1 Choosing the Gates

$|\Phi^+\rangle = \frac{1}{\sqrt{2}}(|00\rangle + |11\rangle)$
$|\Phi^-\rangle = \frac{1}{\sqrt{2}}(|00\rangle - |11\rangle)$
$|\Psi^+\rangle = \frac{1}{\sqrt{2}}(|01\rangle + |10\rangle)$
$|\Psi^-\rangle = \frac{1}{\sqrt{2}}(|01\rangle - |10\rangle)$
$|\Psi\rangle_{1AA'B'B} = |\Psi\rangle_1 |\Phi^+\rangle_{AA'} |\Phi^+\rangle_{BB'}$

$|\Psi\rangle_1$ input
A, A', B' link
B output

$|00\rangle = \frac{1}{\sqrt{2}}(|\Phi^+\rangle + |\Phi^-\rangle)$
$|01\rangle = \frac{1}{\sqrt{2}}(|\Psi^+\rangle + |\Psi^-\rangle)$
$|10\rangle = \frac{1}{\sqrt{2}}(|\Psi^+\rangle - |\Psi^-\rangle)$
$|11\rangle = \frac{1}{\sqrt{2}}(|\Phi^+\rangle - |\Phi^-\rangle)$

$|\Phi^+\rangle_{AA'} |\Phi^+\rangle_{BB'} = \frac{1}{2}(|00\rangle + |11\rangle)(|00\rangle + |11\rangle) = \frac{1}{2}(|0000\rangle + |0011\rangle + |1100\rangle + |1111\rangle)$

$= \frac{1}{2}[$
$|0\rangle (|\Phi^+\rangle + |\Phi^-\rangle) |0\rangle) +$
$|0\rangle (|\Psi^+\rangle + |\Psi^-\rangle) |1\rangle +$
$|1\rangle (|\Psi^+\rangle - |\Psi^-\rangle) |0\rangle +$
$|1\rangle (|\Phi^+\rangle - |\Phi^-\rangle) |1\rangle]$
Reorder the Hilbert spaces: $AA'B'B \to A'B'AB$
$|\Psi\rangle_{A'B'AB} = \frac{1}{2}[$
$|\Phi^+\rangle (|00\rangle + |11\rangle) +$
$|\Phi^-\rangle (|00\rangle - |11\rangle) +$
$|\Psi^+\rangle (|01\rangle + |10\rangle) +$
$|\Psi^-\rangle (|01\rangle - |10\rangle)]$

$|\Psi\rangle_{A'B'1AB} =$
$|\Phi^+\rangle (a|000\rangle + b|100\rangle + a|011\rangle + b|111\rangle) +$ other terms

$\boxed{|\Phi^+\rangle}$

$= \Phi^+[a(\Phi^+ + \Phi^-)|0\rangle + b(\Psi^+ - \Psi^-)|0\rangle + a(\Psi^+ + \Psi^-)|1\rangle + b(\Phi^+ - \Phi^-)|1\rangle]$
$a\Phi^+ |0\rangle$
$a\Phi^- |0\rangle$
$b\Psi^+ |0\rangle$
$-b\Psi^- |0\rangle$
$a\Psi^+ |1\rangle$
$a\Psi^- |1\rangle$
$b\Phi^+ |1\rangle$
$-b\Phi^- |1\rangle$

ϕ^+: $a|0\rangle + b|1\rangle$ no gate
Φ^-: $a|0\rangle - b|1\rangle$ Z gate
Ψ^+: $a|1\rangle + b|0\rangle$ X gate
Ψ^-: $a|1\rangle - b|0\rangle$ Y gate

$\boxed{|\Phi^-\rangle}$

Φ^+: $a|0\rangle - b|1\rangle$: Z gate

Φ^-: $a|0\rangle + b|1\rangle$: no gate

Ψ^+: $-a|1\rangle + b|0\rangle$: Y gate

Ψ^-: $-a|1\rangle - b|0\rangle$: X gate

$\boxed{|\Psi^+\rangle}$

Φ^+: $a\,|1\rangle + b\,|0\rangle$: X gate

Φ^-: $a\,|1\rangle - b\,|0\rangle$: Y gate

Ψ^+: $a\,|0\rangle + b\,|1\rangle$: no gate

$\Psi-$: $a\,|0\rangle - b\,|1\rangle$: Z gate

$\boxed{|\Psi^-\rangle}$

Φ^+: $a\,|1\rangle - b\,|0\rangle$: Y gate

Φ^-: $a\,|1\rangle + b\,|0\rangle$: X gate

Ψ^+: $-a\,|0\rangle + b\,|1\rangle$: Z gate

Ψ^-: $-a\,|0\rangle - b\,|1\rangle$: no gate