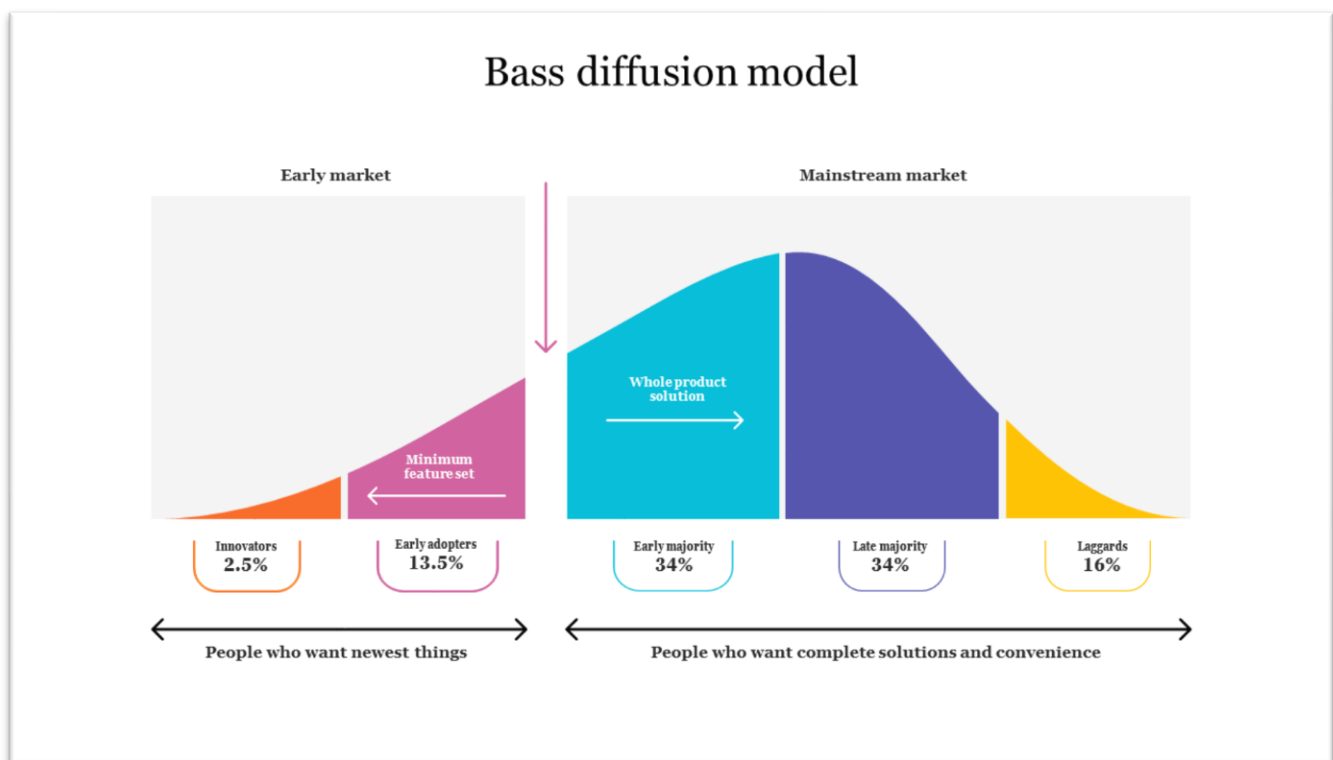


American University of Armenia  
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Homework 1 | Bass Model



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Course: DS233 Marketing Analytics

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## KING SMITH TREADMILL'S BASS MODEL

