Conditional Probability

Probability Rules

- We are in sections 3.1, 3.4 of the textbook.

- A *conditional probability* is the probability that one event will happen *given that* we know another event happened.

- For example, the probability of rolling a 5 on a die is $1/6$. But, if we know the result was odd, the conditional probability of getting a 5 given that the result was odd is $1/3$.

- We write $P(A|B)$ for the conditional probability of event A given that event B happened.

- We have the formula
$$P(A|B) = \frac{P(A \text{ and } B)}{P(B)}.$$

1. Suppose that we roll a die and get an odd number.

   (a) Write down the possible outcomes in this event (getting an odd number).

   (b) Find the conditional probability of getting a 5, given that we got an odd number.

   (c) If the event of rolling an odd number is O and the event of getting a five is T, write the previous probability with P notation.

   (d) Find the conditional probability of getting a 4, given that we got an odd number.

2. We draw one card from a standard deck.

   (a) What is the probability of getting a queen?

   (b) Suppose that we know we got a face card. Write down all the possible outcomes for this event.

   (c) What is the conditional probability that we got a queen, given that we know we got a face card?

   (d) If the event of getting a face card is F and the event of getting a queen is Q, write the previous probability with P notation.

3. Suppose that we know $P(A) = 0.4$, $P(B) = 0.5$, and $P(A \text{ and } B) = 0.2$.

  (a) Find $P(A \text{ or } B)$.

  (b) Find $P(A|B)$

4. Suppose that we know $P(X) = 0.7$, $P(Y) = 0.4$, and $P(X \text{ or } Y) = 0.8$.

  (a) Find $P(X \text{ and } Y)$.

  (b) Find $P(Y|X)$.

5. Suppose that we roll two dice.

  (a) What is the probability that the sum is 5?

  (b) Write out every outcome where at least one of the dice is a 1.

  (c) What is the probability that the sum is 5, given that at least one of the dice is a 1?

  (d) What is the probability that the sum is 5, given that the first die is a 1?

6. For each of these, say whether it is POSSIBLE or IMPOSSIBLE, and explain why in one or two sentences. In all these items, X and Y are events in a random experiment.

   (a) Is is possible to have $P(X|Y) > P(X)$?

   (b) Is it possible to have $P(X|Y) = 0$ and $P(X) > 0$?

   (c) Is it possible to have $P(X|Y) = 1$ and $P(X) < 1$?

7. A survey of 100 students asked whether each student is a smoker. The data is broken down by gender in the table below.

Do you smoke?

	Yes	No	Total
Male	17	42	59
Female	12	29	41
Total	29	71	100

(a) If a person is selected at random from these students, what is the probability that the person is a smoker?

(b) If a person is selected at random from these students, what is the probability that person is female?

(c) If a female is selected at random from these students, what is the probability that person is a smoker?

(d) If a male is selected at random from these students, what is the probability that person is a smoker?

(e) If a smoker is selected at random from these students, what is the probability that person is female?

(f) If a smoker is selected at random from these students, what is the probability that person is male?

8. Suppose that, out of a population of 10,000 cats, 2% have a rare disease. A test has been developed that is 95% accurate in testing for the disease. This means that a cat with the disease has a 95% chance of testing positive, and a cat without the disease has a 95% chance of testing negative.

    (a) Fill in the following table using the information above.

	Test positive	Test negative	Row total
Has disease			
Doesn't have disease			
Column total			**10,000**

    (b) What is the probability that a random cat from the population had a positive test result?

    (c) What is the conditional probability that a cat has the disease, given that it had a positive test result?

    (d) What is the conditional probability that a cat does not have the disease, given that it had a positive test result?

    (e) What is the conditional probability that a cat has the disease, given that it had a negative test result?

    (f) What is the conditional probability that a cat does not have the disease, given that it had a negative test result?