Hypothesis testing when σ is unknown

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- We are in sections 9.5, 9.6 of the textbook
- Sometimes, our sample size is not very large, or we don't know the population standard deviation.
- In this case, we use a t-test instead of a z-test. We use the t table instead of the z table.
- We again use the concept of *degrees of freedom*. The number of degrees of freedom is always $n-1$.
- Compute the t-value using the formula

$$t = (\bar{x} - \mu) / \left(\frac{s}{\sqrt{n}} \right)$$

- Identify the critical value t_c for your level of significance and alternative hypothesis.
- Compare t to t_c.
  - For a two-sided test, accept the null hypothesis if $-t_c < t < t_c$
  - For a right-sided test, accept the null hypothesis if $t < t_c$
  - For a left-sided test, accept the null hypothesis if $t > -t_c$

1. Suppose that we have a sample with $n = 16$. The sample mean is 80 and the sample standard deviation is 8. We want to test the hypothesis

$$H_0 : \mu = 85 \qquad H_1 : \mu < 85$$

   (a) Find the value of t.

   (b) Find the appropriate critical value t_c with a t table for level of significance $\alpha = 0.05$. Use one-sided area because the alternative hypothesis is one-sided.

   (c) In this case, can we reject the null hypothesis?

2. A particular patient should be on blood pressure medicine if their systolic blood pressure is above 140. The following five measurements are made of their blood pressure on different days:

   138 143 141 143 141

   We want to decide if these are enough to conclude their average systolic blood pressure is above 140.

   (a) Let B stand for their blood pressure. Write down the null hypothesis and alternative hypothesis.

   (b) Should we use a t test or z test? Why?

   (c) Find the appropriate t value,

   $$t = (\bar{x} - \mu) / \left(\frac{s}{\sqrt{n}} \right)$$

   You will need to calculate the sample mean, sample standard deviation, and degrees of freedom. You will also need to determine if you need the one-tail area or the two-tail area.

   (d) Find the appropriate critical value t_c with a t table for a 10% level of significance.

   (e) If you had to make a decision based on this, what would your decision be?

3. The element arsenic can be present in ground water in some areas of the country. High levels of arsenic are poisonous, so water is often tested to make sure it is safe. A level of 8 parts per billion is considered acceptable for water that will be used for irrigation.

   A particular well is tested regularly for arsenic levels. Over 35 tests, the sample mean was 7.3 parts per billion, with a sample standard deviation of $s = 1.6$ parts per billion.

   Formulate and apply a hypothesis test to decide if we can be confident that the arsenic levels in this well are less than 8 parts per billion. Use a 1% level of significance in your work.

TABLE 6 **Critical Values for Student's *t* Distribution**

one-tail area	0.250	0.125	0.100	0.075	0.050	0.025	0.010	0.005	0.0005
two-tail area	0.500	0.250	0.200	0.150	0.100	0.050	0.020	0.010	0.0010
d.f. \ *c*	0.500	0.750	0.800	0.850	0.900	0.950	0.980	0.990	0.999
1	1.000	2.414	3.078	4.165	6.314	12.706	31.821	63.657	636.619
2	0.816	1.604	1.886	2.282	2.920	4.303	6.965	9.925	31.599
3	0.765	1.423	1.638	1.924	2.353	3.182	4.541	5.841	12.924
4	0.741	1.344	1.533	1.778	2.132	2.776	3.747	4.604	8.610
5	0.727	1.301	1.476	1.699	2.015	2.571	3.365	4.032	6.869
6	0.718	1.273	1.440	1.650	1.943	2.447	3.143	3.707	5.959
7	0.711	1.254	1.415	1.617	1.895	2.365	2.998	3.499	5.408
8	0.706	1.240	1.397	1.592	1.860	2.306	2.896	3.355	5.041
9	0.703	1.230	1.383	1.574	1.833	2.262	2.821	3.250	4.781
10	0.700	1.221	1.372	1.559	1.812	2.228	2.764	3.169	4.587
11	0.697	1.214	1.363	1.548	1.796	2.201	2.718	3.106	4.437
12	0.695	1.209	1.356	1.538	1.782	2.179	2.681	3.055	4.318
13	0.694	1.204	1.350	1.530	1.771	2.160	2.650	3.012	4.221
14	0.692	1.200	1.345	1.523	1.761	2.145	2.624	2.977	4.140
15	0.691	1.197	1.341	1.517	1.753	2.131	2.602	2.947	4.073
16	0.690	1.194	1.337	1.512	1.746	2.120	2.583	2.921	4.015
17	0.689	1.191	1.333	1.508	1.740	2.110	2.567	2.898	3.965
18	0.688	1.189	1.330	1.504	1.734	2.101	2.552	2.878	3.922
19	0.688	1.187	1.328	1.500	1.729	2.093	2.539	2.861	3.883
20	0.687	1.185	1.325	1.497	1.725	2.086	2.528	2.845	3.850
21	0.686	1.183	1.323	1.494	1.721	2.080	2.518	2.831	3.819
22	0.686	1.182	1.321	1.492	1.717	2.074	2.508	2.819	3.792
23	0.685	1.180	1.319	1.489	1.714	2.069	2.500	2.807	3.768
24	0.685	1.179	1.318	1.487	1.711	2.064	2.492	2.797	3.745
25	0.684	1.198	1.316	1.485	1.708	2.060	2.485	2.787	3.725
26	0.684	1.177	1.315	1.483	1.706	2.056	2.479	2.779	3.707
27	0.684	1.176	1.314	1.482	1.703	2.052	2.473	2.771	3.690
28	0.683	1.175	1.313	1.480	1.701	2.048	2.467	2.763	3.674
29	0.683	1.174	1.311	1.479	1.699	2.045	2.462	2.756	3.659
30	0.683	1.173	1.310	1.477	1.697	2.042	2.457	2.750	3.646
35	0.682	1.170	1.306	1.472	1.690	2.030	2.438	2.724	3.591
40	0.681	1.167	1.303	1.468	1.684	2.021	2.423	2.704	3.551
45	0.680	1.165	1.301	1.465	1.679	2.014	2.412	2.690	3.520
50	0.679	1.164	1.299	1.462	1.676	2.009	2.403	2.678	3.496
60	0.679	1.162	1.296	1.458	1.671	2.000	2.390	2.660	3.460
70	0.678	1.160	1.294	1.456	1.667	1.994	2.381	2.648	3.435
80	0.678	1.159	1.292	1.453	1.664	1.990	2.374	2.639	3.416
100	0.677	1.157	1.290	1.451	1.660	1.984	2.364	2.626	3.390
500	0.675	1.152	1.283	1.442	1.648	1.965	2.334	2.586	3.310
1000	0.675	1.151	1.282	1.441	1.646	1.962	2.330	2.581	3.300
∞	0.674	1.150	1.282	1.440	1.645	1.960	2.326	2.576	3.291

For degrees of freedom *d.f.* not in the table, use the closest *d.f.* that is *smaller*.