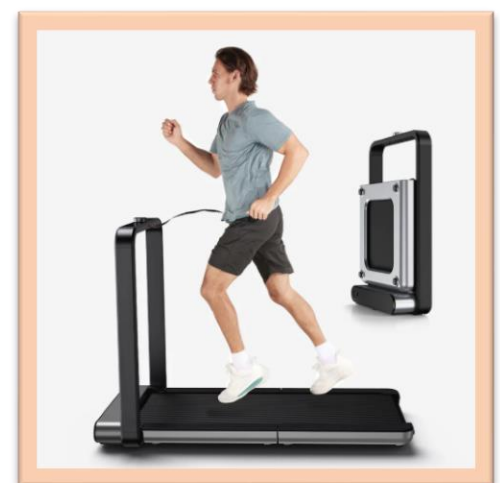
In 2022, Time magazine published a list of the best 100 innovations for that year, which included products in a variety of categories, such as beauty, design, fitness, household, robotics, transportation, etc. I chose an innovation under the fitness category named King Smith WalkingPad X21. I will discuss the features it contains, its look-alike innovation from the past, estimate its Bass Model parameters, make predictions of the diffusion, and eventually estimate the potential market share sales of the product in the United States.



Treadmills have a surprising history of being initially used as torture devices focusing on punishing labor forces in prison. They were initially called “treadwheel” and were invented in China, where the design consisted of two wheels that were connected by interlocking cogs (Papathanasiou, 2022). Treadmills became common in use for health and fitness purposes in 1910, when reports showed that the usage of treadmills benefited heart conditions and reduced the possibility of heart disease (Papathanasiou, 2022). After shifting to the fitness industry, new features were added to modern treadmills, where people can adjust the speed, incline and see how many calories they burned or how many km they ran/walked. The treadmill became one of the most common fitness equipment in the world and started to appear not only in gyms but also in hotels, people’s living spaces, and so on.



The innovation that I chose is the King-Smith WalkingPad X21 (displayed in Figure 1), which is the first double-fold treadmill. WalkingPad is an American company that specializes in the creation of different types of treadmills for running, home fitness, walking & running combined in one and etc. The main unique feature is its ability to double fold into an ultra-thin and flat shape up to 180 degrees. It has a handle that includes an integrated panel display, and with its 9-inch thickness, it can be folded and fit in a closet, under the bed, or can be put in an upright position.



*Figure 1: King Smith WalkingPad*

An additional benefit is that a person can run up to 7.4 mph on it, and it has low sound coming from motors, so it won't disturb surrounding people or neighbors. It is easy and comfortable to use for people living in tight quarters, as it won't take up much additional space.



A look-alike innovation to this WalkingPad are usual treadmills from the past that are also used for health and fitness purposes (displayed in Figure 2).

They have multiple features that are in common, so we can use standard treadmills as a look-alike innovation to the King Smith WalkingPad X21:

**Figure 2:** Ordinary treadmill

1. They share a common purpose and are used to improve health conditions or keep the person in fit shape. Both can be used by fitness enthusiasts who want to walk or run on treadmills and maintain a healthy lifestyle.
2. They have a common design and structure. They both have frames, decks, belt, and use motors and rollers for mechanics. They also share the option of an electrical panel console or handle with a safety key.
3. They have similar features. The common features can include the ability to change speed, see the amount of time running/walking, the distance, the number of burned calories, or count of steps.

I took a time-series data for the look-alike innovation from statista.com. The data contains a series which represent wholesale sales of treadmills for consumers in the U.S. ranging from 2007-2022 and is in million U.S. dollars. As WalkinPad is American company and data shows wholesale sales in the U.S. sector of treadmills, I thought this data is suitable for the analysis.

The data contains only two columns: year, sales in million dollars. It has total of 16 observations. Using the U.S treadmills consumer segment data for sixteen years, the bass model helps to make predictions of the diffusion of innovation and understand the number of adopters by period of King Smith WalkingPad X21 in the United States.

# Hw1\_\_Anna\_\_Shaljyan\_\_Marketing\_\_Analytics

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## Data Manipulation

```
# Reading the second sheet ("Data") of the Excel file
df <- read.xlsx(excel_file, sheet = "Data")

initial_data <- df[2:17, ] #2007-2022
data <- df[8:14, ] #2013-2019

colnames(initial_data) <- c("Year", "Sales of treadmills")
colnames(data) <- c("Year", "Sales of treadmills")

initial_data$Year <- as.integer(initial_data$Year)
initial_data$`Sales of treadmills` <- as.integer(initial_data$`Sales of treadmills`)

data$Year <- as.integer(data$Year)
data$`Sales of treadmills` <- as.integer(data$`Sales of treadmills`)

# View the head of the data
print(initial_data)
```

I read the excel file, the data sheet that contains consumer segment wholesale sales of treadmills for 16 years.

