Probability

> **Probability**
>
> - A *random experiment* is a procedure to obtain one *outcome* from a collection of *possible outcomes*.
>
>   - Flip a coin to get Heads or Tails
>   - Roll a die to get a number from 1 to 6
>   - Draw a card from a deck of cards
>
> - An *event* is a collection of outcomes that we are interested in. For example, we might be interested in drawing a Heart or rolling a 5.
>
> - A *probability* is a number between 0 and 1 that tells how often an event will happen if we perform a random experiment many times.
>
>   - Probability 0 means the event never happens.
>   - Probability 1 means that the event always happens.
>   - Probabilities closer to 1 mean the event happens more often
>   - Probabilities closer to 0 mean the event happens less often
>
> - The **general probability formula:**
>
> $$\text{Probability of an event} = \frac{\text{Number of outcomes in the event}}{\text{Total number of possible outcomes}}.$$
>
> - **How to write probabilities** You can write probabilities as fractions, decimals, or percentages. It is best to write them as fractions in this class, because it is easier for me to give partial credit.

1. Suppose that we roll a single 6 sided die.

   (a) What is the probability that we get a 4?

   (b) What is the probability that we get an even number?

   (c) What is the probability that we get a 3 or a 5?

2. We perform a random experiment by flipping a fair coin 3 times.

    - Write out all the possible outcomes. There will be 8 of them.

    - What is the probability that we get three heads?

    - What is the probability that we get exactly two heads?

    - What is the probability that we get exactly one heads?

    - What is the probability that we get no heads?

3. A lottery machine has small balls numbered from 1 to 20. It selects one ball, and all the balls have the same chance of being chosen

    (a) What is the probability that we get a 9?

    (b) What is the probability that we get an even number?

    (c) What is the probability that we get a number that is strictly smaller than 5?

4. We roll two six sided dice and keep track of the total that we see.

	Second					
First	1	2	3	4	5	6
1						
2						
3						
4						
5						
6						

(a) What is the probability that the total is 3?

(b) What is the probability that the total is 6?

(c) What outcome or outcomes are most likely?

(d) What outcome or outcomes are least likely?

> **A standard deck of cards**
> - A standard American deck of cards has 52 cards.
> - There are 4 suits: spades ♠, clubs ♣, hearts ♡, and diamonds ♢.
> - There are 13 face values: 2 through 10, Jack, Queen, King, and Ace.
> - There is exactly one card for each combination of suit and face value.
> - There are 13 cards of each suit (one of each face value) and 4 cards of each face value (one of each suit).

5. We draw one card at random from a standard deck of cards. All the cards have the same chance of being drawn.

   (a) What is the probability we draw a red card?

   (b) What is the probability we draw a heart?

   (c) What is the probability we draw a card that is a Jack?

   (d) What is the probability we draw a card that is either a Jack, a heart, or both?

   (e) What is the probability that we draw a black Queen?

> **Multiplication principle for independent experiments**
> Suppose we perform an experiment E that consists of two smaller experiments E_1 and E_2 that do not affect each other. The experiment E is called a *compound experiment*.
>
> Then the number of outcomes of E is the product of the number of outcomes of E_1 and the number of outcomes of E_2.
>
> If E_1 has n_1 outcomes and E_2 has n_2 outcomes then E has $n_1 \times n_2$ outcomes.

6. Suppose that we roll a 6 sided die and also flip a fair coin.

    (a) What is the total number of possible outcomes?

    (b) What is the probability that we flip Heads and also roll a 4?

    (c) What is the probability that we flip Heads and also roll an odd number?

7. Suppose that we roll a 6 sided die and also draw a card from a standard deck of cards.

    (a) What is the total number of possible events?

    (b) What is the probability that we roll a 3 and also draw a spade?

    (c) What is the probability that we roll an even number and draw a Queen?

8. We will perform an experiment with dice. You will roll two dice 50 times. Keep track of how often each individual number comes up, and how often each sum comes up.

Individual dice	
1	
2	
3	
4	
5	
6	

Sum of two dice	
2	
3	
4	
5	
6	
7	
8	
9	
10	
11	
12	