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Abstract

We propose a framework of network formation where players can form two types of links: public links observed by everyone and shadow links generally not observed by others. We introduce a novel solution concept called rationalizable conjectural pairwise stability, which generalizes Jackson and Wolinsky (1996)'s pairwise stability notion to accommodate shadow links. We first show that a network is stable if there exist beliefs such that each player conjectures to be in a network that is stable under correct beliefs, and in which she does not want to alter her links unilaterally. We then derive a mechanism to construct a stable network that is not stable under correct beliefs. Third, we establish that the set of stable networks is shrinking in the players' observation radius. Finally, we illustrate our framework in the context of two specific models and show that players may over(under)estimate others' connections and hence under(over)connect.

JEL classification: A14; C70; D82; D85

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1. Introduction

People typically have incomplete information about others' social connections. Expectations about these connections might then influence an individual's decisions with whom to connect.

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Such expectations might have a strong impact when the benefit from linking to another person depends on the social environment. For example, in the presence of homophilic preferences people want to associate with a certain group of people but not with others, such that it may be advantageous if certain connections are not observed by everyone. In the business sphere, people maintain weak tie relationships because they provide access to relevant information. However, they are harder to observe than strong business ties. An individual's decision whether to link to another person in these situations will then heavily depend on her belief about others' unobserved links.

We propose a framework of network formation in which players can form public as well as private relationships to model such situations. While *public links* are observed by everyone, *shadow links* are private in the sense that they are generally not observed by others. We introduce a novel solution concept called *rationalizable conjectural pairwise stability (RCPS)*, which generalizes the pairwise stability (PS) notion of Jackson and Wolinsky (1996, henceforth JW) to incomplete information about the network structure and accommodates shadow links. Our framework provides the foundation to model and understand richer situations of network formation when agents can form different types of relationships that vary in visibility and potentially also in payoff consequences.

We first extend the PS concept of JW to incomplete information about the network structure and shadow links. Players observe public links of everyone in the network but generally do not observe shadow links. Instead, players receive a private signal that may contain information on the shadow links or their own payoff in the network. We then adapt McBride (2006b)'s conjectural pairwise stability for network formation with imperfect monitoring to accommodate two types of links. A tuple of a network and beliefs for each player is *conjectural pairwise stable (CPS)* if no player wants to sever a link, no two players jointly want to form or switch to a public link or a shadow link, and beliefs do not contradict the players' private signals similar to Battigalli and Guaitoli (1988, 1997)'s conjectural equilibrium. We can hence interpret the players' beliefs as based on their monitoring of the network, which is perfect for public relationships but only imperfect for private relationships. Notably, players do not necessarily have correct beliefs about their own payoffs.[1]

This solution concept is weak in the sense that it does not impose any restrictions on beliefs beyond consistency with private signals. In particular, beliefs may not be *rationalizable*. In a second step, we therefore introduce a refinement of CPS that imposes common knowledge of rationality à la Rubinstein and Wolinsky (1994).[2] A network is *rationalizable conjectural pairwise stable (RCPS)* if there exist beliefs for each player such that the tuple of this network and these beliefs is CPS, and additionally the same holds for each network supported by the players' conjectures.

We then provide a sufficient condition for a network to be RCPS. We show that a network is RCPS if there exist consistent beliefs such that each player conjectures to be in a network that is CPS under correct beliefs, and in which she does not want to alter her links unilaterally. The

[1] Allowing for incorrect beliefs about own payoffs appears reasonable in situations where unobserved exogenous variation would make inferences from realized payoffs hard or impossible. Nevertheless, equilibria in which beliefs about own payoffs are correct can be viewed as more robust.

[2] Rubinstein and Wolinsky (1994) study a non-cooperative game in which players receive imperfect signals about others' strategies. In equilibrium, each player's strategy is optimal given her signal and that it is common knowledge that all players maximize utility given their signals. See also Bernheim (1984), Gilli (1999) and Pearce (1984) for related concepts.

first condition ensures rationalizability of beliefs as networks that are stable under correct beliefs are rationalizable. The second condition strengthens the requirements of CPS. Since players may conjecture to be in different networks, CPS under correct beliefs is not sufficient for stability. However, broadly speaking, it is sufficient that the network *appears* stable to everyone in the sense that it would be stable if the own belief was correct and shared by everyone else. Building on this result, we then derive a mechanism that yields an RCPS network that is not CPS under correct beliefs but appears stable to everyone, if such a network exists. Furthermore, we investigate how the set of stable networks changes with the informativeness of private signals. We establish that the set of RCPS networks is (weakly) shrinking in the players' observation radius. Increasing the observation radius implies more restrictions on beliefs such that equilibria which rely on incorrect beliefs may not be rationalizable any more with these more informative private signals. In particular, an RCPS network may only exist when the observation radius is small enough.

Finally, we propose two specific models to illustrate the potential of our concept. The first model considers friendship networks with homophily and perfect substitutes. Similar to de Martí and Zenou (2017), individuals belong to different communities and generally benefit from direct and indirect connections to others, which can be interpreted as benefits from personal interactions. However, homophily means interaction with strong adherents or partisans of the other group is less beneficial or even detrimental. We first show that in equilibrium some agents may overestimate others' connections and hence underconnect (relative to stable networks under correct beliefs), while others underestimate connections and hence overconnect. Second, false yet rationalizable beliefs may lead to segregation of partisans from the rest of society and lower social welfare if only homophily of partisans is large. Each group of partisans falsely believes that the respective other group maintains shadow links to the other members of their community.

Second, we study a connections model in which shadow links represent weak ties and public links represent strong ties. As in JW's connections model, both types of relationship provide discounted benefits from indirect connections to other agents, e.g., information about employment opportunities. Additionally, a strong tie also provides other direct benefits that depend on its exclusivity similar to Morrill (2011), because a personal recommendation becomes less helpful the more people that person recommends. We first show that, if decay is low enough (relative to linking costs), then *stars with peripheral pairs* are efficient and CPS under correct beliefs, and hence also RCPS. In these networks, pairs of strong ties maintain weak ties to a central agent. However, we then show agents may over- or underestimate others' relationships and hence under- or overconnect in equilibrium.

There exists a large and growing literature on network formation, including refinements of PS (e.g., Jackson and Watts, 2002; Jackson and Van den Nouweland, 2005) and extensions to farsighted agents (e.g., Dutta et al., 2005; Herings et al., 2009; Page et al., 2005).[3] These contributions investigate the stability and efficiency of networks under complete information about the network structure. McBride (2006a,b) relaxes this assumption by assuming that players imperfectly monitor the network. In particular, McBride (2006b) studies conjectural pairwise stable networks when players do not observe the connections and payoffs of players that are located far from them in the network, but does not consider rationalizability. To the best of our knowledge, we are the first to study the strategic choice of players to form shadow links and to adapt the rationalizability concept of Rubinstein and Wolinsky (1994) to networks.

[3] We refer to Mauleon and Vannetelbosch (2016), Goyal (2012) and Jackson (2008) for overviews.

The paper is organized as follows. In Section 2 we introduce the model and notation. We define RCPS and present our general results in Section 3. Section 4 presents two specific models on friendship networks with homophily and on weak and strong tie relationships. Section 5 concludes.

2. Model and notation

We consider a set $N = \{1, 2, \dots, n\}$, with $n \geq 3$, of *players* or *agents*. The network relations among these players are captured by a symmetric matrix $g \in \{0, 1, 2\}^{n\times n}$, where each entry g_{ij} such that $i \neq j$ captures the type of direct relation between players $i \in N$ and $j \in N$. We refer to $g_{ij} = 1$ as a *public link* and to $g_{ij} = 2$ as a *shadow link* between i and j, and $g_{ij} = 0$ indicates that no link is present. The collection of all networks is denoted by $\mathcal{G} = \{g \in \{0, 1, 2\}^{n\times n} \mid g_{ij} = g_{ji}, g_{ii} = 0 \text{ for all } i, j \in N\}$. We say that there is a *path (of length* $K - 1$*)* from player i to player $j \neq i$ in network $g \in \mathcal{G}$ if there are players $i = i_1, i_2, \dots, i_{K-1}, i_K = j$ such that $g_{i_k i_{k+1}} \neq 0$ for all $k \in \{1, 2, \dots, K - 1\}$. The number of links in the shortest path from i to j in g is denoted $t_{ij}(g)$ (with $t_{ij}(g) = +\infty$ if there is no path from i to j in g). The number of links or the *degree* of player i in network g is denoted $d_i(g) = \#\{j \in N \mid g_{ij} \neq 0\}$; the number of her public links and shadow links is denoted $d_i^{\text{pub}}(g) = \#\{j \in N \mid g_{ij} = 1\}$ and $d_i^{\text{sha}}(g) = \#\{j \in N \mid g_{ij} = 2\}$, respectively.

For a given network g, the restriction to player i's links is denoted $g_i = (g_{ij})_{j\in N}$, and the restriction to links among players other than i is denoted $g_{-i} = (g_{jj'})_{j,j'\neq i}$. With a slight abuse of notation, we write $g = (g_i, g_{-i})$. The restriction of $\mathcal{G}$ to networks among players other than i is denoted $\mathcal{G}_{-i} = \mathcal{G}|_{N\setminus\{i\}}$, and the restriction to public links is denoted $\mathcal{G}^{\text{pub}} = \mathcal{G}|_{\{0,1\}^{n\times n}}$. Moreover, let $g(ij \to t)$ denote the network obtained from g when changing the relation between i and j from g_{ij} to $t \in \{0, 1, 2\}\setminus\{g_{ij}\}$, i.e.,

$$(g(ij \to t))_{i'j'} = \begin{cases} t, & \text{if } i'j' = ij \text{ or } i'j' = ji \\ g_{i'j'}, & \text{otherwise} \end{cases} \quad \text{for all } i', j' \in N.$$

The *payoff* allocated to each player i across networks is determined by a utility function $u_i : \mathcal{G} \to \mathbb{R}$. We will frequently consider the case when public links and shadow links yield the same payoffs and hence only differ in visibility.