##	Year	Sales of treadmills
## 2	2007	967
## 3	2008	870
## 4	2009	827
## 5	2010	889
## 6	2011	921
## 7	2012	924
## 8	2013	943
## 9	2014	1006
## 10	2015	1006
## 11	2016	1029
## 12	2017	1000
## 13	2018	1034
## 14	2019	1065
## 15	2020	1535
## 16	2021	1744
## 17	2022	1638

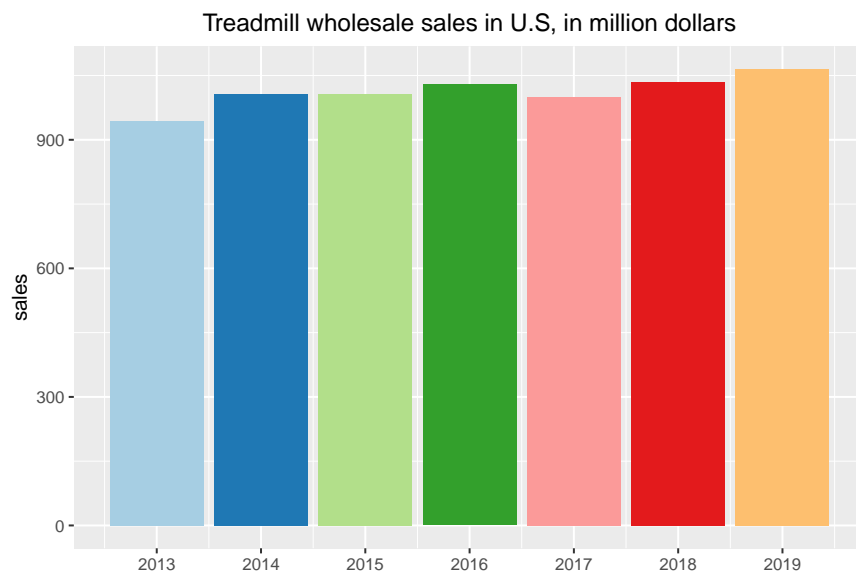
```
print(data)
```

```
##      Year Sales of treadmills
## 8  2013           943
## 9  2014          1006
## 10 2015          1006
## 11 2016          1029
## 12 2017          1000
## 13 2018          1034
## 14 2019          1065
```

Now after modifying data to contain Year and Sales of treadmills we can plot it to have better understanding of the data.

```
# Defining a color palette from RColorBrewer
year_colors <- brewer.pal(7, "Paired") # 7 colors for 7 years

ggplot(data = data, aes(x = Year, y = `Sales of treadmills`, fill = factor(Year))) +
  geom_bar(stat = 'identity') +
  labs(title = 'Treadmill wholesale sales in U.S, in million dollars',
       x = ' ', y = 'sales') +
  scale_x_continuous(breaks = 2013:2019) +
  scale_fill_manual(values = year_colors) +
  theme(plot.title = element_text(hjust = 0.5),
        legend.position = "none")
```



Let's define  $f(t)$  and  $F(t)$  of bass model:

$f(t)$  is the fraction of the total market that adopts at time  $t$

$F(t)$  is the fraction of the total market that has adopted up to and including time  $t$ .

```
bass.f <- function(t,p,q){((p+q)^2/p)*exp(-(p+q)*t)/(1+(q/p)*exp(-(p+q)*t))^2}
bass.F <- function(t,p,q){(1-exp(-(p+q)*t))/(1+(q/p)*exp(-(p+q)*t))}
```

Here  $p$  and  $q$  are innovation and imitation rates, respectively.

