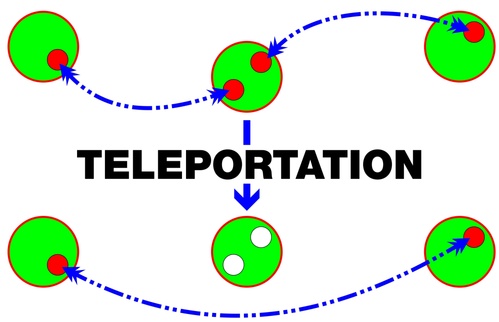

1 Overview

In the figure, the first green circle is the transmitting station, also referred to as endpoint 1. The red dot is the data to be transmitted, $|\Psi\rangle_1$. The transmitting station also holds the link qubit $|\Psi\rangle_A$. The middle green circle is the repeater, which holds the entangled states $|\Psi\rangle_{A'}$ and $|\Psi\rangle_{B'}$. The green circle on the right is endpoint two, $|\Psi\rangle_B$, the receiving station. The red dot there is the received qubit.

The sequence of events is as follows:

1. The transmitting station computes the data qubit
2. The transmitting station entangles its link qubits A and A'
3. The receiving station entangles its link qubits B and B'
4. The repeater measures the link qubits A' and B'
5. The transmitting station measures the the data qubit and its link qubit $|\Psi\rangle_A$
6. The transmitting station sends its measurement results to the receiving station via a non-quantum communication channel
7. The receiving station selects and applies a gate to its link qubit $|\Psi\rangle_B$, reconstructing the transmitted qubit.
8. The receiving station measures the reconstructed data qubit

2 OPENQASM Quantum Circuit Code

3 OPENQASM Quantum Circuit Code

```
OPENQASM 2.0;
include "qelib1.inc";

creg c[5];
qreg one[1];
qreg a[1];
qreg ap[1];
qreg bp[1];
qreg b[1];

gate entangle() q1,q2

x q1;
x q2;
h q1;
cx q1,q2;

gate bell_measure() q1,q2

x q2;
cx q1,q2;
x q2;
x q1;
h q1;
z q1;
```

```
entangle() a,ap;
entangle() bp,b;


”””” + data +
””””
bell_measure() ap,bp;
measure ap-¿c[2];
measure bp-¿c[3];

bell_measure() one,a;
measure one-¿c[0];
measure a-¿c[1];

if (c==1) z b;
if (c==2) x b;
if (c==3) y b;
if (c==4) z b;
if (c==6) y b;
if (c==7) x b;
if (c==8) x b;
if (c==9) y b;
if (c==11) z b;
if (c==12) y b;
if (c==13) x b;
if (c==14) z b;
measure b-¿c[4];
```

4 Dirac Notation

$\begin{bmatrix}1\\0\end{bmatrix} \rightarrow |u\rangle$ $\begin{bmatrix}0\\1\end{bmatrix} \rightarrow |d\rangle$

$\begin{bmatrix}1 & 0\end{bmatrix} \rightarrow \langle u|$

$\begin{bmatrix}1\\0\\0\\0\end{bmatrix} \rightarrow |uu\rangle$ $\begin{bmatrix}0\\1\\0\\0\end{bmatrix} \rightarrow |ud\rangle$ $\begin{bmatrix}0\\0\\1\\0\end{bmatrix} \rightarrow |du\rangle$ $\begin{bmatrix}0\\0\\0\\1\end{bmatrix} \rightarrow |dd\rangle$

For n qubits, if the $b_i, i = 0...n-1$ are the states of the qubits left-to-right, then the corresponding state vector has the j-th element equal to one where the binary representation of j is $b_{n-1}, b_{n-2}...b_0$