Definition 1 *(Perfect substitutes).* We say that public links and shadow links are *perfect substitutes* with respect to $(u_i)_{i\in N}$ if $u_i(g) = u_i(g')$ for all $i \in N$ and all $g, g' \in \mathcal{G}$ such that $g_{jj'} \neq 0$ iff $g'_{jj'} \neq 0$ for all $j, j' \in N$.

Player i knows her own links g_i in network g and receives a *private signal* $s_i(g)$ that may contain information about others' links, the payoffs, etc. Let $(s_i(g))_{jj'}$ denote the information that player i observes about link $g_{jj'}$, for $j, j' \neq i$. Players observe all public links of other players, while shadow links may not be observed, i.e.,

$$(s_i(g))_{jj'} \in \begin{cases} \{g_{jj'}\}, & \text{if } g_{jj'} \in \{0, 1\} \\ \{0, 2\}, & \text{if } g_{jj'} = 2 \end{cases} \quad \text{for all } j, j' \neq i.$$

We will frequently employ private signal functions in the analysis where players observe shadow links of all players that are located at most at distance K from them in the network.

Definition 2 *(Distance-K confirming private signals).* We say that the signal function s_i of player $i \in N$ is *distance-K confirming* if, for all $g \in \mathcal{G}$ and all $j, j' \neq i$ with $g_{jj'} = 2$, $(s_i(g))_{jj'} = 2$ iff the shortest path between either i and j or i and j' has at most length K. We refer to K as the *observation radius*. In particular, s_i *does not contain additional information* if $K = 0$, and s_i is *completely informative* if $K \geq n - 1$.

Note first that we can interpret distance-K confirming private signals as imperfect monitoring of the network structure (McBride, 2006a,b). Second, if $K = 1$, then players only observe shadow links of their neighbors or peers in the network, which yields beliefs that are peer confirming similar to the notion of peer-confirming equilibrium (Lipnowski and Sadler, 2019).[4] To contrast our results to those under complete information, we will assume that private signals contain no additional information in most examples and the specific models in Section 4. The utility functions and the signal functions are common knowledge, but the actual signals are private information.

Let $\Delta(\mathcal{G}_{-i})$ denote the set of probability distributions on $\mathcal{G}_{-i}$. Then

$$U_i(g_i, \mu_i) = \sum_{g'_{-i} \in \mathcal{G}_{-i}} \mu_i(g'_{-i}) u_i(g_i, g'_{-i})$$

denotes the *expected payoff* of player i with links g_i under the (subjective) belief $\mu_i \in \Delta(\mathcal{G}_{-i})$ about the links among other players. We require the players' beliefs not to contradict their signals.

Definition 3 *(Consistent beliefs).* We say that the beliefs $(\mu_i)_{i \in N}$ are *consistent* with the private signals $(s_i(g))_{i \in N}$ obtained in network $g \in \mathcal{G}$ if $s_i(g_i, g'_{-i}) = s_i(g)$ for all $i \in N$ and $g'_{-i} \in \mathcal{G}_{-i}$ such that $\mu_i(g'_{-i}) > 0$.

Finally, we introduce our measures of social welfare. The most common approach in the literature is to use players' expected utilities to measure subjective well-being. Alternatively, we can evaluate social welfare based on actual utilities. Here, the idea is that what matters are the actual benefits from a relationship.

Definition 4 *(Social welfare).*

(i) *Subjective social welfare* of a tuple $(g, (\mu_i)_{i \in N})$ such that $(\mu_i)_{i \in N}$ are consistent with $(s_i(g))_{i \in N}$ is defined as $\sum_{i \in N} U_i(g_i, \mu_i)$.
(ii) *Objective social welfare* of a network $g \in \mathcal{G}$ is defined as $\sum_{i \in N} u_i(g)$.

3. Conjectural pairwise stability and rationalizability

We first consider the case without shadow links and define JW's PS concept under complete information.

Definition 5 *(Pairwise stability, JW).* The network $g \in \mathcal{G}^{\text{pub}}$ is *pairwise stable (PS)* with respect to $(u_i)_{i \in N}$ if

[4] Lipnowski and Sadler (2019) augment a non-cooperative game with an exogenously given network and assume that players have correct conjectures about the strategies of their neighbors, see also Frick et al. (2019) and Battigalli et al. (2020) for related contributions.

(i) for all distinct $i, j \in N$ such that $g_{ij} = 1$,

$$u_i(g) \geq u_i(g(ij \to 0)) \text{ and } u_j(g) \geq u_j(g(ij \to 0)), \text{ and}$$

(ii) for all distinct $i, j \in N$ such that $g_{ij} = 0$,

$$u_i(g) < u_i(g(ij \to 1)) \text{ implies } u_j(g) > u_j(g(ij \to 1)).$$

A network is PS if no player wants to sever a link (condition (i)), and no two players jointly want to form a link (condition (ii)).[5] Next, we extend this notion to incomplete information and shadow links, adapting McBride (2006b)'s conjectural pairwise stability to accommodate two types of links. Following JW and most of the literature, we assume that forming, and now also switching to, a public link or a shadow link requires the consent of both players involved, while severing a link can be done unilaterally. Furthermore, shadow links require the introduction of beliefs or conjectures of the players about other players' links. Players revise their links based on their conjectures about other players' behavior. In equilibrium, these conjectures are consistent with the players' private signals similar to Battigalli and Guaitoli (1988, 1997)'s notion of conjectural equilibrium.

Definition 6 *(Conjectural pairwise stability).* The tuple $(g, (\mu_i)_{i \in N})$ is *conjectural pairwise stable (CPS)* with respect to $(u_i)_{i \in N}$ and $(s_i)_{i \in N}$ if

(i) for all distinct $i, j \in N$ such that $g_{ij} \neq 0$,

$$U_i(g_i, \mu_i) \geq U_i((g(ij \to 0))_i, \mu_i) \text{ and } U_j(g_j, \mu_j) \geq U_j((g(ij \to 0))_j, \mu_j),$$

(ii) for all distinct $i, j \in N$ and $t \in \{1, 2\} \backslash \{g_{ij}\}$,

$$U_i(g_i, \mu_i) < U_i(g(ij \to t)_i, \mu_i) \text{ implies } U_j(g_j, \mu_j) > U_j(g(ij \to t)_j, \mu_j), \text{ and}$$

(iii) $(\mu_i)_{i \in N}$ are consistent with $(s_i(g))_{i \in N}$.

Moreover, g is *CPS under correct beliefs* if additionally $\mu_i(g_{-i}) = 1$ for all $i \in N$.

Similar to PS, a tuple of a network and beliefs is CPS if no player wants to sever a link given her conjecture about other players' links (condition (i)), and no two players jointly want to form or change to a public link or a shadow link given their conjectures (condition (ii)). Additionally, we require that the players' beliefs are consistent with their private signals (condition (iii)).

The next result provides some intuition and follows directly from the above definitions. First, consistency requires correct beliefs with completely informative private signals. Second, if additionally links are perfect substitutes, then stability is as in JW under complete information.

Lemma 1.

(i) The network $g \in \mathcal{G}$ is CPS with respect to $(u_i)_{i \in N}$ and completely informative private signals $(s_i(g))_{i \in N}$ iff it is CPS under correct beliefs.

[5] We follow JW by assuming weak blocking of links, i.e., players only block a link if it would make them strictly worse off.

(ii) Suppose that links are perfect substitutes. The network $g \in \mathcal{G}$ is CPS with respect to $(u_i)_{i \in N}$ and completely informative private signals $(s_i(g))_{i \in N}$ iff $\widetilde{g}$ is PS, where

$$\widetilde{g}_{ij} = \begin{cases} 1, & \text{if } g_{ij} \neq 0 \\ 0, & \text{if } g_{ij} = 0 \end{cases} \text{ for all } i, j \in N.$$

Note also that, if links are not perfect substitutes and private signals are completely informative, then CPS is equivalent to a version of PS where agents can form two different types of links (with different utilities).

CPS is weak in the sense that it does not impose any restrictions on beliefs beyond consistency. In particular, a player's belief may support a network that is not rationalizable, that is, there do not exist beliefs such that the tuple of this network and these beliefs is CPS. We therefore introduce a refinement of CPS that imposes common knowledge of rationality à la Rubinstein and Wolinsky (1994).

Definition 7 *(Rationalizable conjectural pairwise stability).*

(i) The set of networks $G \subseteq \mathcal{G}$ is *rationalizable* with respect to $(u_i)_{i \in N}$ and $(s_i)_{i \in N}$ if for all $g \in G$ there exist beliefs $(\mu_i)_{i \in N}$ such that

  (a) the tuple $(g, (\mu_i)_{i \in N})$ is CPS, and
  (b) for all $i \in N$ and all $g'_{-i} \in \mathcal{G}_{-i}$ such that $\mu_i(g'_{-i}) > 0$, $(g_i, g'_{-i}) \in G$.

(ii) The network $g \in \mathcal{G}$ is *rationalizable conjectural pairwise stable (RCPS)* with respect to $(u_i)_{i \in N}$ and $(s_i)_{i \in N}$ if there exists a rationalizable set $G \subseteq \mathcal{G}$ such that $g \in G$.

First, a set of networks is rationalizable if for each network in the set, there exist beliefs such that the tuple of network and beliefs is CPS. In addition, the support of these beliefs needs to be restricted to this set. Second, a network is RCPS if it is contained in a rationalizable set. In other words, a network is RCPS if there exist beliefs such that the tuple of network and beliefs is CPS, and common knowledge of rationality requires that the same holds for each network supported by these beliefs.

The following example illustrates our concept in the context of perfect substitutes and shows that a network may be RCPS although it would not be stable under complete information.

Example 1. Consider $n = 3$ players and that public links and shadow links are perfect substitutes with payoffs given in Fig. 1. The private signals do not contain additional information. Without shadow links the set of PS networks consists of the "line" networks g^5, g^6 and g^7.