We can experiment with  $p$  and  $q$  values. We can try two methods of parameter estimation: nls (non-linear least squares) and diffusion method.

```
sales_test = initial_data$`Sales of treadmills`
t_testing = 1:length(sales_test)

sales = data$`Sales of treadmills`
t = 1:length(sales)

print(sales_test)
```

Important Note: as in initial sales data of 16 years there were periods with drastic sales falls or sales increases, NLS and diffusion methods were either giving error or estimating inadequately. Because of that I tried to slice the data for smaller range of year to have better and accurate estimations of  $p, q$  and  $m$  (market potential).

```
## [1] 967 870 827 889 921 924 943 1006 1006 1029 1000 1034 1065 1535 1744
## [16] 1638
```

```
print(sales)
```

```
## [1] 943 1006 1006 1029 1000 1034 1065
```

```
# Fit the Bass diffusion model

#bass_m_test <- nls(sales_test ~ m*((p+q)^2/p)*exp(-(p+q)*t_testing))/
#(1+(q/p)*exp(-(p+q)*t_testing))^2, start=c(list(m=sum(sales_test),
#p=0.002, q=0.045)))

# Display the summary

#summary(bass_m_test)
```

As you can see it gives error, because of drastic 200-500 change in values for the last 3 years. Uncomment to see error.

```

# Fit the Bass diffusion model
bass_m <- nls(sales ~ m*((p+q)^2/p)*exp(-(p+q)*t))/(1+(q/p)*exp(-(p+q)*t))^2,
              start=c(list(m=sum(sales), p=0.002, q=0.045)))

# Display the summary
summary(bass_m)

```

### NLS method of parameter estimation (p,q,m)

```

##
## Formula: sales ~ m * (((p + q)^2/p) * exp(-(p + q) * t))/(1 + (q/p) *
##      exp(-(p + q) * t))^2
##
## Parameters:
##      Estimate Std. Error t value Pr(>|t|)
## m 4.264e+04  5.657e+04   0.754   0.493
## p 2.208e-02  2.862e-02   0.772   0.483
## q 4.512e-02  4.892e-02   0.922   0.409
##
## Residual standard error: 23.98 on 4 degrees of freedom
##
## Number of iterations to convergence: 13
## Achieved convergence tolerance: 3.465e-06

```

```

sales = data$`Sales of treadmills`
diff_m = diffusion(sales)
p=round(diff_m$w,4)[1]
q=round(diff_m$w,4)[2]
m=round(diff_m$w,4)[3]
diff_m

```

### Diffusion method of parameter estimation (p,q,m)

```

## bass model
##
## Parameters:
##
##      Estimate p-value
## p - Coefficient of innovation    0.0275    NA
## q - Coefficient of imitation     0.0509    NA
## m - Market potential             34669.9697    NA
##
## sigma: 18.473

```

```

# Getting the model summary
bass_summary <- summary(bass_m)

# Extracting the coefficients (including p and q)
coefficients <- coef(bass_m)

```

```
# Extracting the estimated p and q values
p_estimate <- coefficients["p"]
q_estimate <- coefficients["q"]
m_estimate <- coefficients["m"]
```

```
# Printing the estimated p and q values
cat("Estimated p:", p_estimate, "\n")
```

```
## Estimated p: 0.02208142
```

```
cat("Estimated q:", q_estimate, "\n")
```

```
## Estimated q: 0.04511991
```

```
cat("Estimated m:", m_estimate, "\n")
```

```
## Estimated m: 42644.88
```

Plotting  $f(t)$  and histogram of treadmill sales using NLS parameter estimation.