5 EPR/Bell States

$\Psi^+ = \sqrt{\frac{1}{2}}(|ud\rangle + |du\rangle)$

$\Psi^- = \sqrt{\frac{1}{2}}(|ud\rangle - |du\rangle)$ EPR singlet state

$|\Phi^+\rangle = \sqrt{\frac{1}{2}}(|uu\rangle + |dd\rangle)$

$|\Phi^-\rangle = \sqrt{\frac{1}{2}}(|uu\rangle - |dd\rangle)$

6 Measurement Gate

This is the unitary matrix for the Bell measurement, expressed in the computational basis.

$$U = \begin{bmatrix}\langle\Phi^+|\\ \langle\Phi^-|\\ \langle\Psi^+|\\ \langle\Psi^-|\end{bmatrix} = \begin{bmatrix}1 & 0 & 0 & 1\\ 1 & 0 & 0 & -1\\ 0 & 1 & 1 & 0\\ 0 & -1 & 1 & 0\end{bmatrix}$$

7 Entanglement Swapping

For technical reasons, in the code for the YouTube video, the sign of the second row of the matrix for the measurement gate, $\langle\Psi^-|$, is inverted in the code. The overall sign of a quantum state does not affect measurement results.
$|\Psi\rangle_{AA'B'B} = \frac{1}{2}(|ud\rangle - |du\rangle)(|ud\rangle - |du\rangle) = \frac{1}{2}[|uddu\rangle - |udud\rangle - |dudu\rangle + |duud\rangle]$

$|uu\rangle = \frac{1}{\sqrt{2}}(|\Phi^+\rangle + |\Phi^-\rangle)$
$|ud\rangle = \frac{1}{\sqrt{2}}(|\Psi^+\rangle + |\Psi^-\rangle)$
$|du\rangle = \frac{1}{\sqrt{2}}(|\Psi^+\rangle - |\Psi^-\rangle)$
$|dd\rangle = \frac{1}{\sqrt{2}}(|\Phi^+\rangle - |\Psi^-\rangle)$

$|\Psi\rangle_{AA'B'B} = \frac{1}{2\sqrt{2}}[|u\rangle (|\Phi^+\rangle - |\Phi^-\rangle) |u\rangle - |u\rangle (|\Psi^+\rangle - |\Psi^-\rangle) |d\rangle - |d\rangle (|\Psi^+\rangle + |\Psi^-\rangle) |u\rangle + |d\rangle (|\Phi^+\rangle + |\Phi^-\rangle) |d\rangle]$

$|\Psi\rangle_{A'B'AB} = \frac{1}{2\sqrt{2}}[|\Phi^+\rangle |uu\rangle - |\Phi^-\rangle |uu\rangle - |\Psi^+\rangle |ud\rangle + |\Psi^-\rangle |du\rangle + |\Phi^+\rangle |dd\rangle + |\Phi^-\rangle |dd\rangle]$
$= \frac{1}{2\sqrt{2}}[|\Psi^+\rangle (-|ud\rangle - |du\rangle) + |\Psi^-\rangle (|ud\rangle - |du\rangle) + |\Phi^+\rangle (|uu\rangle + |dd\rangle) + |\Phi^-\rangle (-|uu\rangle + |dd\rangle)]$

$= \frac{1}{2}[-|\Psi^+\rangle_{A'B'} |\Psi^+\rangle_{AB} + |\Psi^-\rangle_{A'B'} |\Psi^-\rangle_{AB} + |\Phi^+\rangle_{A'B'} |\Phi^+\rangle_{AB} - |\Phi^-\rangle_{A'B'} |\Phi^-\rangle_{AB}]$

7.1 Endpoint States after Repeater Measurement

The repeater performs a measurement on its entangled states. The possible outcomes and endpoint states are

Measure($A'B'$)	$\lvert\Psi\rangle_{AB}$
00	$-\lvert\Psi^+\rangle$
01	$\lvert\Psi^-\rangle$
10	$\lvert\Phi^+\rangle$
11	$-\lvert\Phi^-\rangle$

