

Expected Value of a Discrete Variable

- We are in section 5.1 of the textbook.

- A *discrete random variable* is a random process that has a finite number of outcomes, each of which is a number.

    - Roll a die and record the number that comes up
    - Select a card at random and record its value
    - For each class you take, record your grade earned

- Each outcome of the random variable has its own probability. The probabilities have to add up to 1. The probabilities may or may not all be the same. The collection of all the outcomes and their probabilities is called a *discrete probability distribution*.

- The *expected value* is a weighted average of all the outcomes:

    - Multiply each outcome by its probability
    - Add up these products
    - The result is the expected value.
    - We use the notation $E(x)$ for the expected value of a random variable x.

- The formula for the expected value of a discrete random variable x is $\sum x P(x)$.

- The expected value is what we would get if we measured the random variable many times and then averaged our results.

---

1. What is the expected value of the number that comes up when we roll a die?

    (a) Write out the outcomes and their probabilities.

x	1	2	3	4	5	6
$P(x)$						

    (b) Multiply each outcome by its probability and add them up. This is the expected value.

    (c) This means that if I roll a die many times and average the numbers I get, the average will be about _____.

---

2. I put 15 blue balls and 5 red balls into a bag. The blue balls are worth $1.00 and the red balls are worth $5.00. If I draw a ball at random, what is the expected value of the worth of the ball?

   (a) Write out the outcomes and their probabilities.

Color		Blue	Red
Value	x		
Probability	$P(x)$		

   (b) Multiply each outcome by its probability and add them up. This is the expected value.

   (c) This does not mean that any of the balls is exactly worth _____. The expected value is just the average value that I will see if I repeat the experiment many times.

3. I put 10 red balls, 5 blue balls, and 1 yellow ball into a bag. The red balls are worth $1, the blue balls are worth $4, and the yellow ball is worth $50.

   (a) Write out the outcomes and their probabilities.

Color		
Value	x	
Probability	$P(x)$	

   (b) If I draw a ball at random, what is the expected value of the worth of the ball?

4. Determine if each of the following is a valid probability distribution. If it is a valid distribution, find the expected value.

(a)

a	5	10	15
$P(a)$	0.1	0.5	0.15

(b)

b	-1	0	1
$P(b)$	0.25	0.45	0.3

5. Suppose that we play a card game. I shuffle the deck and then you draw one card. If you draw an ace, you win $20. If you don't draw an ace, you don't win anything.

   (a) What is the probability of drawing an ace?

   (b) What is the probability of not drawing an ace?

   (c) What is the expected value of your winnings?

   (d) Suppose that you have to pay $2 to play the game. If you could play it over and over, would you expect to come out ahead? Why or why not?

   (e) Suppose that you only have to pay $1 to play the game. If you could play it over and over at that price, would you expect to come out ahead? Why or why not?

6. A credit hour is a way of measuring how much credit you receive for attending a course; typically this is related to the face-to-face contact hours of the course. You will earn a grade for each course you attend, which is converted to a grade point. At Marshall the grades and grade points are as follows.

Grade earned	Grade point per credit hour
A	4
B	3
C	2
D	1
F	0

Your grade point average (GPA) is computed as a weighted average of grade points for all of the courses you took (weighted by credit hour). Compute the grade points for a class

$$(\text{course credit hours}) * (\text{grade point per credit hour})$$

then divide by the total number of credit hours.

A first-year student at Marshall takes the courses and earns the grades listed below.

Course	Credit hours	Grade	Grade points
Foundations of statistics	3	B	
Found. of stats. lab	1	A	
Principles of selling	3	A	
Calculus 1	5	C	
First-year seminar	3	B	

(a) Compute the GPA for this student.

(b) Rewrite the table of courses and grade points in the form of a probability distribution.

Grade		A	B	C	D	F
Grade point	x					
Probability	$P(x)$					

(c) Use the probability distribution to compute the GPA for this student.