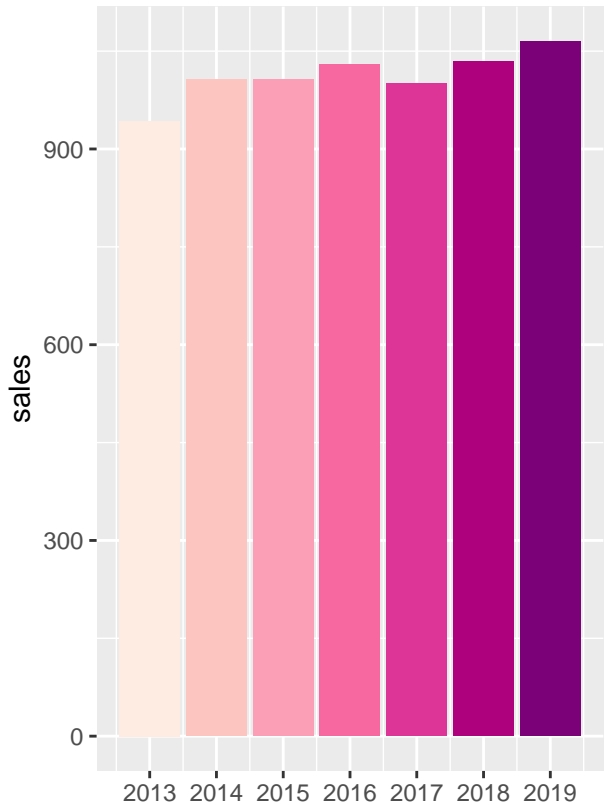
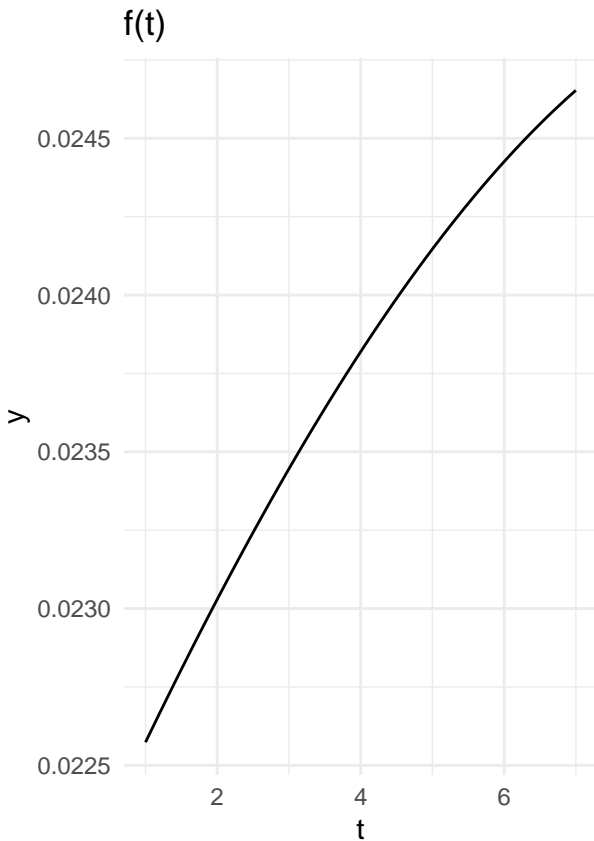
```
# Defining ColorBrewer palette
color_palette <- "RdPu"
```

```
# Plot 1: time_ad
time_ad <- ggplot(data.frame(t = c(1:7)), aes(t)) +
  stat_function(fun = bass.f, args = c(p = 0.02208142, q = 0.04511991),
    aes(color = "Bass Model")) +
  labs(title = 'f(t)') +
  theme_minimal() +
  theme(legend.position = "none") +
  scale_color_manual(values = c("Bass Model" = "black"))
```

```
# Plot 2: treadmill_sales
treadmill_sales <- ggplot(data = data, aes(x = Year, y = `Sales of treadmills`,
  fill = factor(Year))) +
  geom_bar(stat = 'identity') +
  labs(x = " ", y = "sales") +
  scale_fill_brewer(palette = color_palette) +
  scale_x_continuous(breaks = 2013:2019) +
  theme(legend.position = "none")
```

```
# Arranging the plots using ggarrange
ggarrange(time_ad, treadmill_sales)
```