7.2 Teleportation of data qubit

The state of the data qubit is $\Psi_1 = a|u\rangle + b|d\rangle$

7.2.1 Swapped Entanglement

If $|\Psi\rangle_{AB} = -|\Psi^+\rangle$ then
$|\Psi\rangle_{1AB} = -\frac{1}{\sqrt{2}}(a|u\rangle + b|d\rangle)(|ud\rangle + |du\rangle) = \frac{1}{\sqrt{2}}[-a|uud\rangle - a|udu\rangle - b|dud\rangle - b|ddu\rangle]$
$=\frac{1}{2}[|\Psi^+\rangle (-a|u\rangle - b|d\rangle) + |\Psi^-\rangle (-a|u\rangle + b|d\rangle) + |\Phi^+\rangle (-a|d\rangle - b|u\rangle) + |\Phi^-\rangle (-a|d\rangle + b|u\rangle)]$

If $|\Psi\rangle_{AB} = |\Psi^-\rangle$ then
$|\Psi\rangle_{1AB} = \frac{1}{\sqrt{2}}(a|u\rangle + b|d\rangle)(|ud\rangle - |du\rangle) = a|uud\rangle - a|udu\rangle + b|dud\rangle - b|ddu\rangle$
$= \frac{1}{2}[a(|\Phi^+\rangle + |\Phi^-\rangle)|d\rangle - a(|\Psi^+\rangle - |\Psi^-\rangle)|u\rangle - b(|\Psi^+\rangle |\Psi^-\rangle)|d\rangle + b(|\Phi^+\rangle - |\Phi^-\rangle)|u\rangle]$
$= \frac{1}{2}[|\Psi^+\rangle (-a|u\rangle + b|d\rangle) + |\Psi^-\rangle (-a|u\rangle - b|d\rangle) + |\Phi^+\rangle (a|d\rangle - b|u\rangle) + |\Phi^-\rangle (a|d\rangle + b|u\rangle)]$

If $|\Psi\rangle_{AB} = |\Phi^+\rangle$ then
$|\Psi\rangle_{1AB} = \frac{1}{2}[|\Psi^+\rangle (a|d\rangle + b|u\rangle) + |\Psi^-\rangle (a|d\rangle - b|u\rangle) + |\Phi^+\rangle (a|u\rangle + b|d\rangle) + |\Phi^-\rangle (a|u\rangle - b|d\rangle)]$

If $|\Psi\rangle_{AB} = |\Phi^-\rangle$ then
$|\Psi\rangle_{1AB} = \frac{1}{2}[|\Psi^+\rangle (a|d\rangle - b|u\rangle) + |\Psi^-\rangle (a|d\rangle + b|u\rangle) + |\Phi^+\rangle (-a|u\rangle + b|d\rangle) + |\Phi^-\rangle (-a|u\rangle - b|d\rangle)]$

7.2.2 Coding Endpoint Two's Reconstruction of the Teleported Qubit

$$\sigma_x = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$$
$$\sigma_y = \begin{bmatrix} 0 & -i \\ i & 0 \end{bmatrix}$$
$$\sigma_z = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$$

Quantum registers in code:
$1AA'B'B$
01234

Measurement of $A'B'$

A'	B'	b_2	b_3	$\|\Psi\rangle$
0	0	0	0	$-\|\Psi^+\rangle$
0	1	0	1	$\|\Psi^-\rangle$
1	0	1	0	$\|\Phi^+\rangle$
1	1	1	1	$-\|\Phi^-\rangle$

Measurement of $1A$

1	A	b_0	b_1
0	0	0	0
0	1	0	1
1	0	1	0
1	1	1	1

Measurement Combinations

$m(A'B')$	$m(1A)$	Gate	$b_3b_2b_1b_0$	c
00	00	none	0	0
00	01	X	2	
00	10	Z	0001	1
00	11	Y	0011	3
01	00	X	1000	8
01	01	none	10	
01	10	Y	1001	9
01	11	Z	1011	11
10	00	Z	0100	4
10	01	Y	0110	6
10	10	none	0101	5
10	11	X	don't care	
11	00	Y	1100	12
11	01	Z	1110	14
11	10	X	don't care	
11	11	none	1111	15