First, note that rationalizability refines the set of stable networks. The version of g^2 in which players 1 and 2 are connected with a shadow link, $g^{2,s}$, is CPS under correct beliefs for these players and a belief for player 3 that assigns more than weight 2/3 to the empty network g^1 ($\mu_3(g^1_{-3}) > 2/3$ and $\mu_3(g^{2,s}_{-3}) = 1 - \mu_3(g^1_{-3})$). However, $g^{2,s}$ is not RCPS, as the belief of player 3 is not rationalizable.

Second, we show that g^8 is RCPS if all links are shadow links. Consider the set $G = \{g^{i+4,s} \text{ for all } i \in N, g^{8,s}\}$ shown in Fig. 2 and note that $g^{i+4,s}$ is CPS under correct beliefs since g^{i+4} is PS under complete information (Lemma 1) for all $i \in N$. Furthermore, consider $g^{8,s}$ and the following beliefs:

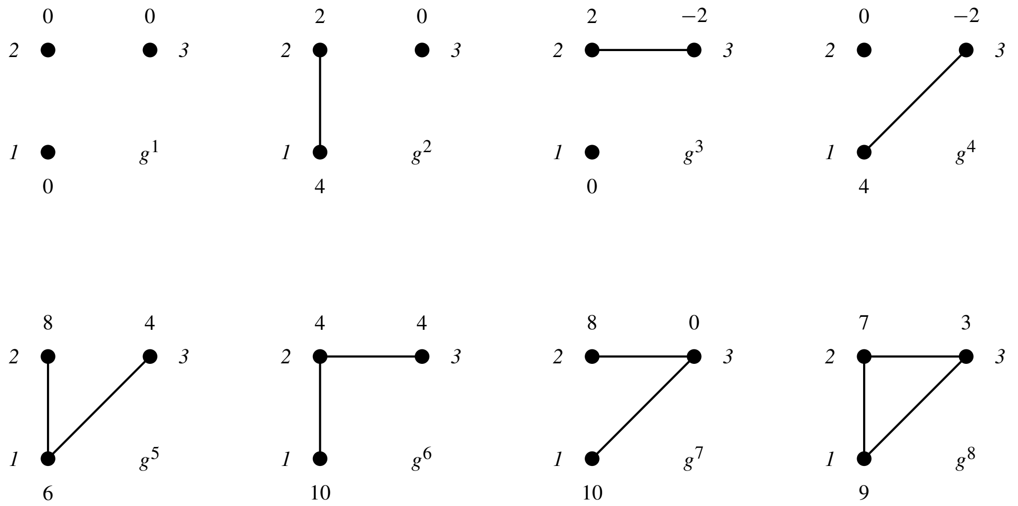

Fig. 1. Payoffs in Example 1. Links are perfect substitutes such that each link in the depicted networks can be either a public link or a shadow link. Player names are given in italics.

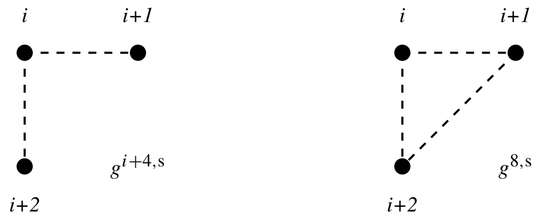

Fig. 2. The rationalizable set G in Example 1. Shadow links are indicated by dashed lines. Player names are given in italics and payoffs are omitted.

$$\mu_i(g_{-i}^{i+4,\mathrm{s}}) \geq 1/2 \text{ and } \mu_i(g_{-i}^{8,\mathrm{s}}) = 1 - \mu_i(g_{-i}^{i+4,\mathrm{s}}) \text{ for all } i \in N.$$

Each player i conjectures that she is in the center of a line network ($g^{i+4,\mathrm{s}}$) with at least probability $1/2$, and otherwise in the actual network $g^{8,\mathrm{s}}$. These beliefs are consistent with the private signals as all links are shadow links, and no player wants to add or sever a link. Therefore, all networks in G are CPS under beliefs with support on G, which implies that G is rationalizable and $g^{8,\mathrm{s}}$ RCPS. Note that objective social welfare in $g^{8,\mathrm{s}}$ (19) is higher than welfare in the line networks g^5, g^6 and g^7 under complete information (18), while subjective social welfare is lower under beliefs that are rationalizable (at most 14.5).

3.1. Equilibrium analysis

We now investigate equilibria in the general model. We say that a player does not object a network if she does not want to alter any of her links unilaterally.

Definition 8 *(No objection).* We say that player $i \in N$ *does not object* the network $g \in \mathcal{G}$ if for all $j \in N$, $j \neq i$, and $t \in \{0, 1, 2\} \backslash \{g_{ij}\}$, $u_i(g) \geq u_i(g(ij \to t))$.

The next lemma says that we can find beliefs such that the tuple of the actual network and these beliefs is CPS if for each player there is a network that she does not object and that does not contradict her signal.

Lemma 2. *There exist beliefs $(\mu_i)_{i\in N}$ such that the tuple $(g,(\mu_i)_{i\in N})$ is CPS with respect to $(u_i)_{i\in N}$ and $(s_i)_{i\in N}$ if for all $i \in N$ there exists $g^i_{-i} \in \mathcal{G}_{-i}$ such that i does not object (g_i, g^i_{-i}) and $s_i(g_i, g^i_{-i}) = s_i(g)$.*

Proof. Consider any $i \in N$ and the belief $\mu_i(g^i_{-i}) = 1$. By assumption, this belief is consistent with $s_i(g)$. Additionally, since i does not object (g_i, g^i_{-i}), we have that for all $j \in N$, $j \neq i$, and $t \in \{0,1,2\}\backslash\{g_{ij}\}$,

$$U_i(g_i,\mu_i) = u_i(g_i, g^i_{-i}) \geq u_i((g_i, g^i_{-i})(ij \to t)) = U_i((g(ij \to t))_i, \mu_i),$$

i.e., the tuple $(g,(\mu_i)_{i\in N})$ is CPS, which finishes the proof. □

A sufficient condition for a network to be RCPS that follows directly from Definition 7 is that it is CPS under correct beliefs, because in this case the set that only contains this network is rationalizable.

Lemma 3. *The network $g \in \mathcal{G}$ is RCPS with respect to $(u_i)_{i\in N}$ and $(s_i)_{i\in N}$ if it is CPS under correct beliefs.*

Note that, if links are perfect substitutes, this implies together with Lemma 1 that there is an RCPS network if there is a PS network under complete information.

To obtain a weaker condition, the next result builds on Lemma 2 and shows that for a network to be RCPS, it is sufficient that there exist beliefs such that each player conjectures to be in a network that is CPS under correct beliefs, provided that she does not object this network and that it does not contradict her signal. From the players' point of view, the network appears stable in the sense that it would be stable if the own belief was correct and shared by everyone else.

Proposition 1. *The network $g \in \mathcal{G}$ is RCPS with respect to $(u_i)_{i\in N}$ and $(s_i)_{i\in N}$ if for all $i \in N$ there exists $g^i_{-i} \in \mathcal{G}_{-i}$ such that*

(i) *i does not object (g_i, g^i_{-i}),*
(ii) *$s_i(g_i, g^i_{-i}) = s_i(g)$, and*
(iii) *(g_i, g^i_{-i}) is CPS under correct beliefs.*

Proof. Consider the set $G = \{g, (g_1, g^1_{-1}), (g_2, g^2_{-2}), \dots, (g_n, g^n_{-n})\}$ and suppose that g^i_{-i} is as desired for all $i \in N$; notice that networks (g_i, g^i_{-i}) and (g_j, g^j_{-j}), $i \neq j$, may be identical. It is left to show that G is rationalizable.

Consider first network g. It follows from (i) and (ii) and Lemma 2 that $(g,(\mu_i)_{i\in N})$ is CPS, where $\mu_i(g^i_{-i}) = 1$ for all $i \in N$. Furthermore, the second condition in Definition 7 (i) is fulfilled as $(g_i, g^i_{-i}) \in G$ for all $i \in N$. Second, consider network (g_i, g^i_{-i}), for any $i \in N$. Condition (iii) implies that both conditions in Definition 7 (i) are fulfilled for correct beliefs, which finishes the proof. □

Note that, if links are perfect substitutes, then condition (iii) means players conjecture to be in a network that would be PS under complete information (Lemma 1). We now build on Proposition 1 and derive a mechanism to construct an RCPS network that is not stable under correct beliefs. To do so, consider a network that is not stable under correct beliefs and suppose each player conjectures to be in a network that is stable under correct beliefs. We know from Proposition 1 that the given network is RCPS if no player objects the network she conjectures to be in. Otherwise, we need to check whether any two players who object have incentives to form or switch to a shadow link or a public link.[6] If not, the given network is RCPS. Otherwise, it is not RCPS, and we can repeat the procedure with a different network and/or different conjectured networks.

Corollary 1. *Fix $(u_i)_{i\in N}$ and $(s_i)_{i\in N}$ and let G^{CB} denote the collection of networks that are CPS under correct beliefs. The following mechanism yields an RCPS network $g \notin G^{CB}$ that is CPS under degenerate beliefs over G^{CB}, if such a network exists:*

1. *Take any network $g \notin G^{CB}$ and, for each $i \in N$, a network $g^i_{-i} \in \mathcal{G}_{-i}$ such that $(g_i, g^i_{-i}) \in G^{CB}$ and $s_i(g_i, g^i_{-i}) = s_i(g)$.*

2a. *If each player i does not object (g_i, g^i_{-i}), then g is as desired. Otherwise, go to Step 2b.*

2b. *If any two distinct players i and j who object (g_i, g^i_{-i}) and (g_j, g^j_{-j}), respectively, do not have incentives to form or switch to a shadow link or a public link given $\mu_i(g^i_{-i}) = 1$ and $\mu_j(g^j_{-j}) = 1$, respectively, then g is as desired. Otherwise, go to Step 3.*

3. *Go back to Step 1 and repeat the procedure with another combination of networks. If no combination with the required properties is left, the mechanism terminates unsuccessfully.*

Proof. Consider any combination of networks $g \notin G^{\mathrm{CB}}$ and $g^i_{-i} \in \mathcal{G}_{-i}$ for each $i \in N$ such that the condition in Step 1 is fulfilled, i.e., $(g_i, g^i_{-i}) \in G^{\mathrm{CB}}$ and $s_i(g_i, g^i_{-i}) = s_i(g)$ for all $i \in N$. If the condition in Step 2a is fulfilled, it follows from Lemma 2 that g is CPS under degenerate beliefs $\mu_i(g^i_{-i}) = 1$ for all $i \in N$, and from Proposition 1 that g is RCPS.