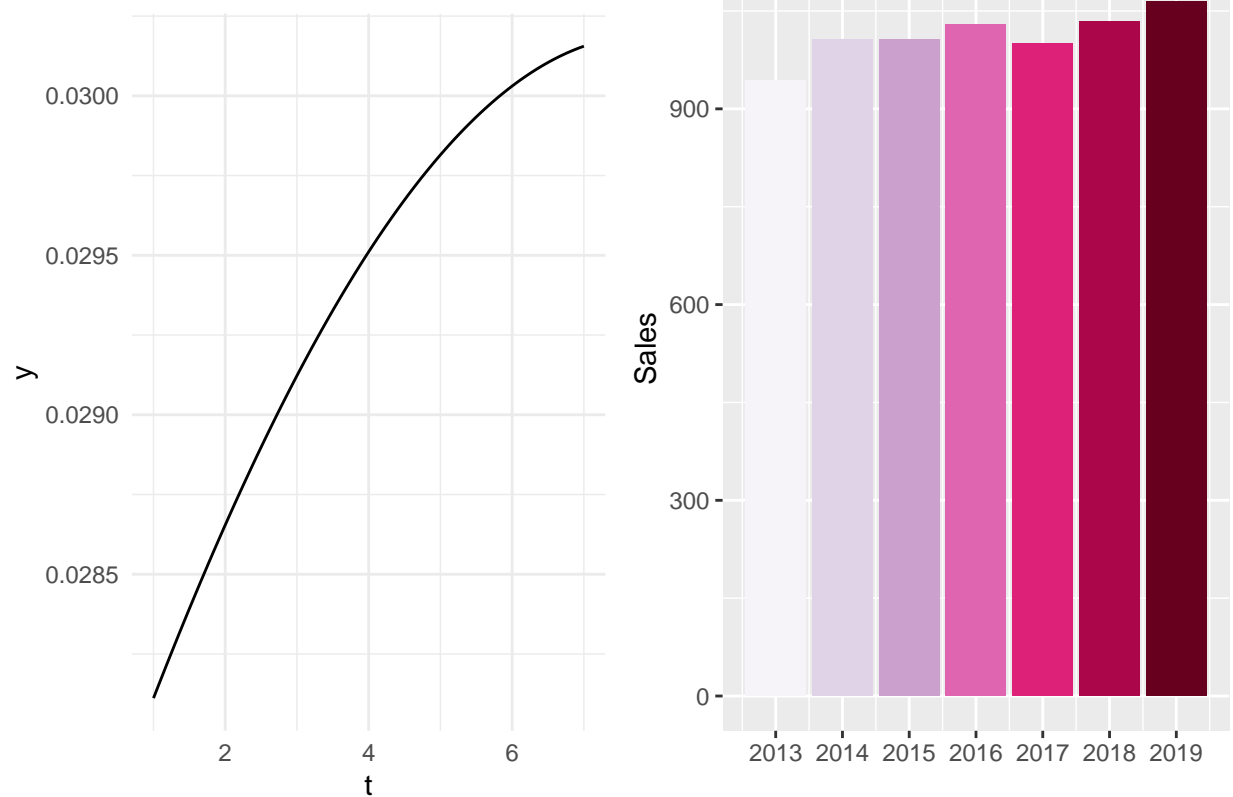
```
# Defining a custom palette
my_palette <- colorRampPalette(brewer.pal(9, "PuRd"))(7)

# Plot 1: time_ad (Black)
time_ad <- ggplot(data.frame(t = c(1:7)), aes(t)) +
  stat_function(fun = bass.f, args = c(p = 0.0275, q = 0.0509),
    color = "black") +
  labs(title = 'f(t)') +
  theme_minimal() +
  theme(legend.position = "none") +
  scale_color_identity()

# Plot 2: treadmill_sales (PuRd palette)
treadmill_sales <- ggplot(data = data, aes(x = Year,
  y = `Sales of treadmills`)) +
  geom_bar(stat = 'identity', fill = my_palette) +
  scale_x_continuous(breaks = 2013:2019) +
  labs(x = " ", y = "Sales")

# Arranging the two plots
ggarrange(time_ad, treadmill_sales)
```

