Two-Qubit Trapped Ion Gate

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Good evening and welcome to my channel. To night I will use the QuTiP quantum library to simulate a two-qubit gate as implemented in the Honey well quantum computer. The gate works by coupling the motions of two trapped ions to their internal electronic states in the Mølmer–Sørensen interaction under the influence of a bichromatic laser beam. The semiclassical Hamiltonian, using the rotating wave approximation in the interaction picture, is

 $H(t) \approx -\hbar\Omega(a^{\dagger}e^{i\epsilon t} + ae^{-i\epsilon t})S_y$ where ϵ is the difference between the photon beat frequency and the phonon frequency and S_y couples the ionic electron energy states: $S_y = \sigma_{1y} + \sigma_{2y}$

I will simulate gate infidelity by including in the Hamiltonian a term proportional to S_x and computing the trace distance between the expected and actual final states. If the ions are initially both in their excited states $|e\rangle$, the expected final state is $(|ee\rangle + i|gg\rangle)/\sqrt{2}$.

The Hamiltonian and states are given in the Wikipedia article here. The use of the trace distance to measure the difference between two quantum states is explained in Nielsen and Chuang, 10-th Anniversary Edition, chapter 9

Here is the Jupyter notebook. Cell 1 imports the necessary libraries, defines the global variables and creates the array of time steps for solving Schrodinger's equation. Cell 2 is a utility function for projecting the output combined states onto two-qubit states in the computational basis. Cell four sets of the time-dependence of the three terms in the Hamiltonian. Cell four creates the Hamiltonian and sets the initial state. Cell five runs the simulation for a range of gate infidelity values and plots the evolution of the quantum state as well as the dependence of errors in the final state on the degree of gate infidelity. In the first of these two plots, you can see that in the final state there is an equal mixture of $|gg\rangle$ and $|ee\rangle$. In the second plot, you can see that the trace distance is quite sensitive to the degree of gate infidelity.