Otherwise, if the condition in Step 2a is not fulfilled, consider the set $G = \{g, (g_1, g^1_{-1}), (g_2, g^2_{-2}), \dots, (g_n, g^n_{-n})\}$ and degenerate beliefs $\mu_i(g^i_{-i}) = 1$ for all $i \in N$. Players do not have incentives to delete a link, as they conjecture to be in a network that is CPS under correct beliefs. Hence, g is CPS under these beliefs if any two distinct players i and j do not have incentives to form or switch to a shadow link or a public link. In particular, forming or switching to a link requires them to object (g_i, g^i_{-i}) and (g_j, g^j_{-j}), respectively. Thus, if the condition in Step 2b is fulfilled, g is CPS under degenerate beliefs over G^{CB}. In this case, G is rationalizable and g RCPS, as $(g_i, g^i_{-i}) \in G^{\mathrm{CB}}$ for all $i \in N$.

Finally, note that the procedure terminates after finitely many iterations and iterates, for each candidate network $g \notin G^{\mathrm{CB}}$, over all consistent and degenerate beliefs over G^{CB}, which finishes the proof. □

Note that Step 2a serves to illustrate the connection to Proposition 1, it is not essential to the mechanism and may hence be skipped. To illustrate these results, we now revisit Example 1.

[6] Note that no player has incentives to delete a link, as the conjectured networks are CPS under correct beliefs.

Example 2. Consider $n = 3$ players and the payoffs and private signals from Example 1. Recall that without shadow links the set of PS networks consists of the line networks g^5, g^6 and g^7, and that $g^{8,s}$ (g^8 with all shadow links) is RCPS. We now derive this result applying the mechanism described in Corollary 1. Note that the set $G^{\text{CB}} = \{g \mid \widetilde{g} \in \{g^5, g^6, g^7\}\}$ contains any "version" of the PS networks g^5, g^6 and g^7 (Lemma 1); in particular, any network $g \in G^{\text{CB}}$ is RCPS (Lemma 3). Now, consider the network $g^{8,s} \notin G^{\text{CB}}$ together with the conjectured networks $g^i_{-i} = g^{i+4,s}_{-i}$ for all $i \in N$ shown in Fig. 2. It is straightforward to verify that these networks satisfy the conditions in Step 1 and 2a of the mechanism described in Corollary 1, implying that $g^{8,s}$ is RCPS. Alternatively, one could also verify that the conjectured networks satisfy the conditions in Proposition 1.

Finally, we investigate how the set of stable networks changes with the informativeness of the agents' private signals. Consider distance-K confirming private signals, such that the observation radius K measures informativeness. We show that decreasing the observation radius "coarsens" RCPS. Suppose that the network g is RCPS, then there exists a rationalizable set $G \ni g$. Now, decreasing the observation radius implies less restrictions on beliefs to be consistent with the private signals. Hence, G is still rationalizable, and g RCPS. This proves the following result. Let RCPS(K) denote the set of RCPS networks with respect to $(u_i)_{i\in N}$ and distance-K confirming private signals.

Proposition 2. *Consider distance-K confirming private signals. Then the set of RCPS networks with respect to $(u_i)_{i\in N}$ is (weakly) shrinking in K, i.e., RCPS$(K') \subseteq$ RCPS(K) if $K' > K$.*

To illustrate Proposition 2, note that the network $g^{8,s}$ in Example 1 is not RCPS any more if we increase the observation radius from 0 to 1. We know that under complete information, a PS network may not exist, and the same holds for RCPS with completely informative private signals. The next example shows that an RCPS network may only exist when the observation radius is small enough.

Example 3. Consider $n = 5$ players and that public links and shadow links are perfect substitutes with payoffs given in Fig. 3. The private signals are distance-K confirming. Suppose further that there exists an improving path (under complete information) from any other network to one of the networks depicted in Fig. 3.[7]

Then there exists an RCPS network iff $K = 0$. In this case, the networks $g^{4,1}$ and $g^{4,2}$ shown in Fig. 4 are RCPS. To see this, first set $G = \{g^{4,1}, g^{4,2}\}$. Second, take $g^{4,1}$ and assign correct beliefs to everyone except the player in the center, i.e., $\mu_i(g^{4,1}_{-i}) = 1$ for $i \neq 1$. To player 1 we assign the belief that she is in $g^{4,2}$, $\mu_1(g^{4,2}_{-1}) = 1$. Under these beliefs, $g^{4,1}$ is CPS. Note that the belief of player 1 would not be consistent if $K > 0$. Analogously, also $g^{4,2}$ is CPS under beliefs with support on G. Therefore, G is rationalizable and its networks RCPS.

[7] An *improving path (under complete information)* from a network $g \in \mathcal{G}^{\text{pub}}$ to a network $g' \neq g$ is a finite sequence of networks $g = g_1, g_2, \ldots, g_K = g'$ such that for all $k = 1, 2, \ldots, K-1$ either (i) $g_{k+1} = g_k(ij \to 0)$ for some distinct $i, j \in N$ such that $u_i(g_{k+1}) > u_i(g_k)$ or $u_j(g_{k+1}) > u_j(g_k)$, or (ii) $g_{k+1} = g_k(ij \to 1)$ for some distinct $i, j \in N$ such that $u_i(g_{k+1}) > u_i(g_k)$ and $u_j(g_{k+1}) \geq u_j(g_k)$.

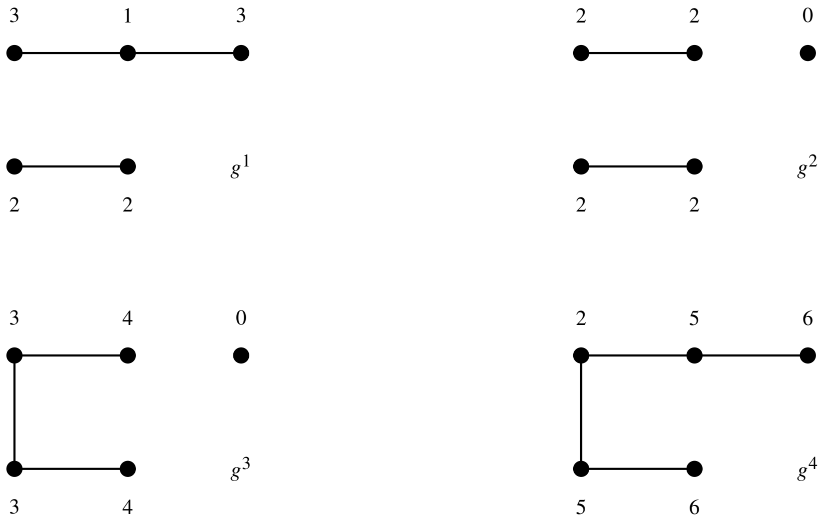

Fig. 3. Payoffs in Example 3 (up to permutations of the players). Links are perfect substitutes such that each link in the depicted networks can be either a public link or a shadow link.

Fig. 4. The rationalizable set G in Example 3. Public links are indicated by solid lines and shadow links by dashed lines. Player names are given in italics. Payoffs are omitted.

4. Two specific models

We propose two specific models of network formation with shadow links. These models illustrate different ways how shadow links can enrich the framework. The first model considers friendship networks with homophily and perfect substitutes. Second, we study a connections model in which shadow links represent weak tie relationships and public links represent strong tie relationships.

4.1. Segregation and homophily in friendship networks

Our social identity is determined by all the social groups or communities that we belong to. Communities may be defined along categories such as language, religion, education, political ideology, support of sports teams, etc. The degree of identification with the communities we belong to likely affects our social interaction with people from other communities. In particular, people may be homophilic in the sense that they do not even want to indirectly associate with strong adherents of another community, e.g., because this may result in conflict/unpleasant arguments during occasional encounters at social events organized by common friends.

To capture this, we consider a variation of the connections model of JW in which agents belong to different communities similar to de Martí and Zenou (2017).[8] Additionally, we introduce moderate and partisan types. Individuals generally benefit from direct and indirect connections to others, which can be interpreted as benefits from personal interactions (meeting/socializing/discussing with friends, etc.). These benefits decay with distance, reflecting that indirect connections' influence is through friends. However, connections to partisans of the other group are less beneficial or even detrimental. Forming links is costly, reflecting the time necessary to maintain friendships. We further assume that links are perfect substitutes. Here, shadow links may serve the purpose of concealing relations with partisans of the own group, to not deter the possibility of connections to moderates of the other group.