## Plotting $f(t)$ and histogram of treadmill sales using diffusion parameter estimation

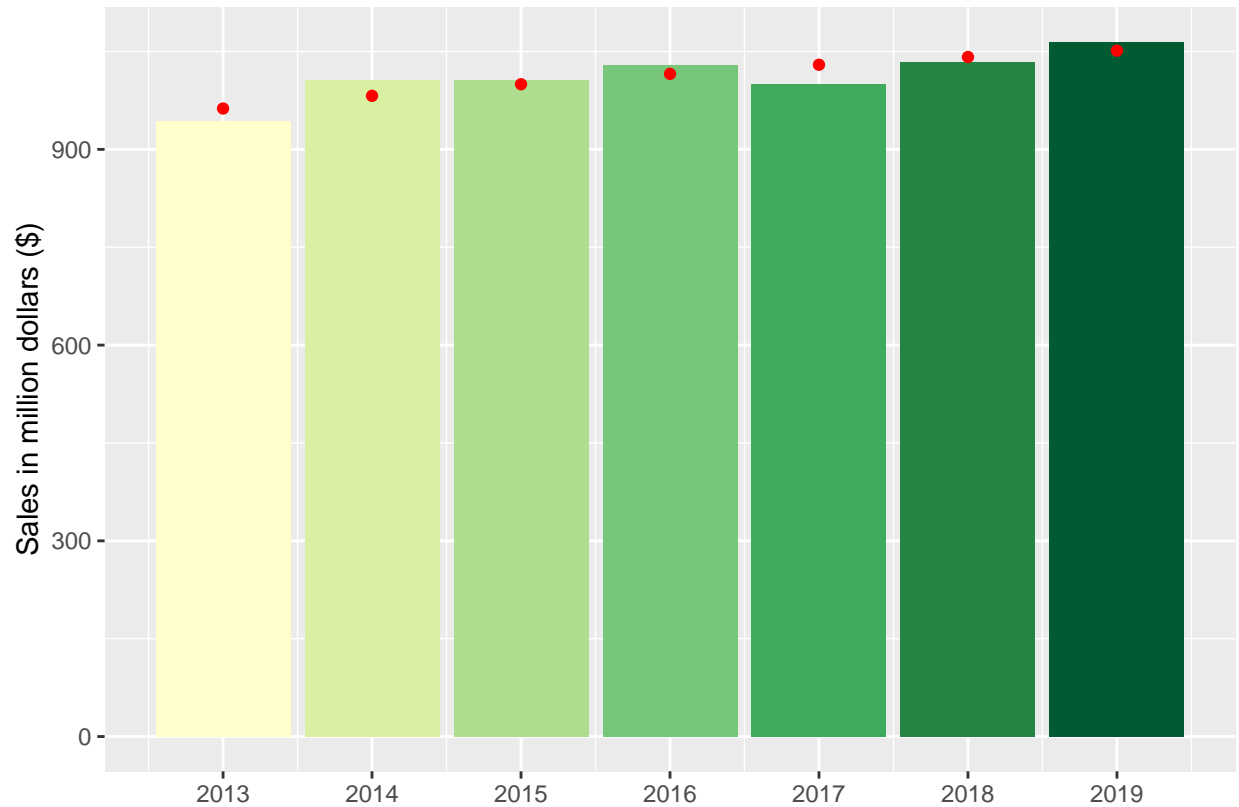


```
data$pred_sales <- bass.f(1:7, p = 0.02208142, q = 0.04511991) * 42644.88

# Defining the ColorBrewer palette
my_palette <- brewer.pal(7, "YlGn")

# Create the ggplot visualization
ggplot(data = data, aes(x = Year, y = `Sales of treadmills`)) +
  geom_bar(stat = 'identity', fill = my_palette) +
  geom_point(mapping = aes(x = Year, y = pred_sales), color = 'red') +
  labs(x = ' ', y = 'Sales in million dollars ($)') +
  scale_x_continuous(breaks = 2013:2019, labels = 2013:2019) +
  scale_fill_identity()
```

