Estimating a mean when σ is known - Confidence intervals

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- We are in section 8.1 of the textbook.
- Suppose we want to estimate the average value of a population using the average value of a sample.
- We know the sample size is n and the population standard deviation is σ.
- **Rule of thumb:** this method is reliable as long as $n \geq 30$ or the population data has a normal distribution.
- First, we choose a confidence level, usually 95% or 99%. We find the corresponding **critical value** using the a z-table or the chart below.

Confidence Interval Critical Values z_c

Level of Critical Confidence c	Value of z_c
0.70, or 70%	1.04
0.75, or 75%	1.15
0.80, or 80%	1.28
0.85, or 85%	1.44
0.90, or 90%	1.645
0.95, or 95%	1.96
0.98, or 98%	2.33
0.99, or 99%	2.58

- We find the **margin of error**, E, using the formula

$$E = z_c\left(\frac{\sigma}{\sqrt{n}}\right)$$

- If the sample average is $\bar{x}$, the confidence interval is $(\bar{x}-E, \bar{x}+E)$.
- The meaning of the confidence interval is: if we repeatedly took random samples of size n, we would get a list of different confidence intervals. Every sample would have a different confidence interval with the same width. The confidence level tells us how often the confidence interval we get would contain the actual population average.

1. Suppose that we know a distribution is normal and has standard deviation 14. We take a sample of size 64, and our sample average is 70. What can we say about the actual mean, which we don't know?

   (a) What is the point estimate for the population mean?

   (b) What is the standard deviation of our sampling distribution? Use the formula $\frac{\sigma}{\sqrt{n}}$.

   (c) What is the critical value z_c for 95% confidence? Use the table on page 1.

   (d) What is the margin of error E of our 95% confidence interval, E? Use the formula

   $$E = z_c \left(\frac{\sigma}{\sqrt{n}} \right)$$

   (e) What is our confidence interval for 95% confidence? Use the formula $(\bar{x} - E, \bar{x} + E)$.

2. Suppose that we know a distribution is normal and has standard deviation 6. We take a sample of size 100, and our sample average is 93. What can we say about the actual mean, which we don't know?

   (a) What is the point estimate for the population mean?

   (b) What is the standard deviation of our sampling distribution? Use the formula $\frac{\sigma}{\sqrt{n}}$.

   (c) What is the critical value z_c for 99% confidence? Use the table on page 1.

   (d) What is the margin of error for our 99% confidence interval, E? Use the formula

   $$E = z_c \left(\frac{\sigma}{\sqrt{n}} \right)$$

   (e) What is our confidence interval for 99% confidence? Use the formula $(\bar{x} - E, \bar{x} + E)$.

3. Suppose that we know a distribution is normal and has standard deviation 20. We take a sample of size 80, and our sample average is 210. What can we say about the actual mean, which we don't know?

   (a) What is the point estimate for the population mean?

   (b) What is the standard deviation of our sampling distribution? Use the formula $\frac{\sigma}{\sqrt{n}}$.

   (c) What is the critical value z_c for 98% confidence? Use the table on page 1.

   (d) What is the margin of error for our 98% confidence interval, E? Use the formula

   $$E = z_c \left(\frac{\sigma}{\sqrt{n}} \right)$$

   (e) What is our confidence interval for 98% confidence? Use the formula $(\bar{x} - E, \bar{x} + E)$.