Formally, each agent $i \in N$ belongs to a *community (or group)* $\gamma_i \in \{A, B\}$ and is of *type* $\theta_i \in \{m, p\}$, where $\theta_i = m$ if i is a *moderate* and $\theta_i = p$ if i is a *partisan*. We denote the set of agents who belong to community γ by $N_\gamma = \{i \in N \mid \gamma_i = \gamma\}$, the subset of type θ of this group by $N_\gamma^\theta = \{i \in N_\gamma \mid \theta_i = \theta\}$, and the cardinalities by $n_\gamma = \#N_\gamma$ and $n_\gamma^\theta = \#N_\gamma^\theta$, respectively. Preferences are given by:

$$u_i(g) = \sum_{j \in N_{\gamma_i} \setminus \{i\}} \delta^{t_{ij}(g)} + \sum_{j \notin N_{\gamma_i}} \phi_{ij} \cdot \delta^{t_{ij}(g)} - c \cdot d_i(g),$$

where $0 < \delta < 1$ captures the idea that the value of a connection is proportional to proximity, $c > 0$ denotes the cost of forming a link incurred by each of the players, and ϕ_{ij} captures homophily. In particular, we assume that $\phi_{ij} = 1$ if $\theta_j = m$, $\phi_{ij} = \overline{\phi} < c/\delta$ if $\theta_i = m$ and $\theta_j = p$, and $\phi_{ij} = \underline{\phi} < \overline{\phi}$ if $\theta_i = \theta_j = p$. That is, interaction with partisans of the other group is less beneficial, both for moderates and even more so for partisans. We further assume that $c < \delta - \delta^2$, so that agents prefer to be directly connected to everyone within their group in absence of connections across groups. Moreover, the private signals do not contain additional information. Note that we recover the classical version of the connections model studied in JW if all agents belong to the same community, $N = N_A$. To avoid some special cases, we consider $n_A \neq n_B$ and $n_\gamma^\theta \geq 1$ for all $\gamma \in \{A, B\}$ and $\theta \in \{m, p\}$, such that the communities are not of equal size and consist of agents of both types. Finally, we employ a refinement of our solution concept in which agents may sever multiple links at the same time, which avoids that they maintain harmful (indirect) connections to partisans of the other group in equilibrium.

We begin our analysis by studying stable networks under complete information. We first show that both communities may stay segregated if homophily (even of moderates) is large (low $\overline{\phi}$). In this case, moderates refrain from connections to moderates of the other group because such connections would imply indirect connections to partisans of that group. On the other hand, there exists an equilibrium in which both groups and the moderates are each fully intraconnected, i.e., any pair of agents from one of the groups or the moderates is directly connected, if homophily is rather small (high $\underline{\phi}$). Finally, we show that the partisans of one group may stay segregated from the rest of society, while the other group is fully intraconnected, if homophily of partisans is large (low $\underline{\phi}$) but that of moderates is small (high $\overline{\phi}$). Those partisans then refrain from connecting to the moderates of their own group to avoid indirect connections to partisans of the other group.

8 See also Bjerre-Nielsen (2020) for a related model of network formation with multiple types.

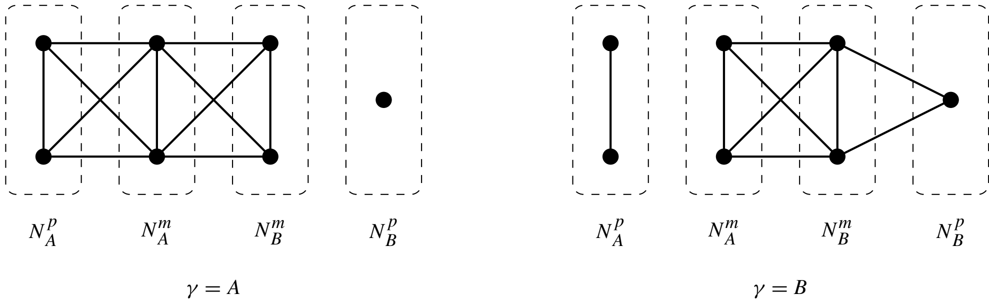

Fig. 5. PS networks in Proposition 3 (iii) for $n = 7$.

Proposition 3.

(i) The network g in which the agents in each of the sets N_A and N_B are fully intraconnected and no other links are present is PS iff

$$\overline{\phi} < \max_{\gamma} \frac{c - \delta - (n_\gamma^m - 1)\delta^2}{n_\gamma^p \delta^2}.$$

Otherwise, any PS network is such that the sets N_A and N_B are (partly) connected.

(ii) The network g in which the agents in each of the sets $N_A^m \cup N_B^m$, N_A and N_B are fully intraconnected and no other links are present is PS iff

$$\overline{\phi} \geq \max_{\gamma} \frac{n_\gamma^m (c - \delta)}{n_\gamma^p \delta^2} \text{ and } \underline{\phi} \geq \max_{\gamma \neq \gamma'} \frac{n_\gamma^m (c - \delta) - n_{\gamma'}^m \delta^2}{n_{\gamma'}^p \delta^3}.$$

(iii) The network g in which the agents in each of the sets $N_A^m \cup N_B^m$, N_γ and $N_{\gamma'}^p$, with $\gamma, \gamma' \in \{A, B\}$ such that $\gamma \neq \gamma'$, are fully intraconnected and no other links are present is PS iff

$$\overline{\phi} \geq \frac{n_\gamma^m (c - \delta)}{n_\gamma^p \delta^2} \text{ and } \underline{\phi} < \frac{c - \delta - (n_A^m + n_B^m - 1)\delta^2}{n_\gamma^p \delta^3}.$$

Proof. See Appendix A. □

Note that the upper bound on $\underline{\phi}$ in part (iii) is negative. Fig. 5 illustrates the stable networks in this case. Next, we consider shadow links and perfect substitutes. We now refer to a set of players as fully intraconnected if each pair of players from the set is connected by either a public link or a shadow link. We build on Proposition 3 (iii) and derive conditions such that (some) partisans of both groups stay segregated from the moderates, because they falsely believe that the partisans of the other group are connected to the moderates. In particular, some partisans overestimate others' connections and hence underconnect (relative to PS networks), while others underestimate connections and hence overconnect.

Corollary 2. *For any $\hat{N}_A^p \subseteq N_A^p$ and $\hat{N}_B^p \subseteq N_B^p$, the network g in which*

(i) the agents in each of the sets $N_A^m \cup N_B^m$, N_A^p and N_B^p are fully intraconnected,
(ii) each pair $(i, j) \in \hat{N}_\gamma^p \times N_\gamma^m$, for $\gamma \in \{A, B\}$, is connected with shadow links,

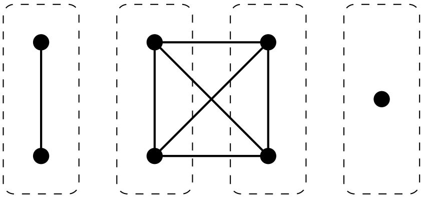

Fig. 6. RCPS network in Corollary 2 in which $\hat{N}_A^p = \hat{N}_B^p = \emptyset$, for $n = 7$. Public links are indicated by solid lines and shadow links by dashed lines.

and no other links are present is RCPS iff

$$\overline{\phi} \geq \max_{\gamma} \frac{n_\gamma^m (c-\delta)}{n_\gamma^p \delta^2} \text{ and } \underline{\phi} < \frac{c - \delta - (n_A^m + n_B^m - 1)\delta^2}{\min\{n_A^p, n_B^p\}\delta^3}.$$

Proof. See Appendix A. □

Corollary 2 shows that, if homophily of partisans is large but that of moderates is small, part of the partisans of both groups may stay segregated from the moderates, because they suspect the partisans of the respective other group to be connected to the moderates. Those partisans falsely believe that all partisans of the respective other group maintain shadow links to the moderates – they overestimate others' links and hence underconnect. Conversely, the partisans who are connected to the moderates with shadow links falsely believe that all partisans of the respective other group are segregated from the moderates – they underestimate others' links and hence overconnect. Thus, all partisans believe to be in a network whose complete information equivalent network is PS (a version of Proposition 3 (iii)) and have no incentives to add or cut links to the moderates. The moderates hold correct beliefs and also have no incentives to sever any links. These beliefs are rationalizable and the network hence RCPS. Note that this requires links between partisans and moderates to be shadow links.

Remark 1. Corollary 2 includes the network in which both groups of partisans are entirely segregated from the moderates and all links are public links as a special case ($\hat{N}_A^p = \hat{N}_B^p = \emptyset$), see Fig. 6 for an illustration. Note first that all partisans overestimate others' links and hence underconnect. Second, due to segregation, expected and actual payoffs nevertheless coincide. Third, this network is not PS under complete information, as moderates and partisans of each group would have incentives to add links between each other.

Finally, we establish that entire segregation of both groups of partisans reduces social welfare, as compared to the case when one group of partisans is connected to the moderates (Proposition 3 (iii)), if homophily of moderates is not too large. Note that subjective and objective social welfare coincide under the beliefs that we employ above to rationalize the network with segregation of both groups. We therefore restrict attention to objective social welfare.

Proposition 4. *Suppose that the agents in each of the sets $N_A^m \cup N_B^m$, N_A^p and N_B^p are fully intraconnected and no other links are present. Objective social welfare is strictly higher when additionally also N_A is fully intraconnected iff*

$$\overline{\phi} > \frac{2n_A^m(c-\delta)}{n_B^m\delta^2} - 1.$$

Proof. See Appendix A. □

Hence, false yet rationalizable beliefs may lead to segregation of partisans from moderates and lower social welfare under incomplete information about the network structure if *only* homophily of partisans is large.

4.2. A connections model with weak and strong ties

Consider agents who may form weak and strong tie relationships. Both types of relationship provide discounted benefits from indirect connections to other agents, e.g., information about relevant employment opportunities. Additionally, strong ties provide other direct benefits, e.g., personal recommendations or forwarding of applications. However, the value of a strong tie depends on its exclusivity: a personal recommendation becomes less helpful the more people that person recommends. Hence, while weak ties are only associated with positive externalities, strong ties are also associated with negative externalities. We represent weak ties by shadow links and strong ties by public links. This reflects the idea that a strong tie is easier to observe than a weak tie, e.g., because people in a strong business relationship often interact in meetings or over lunch. We hence also assume that strong ties are more costly to maintain than weak ties.[9]

Our model combines the connections model of JW with the degree-based model of Morrill (2011); see also Möhlmeier et al. (2016) for a closely related model under complete information. Preferences are given by:

$$u_i(g) = \sum_{j\neq i} \delta^{t_{ij}(g)} + \sum_{j:g_{ij}=1} \frac{1}{d_j^{\text{pub}}(g)} - c^{\text{pub}} \cdot d_i^{\text{pub}}(g) - c^{\text{sha}} \cdot d_i^{\text{sha}}(g),$$

where $0 < \delta < 1$ captures discounted benefits from indirect connections, and $c^{\text{pub}} > c^{\text{sha}} > 0$ denote the costs of forming a public link and a shadow link, respectively, incurred by each of the players. Links are thus not perfect substitutes. Moreover, the private signals do not contain additional information. To simplify the analysis, we assume $n \geq 4$, $c^{\text{sha}} = c^{\text{pub}}/2$ and consider intermediate linking costs, $\delta > c^{\text{sha}} > 1/2$.