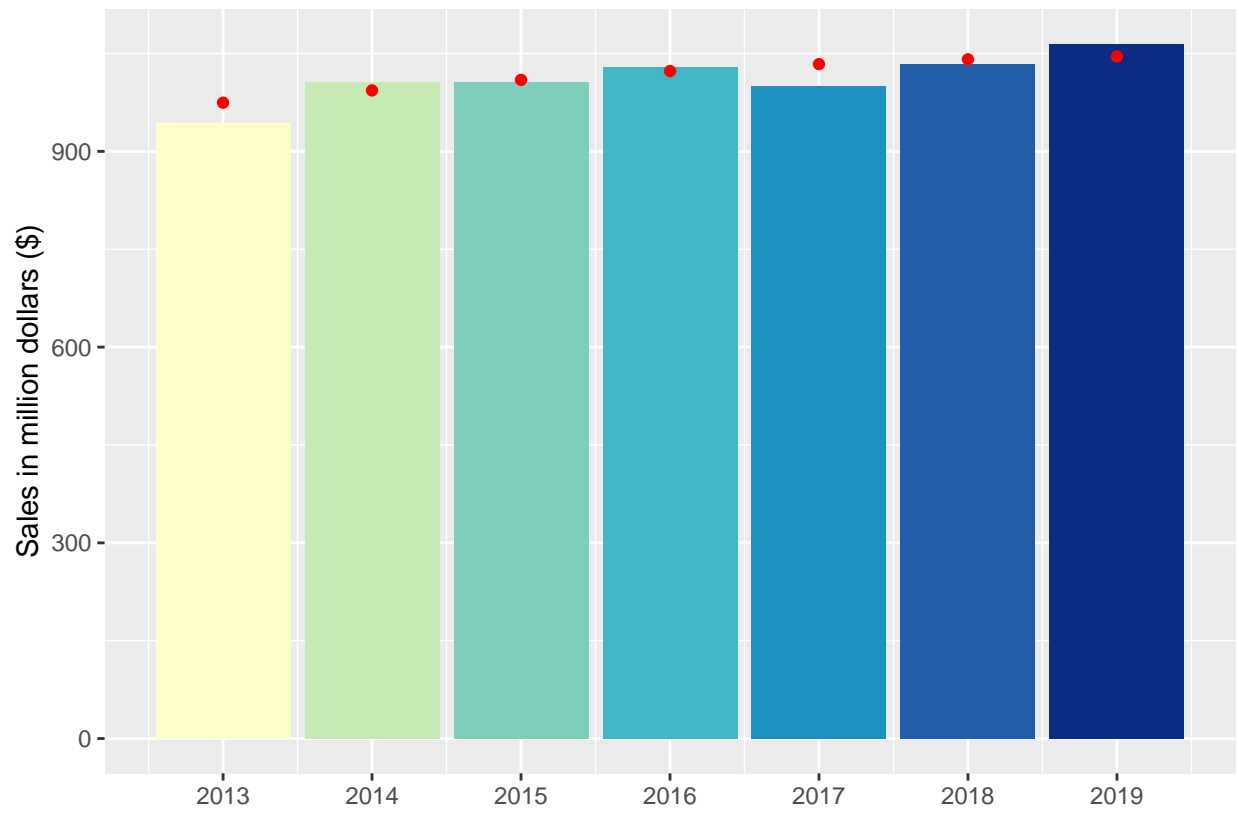
Estimating the number of adopters by period with NLS parameter estimation  $p$ ,  $q$  and  $m$ .



```
data$pred_sales <- bass.f(1:7, p = 0.0275, q = 0.0509) * 34669.9697

# Create the ggplot
ggplot(data = data, aes(x = Year, y = `Sales of treadmills`)) +
  geom_bar(stat = 'identity', fill = brewer.pal(7, "YlGnBu")) +
  geom_point(mapping = aes(x = Year, y = pred_sales), color = 'red') +
  labs(x = ' ', y = 'Sales in million dollars ($)') +
  scale_x_continuous(breaks = 2013:2019, labels = 2013:2019) +
  scale_fill_identity()
```

Estimating the number of adopters by period with diffusion parameter estimation  $p$ ,  $q$  and  $m$ .



### **Sources used:**

For the Time's magazine innovation:

<https://time.com/collection/best-inventions-2022/6228299/king-smith-walkingpad-x21/>

For the history of treadmills (article):

Papathanasiou, K. (2022, December 2). The Treadmill's surprising history as a torture device. The Vale Magazine.

<https://thevalemagazine.com/2022/11/30/the-treadmills-surprising-history-as-a-torture-device/>

For standard treadmill time series data:

<https://www.statista.com/statistics/236131/us-wholesale-sales-of-treadmills-consumer-segment/>