INTRODUCTION

In this project the system is a continuous bioreactor with two different inflows: one contains a highly concentrated solution of coffee ($C_{in,1}$ = 50 mg/dl) and the other contains water ($C_{in,2}$ = 0).
The aim of this project is to control the concentration of coffee in the reactor by using the two inflows. So, in this case, we are talking about a MISO (Multi Input Single Output) system where the inputs are the inflows of coffee ($C_{in,1}$= u_1) and water ($C_{in,1}$= u_2), the output is the coffee concentration in the reactor (C = *y*) and we want to track the desired output y_r(variable over time). The inputs are subject to a constraint: in fact, the minimum and maximum inflows are respectively 0 and 66 ml/min at every instant of time (the sensing and the control are not continuous but digital with a sampling time of 10 sec).
The project is divided into different tasks:

1. PID control with only 1 control input
2. Non-linear model derivation (to describe the bioreactor) and its analytical linearization
3. MPC for both the linearized model of the reactor and the ‘real’ (non-linear) reactor
4. MPC with integral action (with and without the input constraints)

TASK 3.1: MPC for the Linearized Reactor

In this case the MPC is used to control the reactor described with the linearized state space model, introduced in the previous task.
First, we choose the prediction horizon PH and the cost function which is used to compute the optimal sequence of N control actions (until PH):

J = $\sum_{i=1}^{N}(\hat{y}(k+i) - y_r(k+i))^T Q\,(\hat{y}(k+i) - y_r(k+i)) + \sum_{i=0}^{N-1}(u(k+i) - u_r)^T R\,(u(k+i) - u_r)$

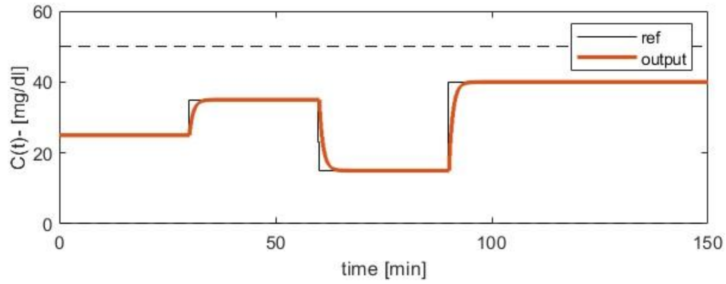
The control action is subject to saturation ($0 \le u(k+i) \le u_{max}$) for the generic instant of time *i*, so the linear constraints are: $Fu(k+i) \le f$, with $F = [-1\;0; 1\;0;\;0-1; 0\;1]$and $f = \;[0;\;u_{max}; 0;\;u_{max}]$.
Then, we introduce the matrix representation (condensed form) for all the terms:

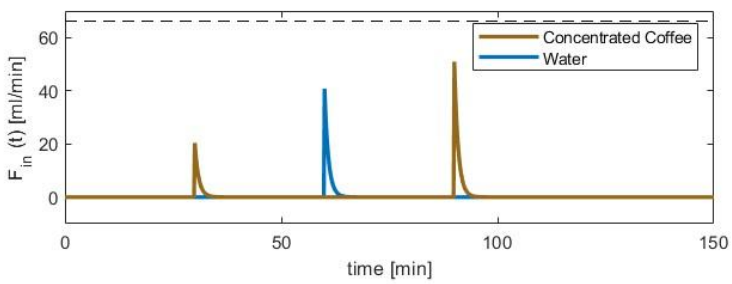
- system evolution: $\hat{X}(k) = \;A_{cond}x(k) + B_{cond}U(k),\;\hat{Y}(k) = C_{cond}\hat{X}(k) = \;A_{call}x(k) + \;B_{call}U(k)$
- linear constraints: $F_{call}\;U(k) \le f_{call}$
- cost function: $J = \left(\hat{Y}(k) - Y_r(k)\right)^T Q_{call}(\hat{Y}(k) - Y_r(k)) + \;\left(U(k) - U_r(k)\right)^T R_{call}\left(U(k) - U_r(k)\right)$

A_{cond} and B_{cond} are calculated such that: $\hat{x}(k+i) = \;A^i x(0) + \;\sum_{j=1}^{i} A^{i-j} B u(k+j-1)$ for every general instant of time *i*. In this case A=0 so A_{cond}= column vector of N zeros, B_{cond}= diagonal block matrix with B as the elements of the diagonal, C = 1 so C_{cond}= identity matrix of dimension NxN.
So, we calculate the calligraphic matrices:

- A_{call}= CA_{cond} and B_{call}= CB_{cond}
- Q_{call}, R_{call}, F_{call} and f_{call}= diagonal block matrices with *Q, R, F* and *f* as the elements of the diagonal

Replacing $\hat{Y}(k)$ with the outcome prediction equation the MPC optimization problem becomes a QP (quadratic programming) problem in the form: $min(J(x))\;with\,J\; = \frac{1}{2}{x_{qp}}^T H_{qp} x_{qp} + {f_{qp}}^T x_{qp}$ and linear constraints. In this case: the optimization variable $x_{qp}\; = \;U(k)$, $H_{qp}\; = B^T{}_{call} Q_{call} B_{call} + R_{call}$ and $f_{qp} = \;B^T{}_{call} Q_{call}(A_{call}\hat{x}(k) - Y_r(k)) - R_{call} U_r(k)$.
In the end we can easily find U(k) with the Matlab function 'quadprog' and we can apply only the first 2 elements of the vector (which represent the current inputs), finding the current output y(k).
The matrices of weights Q and R (the tuning parameters) have a very important role because they regulate controller aggressiveness. In this case we choose Q = 5 and R = [1 0; 0 1]. The results of the MPC are shown in the following plots of the output (concentration of coffee) and the inputs (inflows of coffee and water):





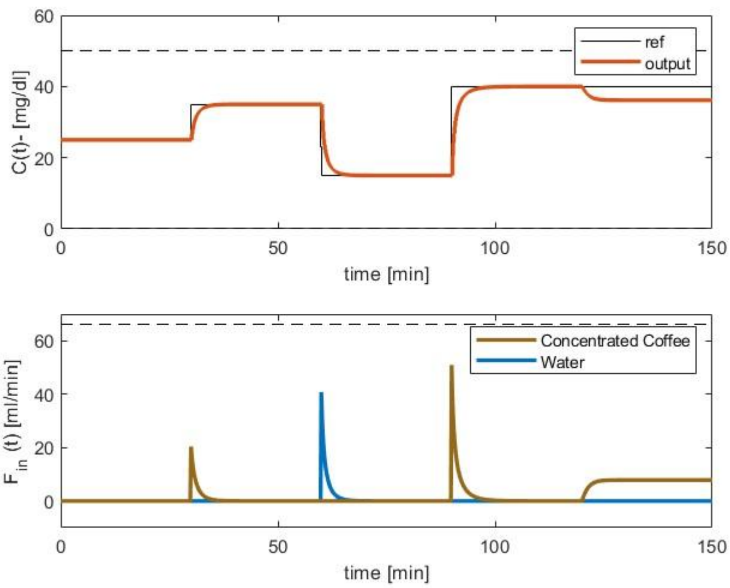
As we can see, the requests ($\pm$ settling time $\leq$ 10 min and overshoot $\leq$ 5%) are satisfied and the inputs were not saturated. There is an inflow of coffee when the reference increases and one of water when the reference decreases. Otherwise, both inflows are null, as it should be. In this case the controller is also able to reach offset free reference tracking, so the error is 0 in the steady state. In fact, we are not considering the disturbance that acts on the system: the output is not influenced by it, so the reference is tracked very well.

TASK 3.2: MPC for the nonlinear reactor

In this task we apply the MPC (implemented in the previous point) to the initial nonlinear reactor and we analyze if it works well. In this case we are considering also the constant disturbance that acts in the system after t = 120s, so the state update becomes x(k+1) = Ax(k) + Bu(k) + Md(k). We have to introduce $D(k) = [d(k);\ \dots;\ d(k+N-1)]$ and the two block matrices M_{cond}(defined as B_{cond}, with B replaced with M) and $M_{call} = \ CM_{cond}$ which slightly change the optimization problem. In fact, also in this case, we can write the equivalent QP problem and the only term that changes from the previous task is

$$f_{qp} \ = {B^T}_{call} Q_{call} (A_{call} \hat{x}(k) + M_{call} D(k) - \ Y_r(k)) - R_{call} U_r(k)$$

The aim is to evaluate if the controller of the previous task works also for the real system, so the parameters Q and R are tuned in the same way. The results are shown in the following plot:



As we can see the output tracks well the reference until the steady state is reached but, after t = 120 sec, the disturbance starts to act on the system and, even if the controller tries to reach the reference by applying an inflow of coffee, the offset free reference tracking is not achieved (in steady state we have a constant difference between the output and the reference).