We focus on low decay of indirect benefits, such that – reminiscent of the star networks in the connections model of JW – networks in which pairs of strong ties maintain weak ties to a central agent will play a central role in our analysis. Let $\lfloor x \rfloor$ denote the greatest integer less or equal to x.

Definition 9 *(Star with peripheral pairs).* A network g is a *star with peripheral pairs* if

(i) there exists $i \in N$ such that $d_i^{\text{sha}}(g) = \lfloor \frac{n-1}{2} \rfloor$ (the *center*),
(ii) $d_j^{\text{pub}}(g) = 1$ for all (except one, if n is odd) $j \in N$, and
(iii) $g_{jk} \neq 0$ for any $j, k \neq i$ implies that $g_{jk} = 1$ and ($g_{ij} = 2$ or $g_{ik} = 2$, but not both).

[9] Note that otherwise agents would only form strong ties.

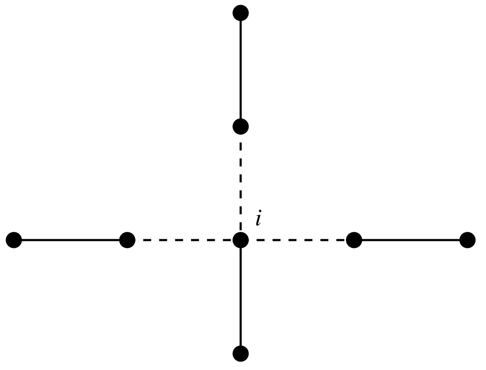

Fig. 7. Star with peripheral pairs and center i for $n = 8$. Public links are indicated by solid lines and shadow links by dashed lines.

Note that a star with peripheral pairs is a tree, see Fig. 7 for an illustration. We first investigate efficient networks. Let

$$F(\delta, n) \equiv \delta(1-\delta)\left(1 + (1 + 1_{\{n \text{ odd}\}})\delta + \left\lfloor \frac{n}{2} - 2 \right\rfloor (\delta + \delta^2)\right),$$

with $1_{\{n \text{ odd}\}} = 1$ if n is odd and $1_{\{n \text{ odd}\}} = 0$ otherwise, denote the total gain in benefits from adding another shadow link that involves the center, and note that $F(\delta, n) \to 0$ as $\delta \to 1$. We show that, if decay is low enough (relative to costs), then stars with peripheral pairs uniquely maximize objective social welfare. First, in the cost range that we consider, each player will maintain one strong tie (if possible). Second, similar to the connections model of JW, connecting these peripheral pairs via a center agent is efficient if decay is low enough.

Proposition 5. *Let $\bar{\delta}(c^{sha}, n) \in (1/2, 1)$ denote the smallest value of δ such that*

$$1 + \delta(1-\delta) - F(\delta, n) \geq c^{sha} \geq F(\delta, n).$$

If $\delta \geq \bar{\delta}(c^{sha}, n)$, then g maximizes objective social welfare iff g is a star with peripheral pairs.

Proof. See Appendix A. □

Next, we investigate stable networks under correct beliefs, which are also RCPS (Lemma 3). Note that, as above, each player will maintain one public link (if possible). We first show that any tree with this property is stable if decay is low enough relative to costs, because in this case adding additional shadow links to reduce the decay does not pay off. Second, we determine the weaker condition under which stars with peripheral pairs are stable.

Proposition 6.

(i) Any tree g such that $d_i^{pub}(g) = 1$ for all (except one, if n is odd) $i \in N$ is CPS under correct beliefs if

$$c^{sha} \geq \left(1 - \delta^{\lfloor n/2 \rfloor}\right) \frac{\delta - \delta^{\lfloor (n+1)/2 \rfloor}}{1-\delta}.$$

(ii) *A star with peripheral pairs g is CPS under correct beliefs iff*

$$c^{sha} \geq (1-\delta^2)(\delta + 1_{\{n\geq 6\}}\delta^2).$$

Proof. (i) Consider a tree g such that $d_i^{\text{pub}}(g) = 1$ for all (except one, if n is odd) $i \in N$ and suppose that beliefs are correct. Note first that no player has an incentive to cut a link, as $1 > \delta > c^{\text{sha}}$. Furthermore, players also have no incentives to switch from a public link to a shadow link or vice versa, as $1 > c^{\text{sha}} > 1/2$. For any agent, the gain from adding a shadow link is at most

$$2\sum_{k=1}^{\lfloor (n-1)/2 \rfloor} \delta^k + 1_{\{n \text{ even}\}}\delta^{n/2} - c^{\text{sha}} - \sum_{k=1}^{n-1}\delta^k. \tag{1}$$

Note that, for at least one agent in any pair of agents, the gain from adding a public link is strictly lower than (1), as the other agent already maintains one public link and $c^{\text{sha}} > 1/2$. Hence, g is CPS under correct beliefs if

$$2\sum_{k=1}^{\lfloor (n-1)/2 \rfloor} \delta^k + 1_{\{n \text{ even}\}}\delta^{n/2} - c^{\text{sha}} - \sum_{k=1}^{n-1}\delta^k \leq 0 \Leftrightarrow c^{\text{sha}} \geq \left(1-\delta^{\lfloor n/2 \rfloor}\right)\sum_{k=1}^{\lfloor (n-1)/2 \rfloor}\delta^k$$
$$\Leftrightarrow c^{\text{sha}} \geq \left(1-\delta^{\lfloor n/2 \rfloor}\right)\frac{\delta - \delta^{\lfloor (n+1)/2 \rfloor}}{1-\delta}.$$

(ii) Consider a star with peripheral pairs g and suppose that beliefs are correct. Analogously to part (i), agents have no incentives to cut a link or switch from a public link to a shadow link or vice versa. Since the center agent has no incentives to add a shadow link with a peripheral agent, the additional shadow link that is most beneficial to both agents is between two agents at distance two (one agent at distance one and one at distance two, if $n < 6$) from the center. Adding this link is not beneficial iff

$$\delta^3 + 1_{\{n\geq 6\}}\delta^4 \geq \delta + 1_{\{n\geq 6\}}\delta^2 - c^{\text{sha}} \Leftrightarrow c^{\text{sha}} \geq (1-\delta^2)(\delta + 1_{\{n\geq 6\}}\delta^2). \tag{2}$$

Analogously to part (i), the gain from adding a public link is strictly lower than (2), which finishes the proof. □

Since $F(\delta, n) \geq (1-\delta^2)(\delta + 1_{\{n\geq 6\}}\delta^2)$, a star with peripheral pairs is stable under correct beliefs whenever it is efficient:

Corollary 3. *If $\delta \geq \bar{\delta}(c^{sha}, n)$, then a star with peripheral pairs g is CPS under correct beliefs.*

Next, we show that agents may both over- and underestimate others' relationships in equilibrium. If stars with peripheral pairs are stable under correct beliefs, then a network is RCPS as long as each player's signal does not contradict being in such a network.

Corollary 4. *Any network g such that $d_i^{pub}(g) = 1$ for all (except one, if n is odd) $i \in N$, and $d_i^{sha}(g) \in \{0, 1, \lfloor \frac{n-1}{2} \rfloor\}$ if $d_i^{pub}(g) = 1$ and $d_i^{sha}(g) = 1$ otherwise, is RCPS if*

$$c^{sha} \geq (1-\delta^2)(\delta + 1_{\{n\geq 6\}}\delta^2).$$

Proof. Suppose that $c^{\text{sha}} \geq (1-\delta^2)(\delta + 1_{\{n\geq 6\}}\delta^2)$ and fix any network g such that $d_i^{\text{pub}}(g) = 1$ for all (except one, if n is odd) $i \in N$, and $d_i^{\text{sha}}(g) \in \{0, 1, \lfloor \frac{n-1}{2} \rfloor\}$ if $d_i^{\text{pub}}(g) = 1$ and $d_i^{\text{sha}}(g) = 1$ otherwise. There exists a star with peripheral pairs $g^i \in \mathcal{G}$ such that $(g_i, g^i_{-i}) = g^i$ for all $i \in N$. Note that we can choose these networks such that if $d_i^{\text{sha}}(g) = d_j^{\text{sha}}(g) = 0$ for two distinct agents $i, j \in N$ and $d_j^{\text{sha}}(g^i) = \lfloor \frac{n-1}{2} \rfloor$ (j is the center in g^i), then $d_i^{\text{sha}}(g^j) \in \{0, 1\}$ (i is not the center in g^j).

Next, consider the beliefs $\mu_i(g^i_{-i}) = 1$ for all $i \in N$. First, analogously to the proof of Proposition 6 (ii), agents have no incentives to cut a link or switch from a public link to a shadow link or vice versa under these beliefs. Second, the only agents who may have incentives to add a shadow link under these beliefs are agents i such that $d_i^{\text{sha}}(g) = 0$, with the center agent j in g^i. However, by construction, i is not the center in g^j, and hence agent j has no incentives to add this link. Third, for at least one agent in any pair of agents, the gain from adding a public link is strictly lower than that from adding a shadow link, as the other agent already maintains one public link and $c^{\text{sha}} > 1/2$. Hence, the tuple $(g, (\mu_i)_{i\in N})$ is CPS. Moreover, since g^i is CPS under correct beliefs for all $i \in N$ (Proposition 6 (ii)), the set $G = \{g, g^i \text{ for all } i \in N\}$ is rationalizable and g RCPS, which finishes the proof. □

Hence, on the one hand, agents may overestimate others' relationships in equilibrium and not want to add any shadow links even though no such links are present. On the other hand, they may underestimate others' relationships, such that two (or more) agents believe to be the center agent and maintain shadow links to about half of the other agents. The following example illustrates these results.

Example 4. Consider $n = 6$ players. A star with peripheral pairs g^1 is CPS under correct beliefs, and hence RCPS, iff $c^{\text{sha}} \geq (1-\delta^2)(\delta + \delta^2)$, see Fig. 8 for an illustration. Furthermore, g^1 both maximizes objective social welfare and is CPS under correct beliefs if $\delta \geq \bar{\delta}(c^{\text{sha}}, 6)$, where $\bar{\delta}(c^{\text{sha}}, 6) \in (1/2, 1)$ denotes the smallest value of δ such that

$$1 + \delta(1-\delta)(1-(1+\delta)^2) \geq c^{\text{sha}} \geq \delta(1-\delta)(1+\delta)^2.$$

The networks g^2 to g^4 shown in the figure are also RCPS if $c^{\text{sha}} \geq (1-\delta^2)(\delta+\delta^2)$ because players may conjecture to be in (a permutation of) g^1. In g^2 and g^3, all players under- and overestimate the number of shadow links other agents maintain and hence over- and underconnect (relative to stable networks under correct beliefs), respectively. Furthermore, in g^4, some players underestimate their strong ties' shadow links and overconnect, while other players overestimate their strong ties' shadow links and underconnect.

5. Conclusion

We propose a framework of network formation in which players can form public links as well as shadow links, which are generally not observed. Instead, players receive a private signal that may contain information on the shadow links or their own payoff in the network. We introduce the novel solution concept RCPS, which generalizes PS to incomplete information and shadow links. RCPS requires deterrence of pairwise deviations and rationalizability of players' beliefs à la Rubinstein and Wolinsky (1994).

We first show that a network is RCPS if there exist beliefs such that each player conjectures to be in a network that is CPS under correct beliefs, and in which she does not want to alter her

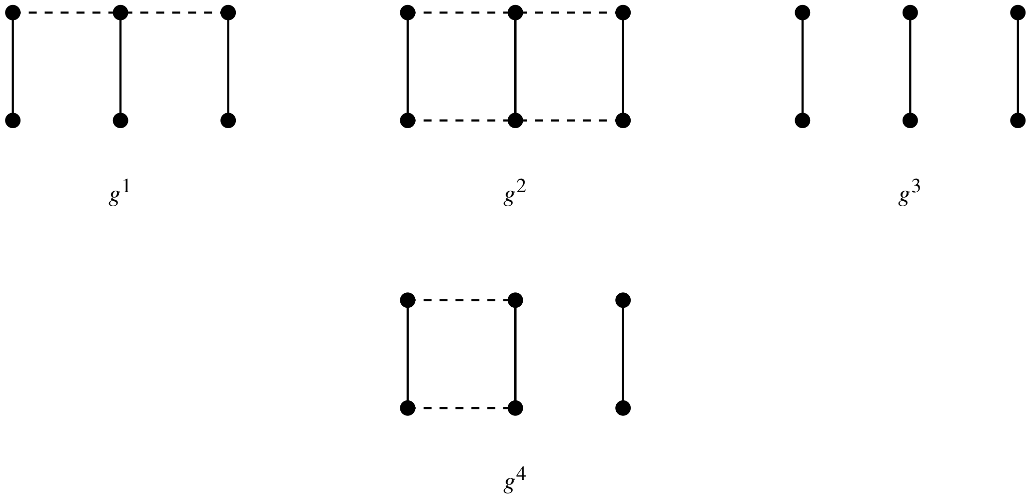

Fig. 8. RCPS networks in Example 4 for $n = 6$ and $c^{\text{sha}} \geq (1 - \delta^2)(\delta + \delta^2)$. Public links are indicated by solid lines and shadow links by dashed lines.

links unilaterally. We then derive a mechanism to construct a stable network that is not stable under correct beliefs. Furthermore, we establish that the set of RCPS networks is shrinking in the players' observation radius.

Our framework provides the foundation to model and understand richer situations of network formation. To illustrate this potential, we propose two specific models on friendship networks with homophily and on weak and strong tie relationships. We show agents may over- or underestimate others' connections and hence under- or overconnect in equilibrium, respectively.

Future research should further exploit the potential of our framework to enrich various settings of network formation. Another avenue is to extend RCPS beyond pairwise deviations and myopic players. Furthermore, experimental investigations on network formation with incomplete information and shadow links would advance our understanding of people's behavior in such situations.

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Appendix A

Proof of Proposition 3. (i) First, notice that if there are no links across groups in g, i.e., $g_{ij} = 0$ if $\gamma_i \neq \gamma_j$, and g is PS, then N_A and N_B are fully intraconnected, as $c < \delta - \delta^2$. Hence, it is left to show that there are not two agents who want to add a link. Agent $i \in N_{\gamma_i}$ strictly prefers not to add a link with $j \in N^m_{\gamma_j}$ with $\gamma_i \neq \gamma_j$ iff

$$u_i(g) > u_i(g(ij \to 1)) \Leftrightarrow (n_{\gamma_i} - 1)(\delta - c) > n_{\gamma_i}(\delta - c) + (n^m_{\gamma_j} - 1 + \overline{\phi} n^p_{\gamma_j})\delta^2$$
$$\Leftrightarrow \frac{c - \delta - (n^m_{\gamma_j} - 1)\delta^2}{n^p_{\gamma_j}\delta^2} > \overline{\phi}. \tag{3}$$

Notice that (3) only needs to hold for $\gamma_j = A$ or $\gamma_j = B$, which yields the desired upper bound on $\overline{\phi}$. As $\underline{\phi} < \overline{\phi}$, agent $i \in N_{\gamma_i}$ also has no incentives to add a link with $j \in N^p_{\gamma_j}$ with $\gamma_i \neq \gamma_j$, which finishes the first part.

(ii) Consider any $\gamma \in \{A, B\}$ and $\gamma' \neq \gamma$. Notice that all links are either between agents from the same group or between moderates. Hence, $c < \delta - \delta^2$ implies that severing a link is beneficial only if it cuts indirect connections to partisans of the other group. First, for $i \in N^m_\gamma$, this implies cutting all links to agents $j \in N^m_{\gamma'}$. Agent i has no incentives to do so iff

$$u_i(g) \geq u_i(g(ij \to 0\ \forall j \in N^m_{\gamma'}))$$
$$\Leftrightarrow (n_\gamma - 1 + n^m_{\gamma'})(\delta - c) + \overline{\phi} n^p_{\gamma'}\delta^2 \geq (n_\gamma - 1)(\delta - c)$$
$$\Leftrightarrow \overline{\phi} \geq \frac{n^m_{\gamma'}(c - \delta)}{n^p_{\gamma'}\delta^2}. \tag{4}$$

Second, for $i \in N^p_\gamma$, this implies cutting all links to agents $j \in N^m_\gamma$. Agent i has no incentives to do so iff

$$u_i(g) \geq u_i(g(ij \to 0\ \forall j \in N^m_{\gamma}))$$
$$\Leftrightarrow (n_\gamma - 1)(\delta - c) + n^m_{\gamma'}\delta^2 + \underline{\phi} n^p_{\gamma'}\delta^3 \geq (n^p_\gamma - 1)(\delta - c)$$
$$\Leftrightarrow \underline{\phi} \geq \frac{n^m_\gamma(c - \delta) - n^m_{\gamma'}\delta^2}{n^p_{\gamma'}\delta^3}. \tag{5}$$

Taking the maximum over $\gamma \in \{A, B\}$, $\gamma' \neq \gamma$, in (4) and (5) yields the desired lower bounds on $\overline{\phi}$ and $\underline{\phi}$. Furthermore, as moderates are already fully intraconnected, adding a link involves a partisan. Agent $i \in N^m_{\gamma_i}$ has no incentives to add a link with $j \in N^p_{\gamma_j}$ with $\gamma_i \neq \gamma_j$ iff

$$u_i(g) > u_i(g(ij \to 1))$$
$$\Leftrightarrow (n_{\gamma_i} - 1 + n^m_{\gamma_j})(\delta - c) + \overline{\phi} n^p_{\gamma_j}\delta^2 > (n_{\gamma_i} - 1 + n^m_{\gamma_j})(\delta - c) + \overline{\phi}\delta - c + \overline{\phi}(n^p_{\gamma_j} - 1)\delta^2$$
$$\Leftrightarrow c > \overline{\phi}(\delta - \delta^2), \tag{6}$$

which is implied by $\overline{\phi} < c/\delta$. As $\underline{\phi} < \overline{\phi}$, also agent $i \in N^p_{\gamma_i}$ has no incentives to add a link with $j \in N^p_{\gamma_j}$ with $\gamma_i \neq \gamma_j$, which finishes the second part.

(iii) Consider any $\gamma \in \{A, B\}$ and $\gamma' \neq \gamma$. Analogue to the second part, severing a link is beneficial only if it cuts indirect connections to partisans of the other group. Agent $i \in N^m_{\gamma'}$ has no incentives to cut all links to agents $j \in N^m_{\gamma}$ iff

$$\begin{aligned} & u_i(g) \geq u_i(g(ij \to 0\ \forall j \in N^m_\gamma)) \\ \Leftrightarrow & (n_{\gamma'} - 1 + n^m_\gamma)(\delta - c) + \overline{\phi} n^p_\gamma \delta^2 \geq (n_{\gamma'} - 1)(\delta - c) \\ \Leftrightarrow & \overline{\phi} \geq \frac{n^m_\gamma (c-\delta)}{n^p_\gamma \delta^2}, \end{aligned}$$

which is the desired lower bound on $\overline{\phi}$. It is left to show that there are not two agents who want to add a link. Agent $i \in N^m_{\gamma'}$ prefers to form a link with $j \in N^p_{\gamma'}$. However, j strictly prefers not to form a link with i iff

$$\begin{aligned} & u_j(g) > u_j(g(ij \to 1)) \\ \Leftrightarrow & (n^p_{\gamma'} - 1)(\delta - c) > n^p_{\gamma'}(\delta - c) + (n^m_A + n^m_B - 1)\delta^2 + \underline{\phi} n^p_\gamma \delta^3 \\ \Leftrightarrow & \frac{c - \delta - (n^m_A + n^m_B - 1)\delta^2}{n^p_\gamma \delta^3} > \underline{\phi}, \end{aligned} \tag{7}$$

which is the desired upper bound on $\underline{\phi}$. Condition (7) implies that j also has no incentives to add a link with an agent $k \in N_\gamma$. Finally, analogue to (6), $\overline{\phi} < c/\delta$ implies that $j \in N^m_{\gamma'}$ has no incentives to add a link with $i \in N^p_\gamma$, which finishes the proof. □

Proof of Corollary 2. As links are perfect substitutes, we can assume without loss of generality that g consists of shadow links only. Let g^γ, $\gamma \in \{A, B\}$, denote the network in which the agents in each of the sets $N^m_A \cup N^m_B$, N_γ and $N^p_{\gamma'}$, $\gamma' \neq \gamma$, are fully intraconnected with shadow links and no other links are present. We show that the set $G = \{g, g^A, g^B\}$ is rationalizable, which then implies that g is RCPS. Note that g^A is CPS under correct beliefs by Lemma 1 and Proposition 3 (iii) iff

$$\overline{\phi} \geq \frac{n^m_A (c-\delta)}{n^p_A \delta^2} \text{ and } \underline{\phi} < \frac{c - \delta - (n^m_A + n^m_B - 1)\delta^2}{n^p_A \delta^3}. \tag{8}$$

Analogously, g^B is CPS under correct beliefs iff

$$\overline{\phi} \geq \frac{n^m_B (c-\delta)}{n^p_B \delta^2} \text{ and } \underline{\phi} < \frac{c - \delta - (n^m_A + n^m_B - 1)\delta^2}{n^p_B \delta^3}. \tag{9}$$

Combining (8) and (9) yields the desired bounds on $\overline{\phi}$ and $\underline{\phi}$, because the nominators are negative since $c < \delta - \delta^2$.

Finally, consider g and the following beliefs: $\mu_i(g_{-i}) = 1$ if $i \in N^m_A \cup N^m_B$, $\mu_i(g^B_{-i}) = 1$ if $i \in \hat{N}^p_B \cup N^p_A \backslash \hat{N}^p_A$, and $\mu_i(g^A_{-i}) = 1$ if $i \in \hat{N}^p_A \cup N^p_B \backslash \hat{N}^p_B$. These beliefs are consistent and assign probability 1 to networks from G. Notice that partisans have no incentives to sever any links under these beliefs. Furthermore, it follows from the proof of Proposition 3 (iii) that moderates have no incentives to sever any links and that partisans $i \in N^p_A \backslash \hat{N}^p_A \cup N^p_B \backslash \hat{N}^p_B$ have no incentives to form a link with a moderate of their group if conditions (8) and (9) hold. Hence, $(g, (\mu_i)_{i\in N})$ is CPS, which finishes the proof. $\square$

Proof of Proposition 4. First, objective social welfare in the network in which the agents in each of the sets $N_A^m \cup N_B^m$, N_A and N_B^p are fully intraconnected and no other links are present is

$$\begin{aligned} & n_A^m(n_A + n_B^m - 1)(\delta - c) + n_B^m\left((n_A^m + n_B^m - 1)(\delta - c) + \overline{\phi} n_A^p \delta^2\right) \\ & + n_A^p\left((n_A - 1)(\delta - c) + n_B^m \delta^2\right) + n_B^p(n_B^p - 1)(\delta - c). \end{aligned} \tag{10}$$

Second, objective social welfare in the network in which the agents in each of the sets $N_A^m \cup N_B^m$, N_A^p and N_B^p are fully intraconnected and no other links are present is

$$\begin{aligned} & (n_A^m + n_B^m)(n_A^m + n_B^m - 1)(\delta - c) + n_A^p(n_A^p - 1)(\delta - c) \\ & + n_B^p(n_B^p - 1)(\delta - c). \end{aligned} \tag{11}$$

Finally, subtracting (11) from (10) yields

$$2n_A^m n_A^p(\delta - c) + (1 + \overline{\phi}) n_B^m n_A^p \delta^2 > 0 \Leftrightarrow \overline{\phi} > -\frac{2n_A^m(\delta - c)}{n_B^m \delta^2} - 1,$$

which finishes the proof. $\square$

Proof of Proposition 5. Observe first that $\delta > c^{\text{sha}} > 1/2$ implies that the network maximizing objective social welfare g is connected. Second, each agent will maintain at most one public link (otherwise switching one of them to a shadow link increases objective social welfare by at least $2c^{\text{sha}} - 1 > 0$). Third, if agents j and k are connected, then either $d_j^{\text{pub}}(g) = 1$ or $d_k^{\text{pub}}(g) = 1$ (otherwise switching their link to a public link increases objective social welfare by $2 - 2c^{\text{sha}} > 0$). Hence, there are at most $\lfloor \frac{n}{2} \rfloor$ public links in g, such that total direct benefits are at most $2\lfloor \frac{n}{2} \rfloor (1 - c^{\text{sha}})$.

Provided δ is large enough, g will be a tree and contain $\lfloor \frac{n}{2} \rfloor$ public links, as $c^{\text{sha}} < 1$. Note because each agent maintains at most one public link, the diameter is at least 4 (3 if $n \in \{4, 5\}$). Note also that a tree contains $n - 1$ links. It is thus left to determine how the remaining $n - 1 - \lfloor \frac{n}{2} \rfloor = \lfloor \frac{n-1}{2} \rfloor$ shadow links connect the $\lfloor \frac{n}{2} \rfloor$ pairs connected with a public link (and the singleton, if n is odd). Each public link yields a direct benefit of $2(1 + \delta)$, and a shadow link between two pairs with a public link yields $2(\delta + 2\delta^2 + \delta^3)$. An indirect connection between two pairs with a public link yields at most $2\delta(\delta + 2\delta^2 + \delta^3)$. Furthermore, if n is odd, then a shadow link between the singleton and one of the pairs yields direct and indirect benefits of at most $2(\delta + \delta^2)\left(1 + \lfloor \frac{n}{2} - 1 \rfloor \delta\right)$. Hence, total benefits are at most equal to

$$\begin{aligned} & \left\lfloor \frac{n}{2} \right\rfloor \cdot 2(1 + \delta) + \left\lfloor \frac{n}{2} - 1 \right\rfloor \cdot 2(\delta + 2\delta^2 + \delta^3) + \left\lfloor \frac{n}{2} - 1 \right\rfloor \left\lfloor \frac{n}{2} - 2 \right\rfloor \cdot \delta(\delta + 2\delta^2 + \delta^3) \\ & + 1_{\{n \text{ odd}\}} \cdot 2(\delta + \delta^2)\left(1 + \left\lfloor \frac{n}{2} - 1 \right\rfloor \delta\right) \\ = & \left\lfloor \frac{n}{2} \right\rfloor \cdot 2(1 + \delta) + \left\lfloor \frac{n}{2} - 1 \right\rfloor \left(2(\delta + 2\delta^2 + \delta^3) + \left\lfloor \frac{n}{2} - 2 \right\rfloor \cdot \delta(\delta + 2\delta^2 + \delta^3)\right) \\ & + 1_{\{n \text{ odd}\}} \cdot 2(\delta + \delta^2)\left(1 + \left\lfloor \frac{n}{2} - 1 \right\rfloor \delta\right), \end{aligned}$$

which corresponds to the total benefits from a star with $\lfloor \frac{n}{2} \rfloor$ peripheral pairs. As any other tree with $\lfloor \frac{n}{2} \rfloor$ public links and $d_j^{\text{pub}}(g) \leq 1$ for all $j \in N$ yields strictly lower benefits from indirect connections, a star with $\lfloor \frac{n}{2} \rfloor$ peripheral pairs uniquely maximizes objective social welfare.

It is left to determine the threshold on δ for g to be a tree with $\lfloor \frac{n}{2} \rfloor$ public links. Let i denote the center and j an agent at distance two from the center. First, note that adding a public link will decrease objective social welfare. Adding a shadow link between j and another agent $k \neq i$ increases objective social welfare by at most

$$2\left((1-\delta^2)(\delta+\delta^2) - c^{\text{sha}}\right). \tag{12}$$

Adding a shadow link between j and i increases objective social welfare by

$$2\delta(1-\delta)\left(1 + (1 + 1_{\{n \text{ odd}\}})\delta + \left\lfloor \frac{n}{2} - 2 \right\rfloor (\delta+\delta^2)\right) - 2c^{\text{sha}} = 2F(\delta, n) - 2c^{\text{sha}},$$

which (weakly) exceeds (12). Since the increase in objective social welfare decreases with the number of shadow links that we add, the network maximizing objective social welfare g is a tree if $c^{\text{sha}} \geq F(\delta, n)$. Second, deleting a public link may increase objective social welfare, but only if we then add a shadow link instead. As seen above, we only need to check whether deleting the public link of agent j and instead adding a shadow link between j and i increases objective social welfare. This is not the case if

$$c^{\text{sha}} \leq 1 - \delta(1-\delta)\left((1 + 1_{\{n \text{ odd}\}})\delta + \left\lfloor \frac{n}{2} - 2 \right\rfloor (\delta+\delta^2)\right) = 1 + \delta(1-\delta) - F(\delta, n).$$

Finally, $F(\delta, n) \to 0$ as $\delta \to 1$ implies that the conditions are satisfied if δ is large enough, which finishes the proof. □

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