

# The Evolution of Conventions in the Presence of Social Competition

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## Abstract

We study the long run convention emerging from stag hunt interactions when agents occasionally revise their action over time adopting a perturbed myopic best response rule, with the novelty of introducing social competition in the form of assignment of prizes to agents depending on the payoff ranking resulting from the stag hunt interaction. We find that social competition plays a crucial and articulated role in the selection of the long run convention: indeed, a high enough reward from competition selects the payoff-dominant convention when competition is harsh, and the maximin convention when competition is mild.

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**Keywords:** risk-dominant; payoff-dominant; maximin; stag hunt; stochastic stability.

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# 1 Introduction

A convention can be understood as an equilibrium of a coordination game played by the individuals belonging to a group or a society: once a particular way of doing things becomes established as a rule, it continues in force because we prefer to conform to the rule given the expectation that others are going to conform (Lewis, 2008).<sup>1</sup> An important issue that can be studied in this setting is the tension between Pareto dominance and safety: while a convention pays a higher payoff to every individual if coordination is actually achieved, another convention is less risky since it performs better if miscoordination occurs. The stylized game that is used to capture this conflict is the *stag hunt* game, which is often seen as a paradigmatic representation of the obstacles to achieve social cooperation (Skyrms, 2004).

In this paper we study the evolution of conventions in a population of agents playing a stag hunt game, where each agent only occasionally revises his action, and when he does so, he chooses a myopic best reply. We model agents' interactions as a pairwise match of randomly drawn individuals, but in addition we allow for some degree of stability of the interaction. We derive the emerging long-run convention applying the notion of stochastic stability, which was introduced by Foster and Young (1990) and then developed by Young (1993) and Kandori et al. (1993) building on technical results in Freidlin and Wentzell (1984). Basically, when agents are allowed to make mistakes with a tiny probability, stochastic stability selects those equilibria that are relatively easiest to reach in terms of mistakes – also referred to as errors or mutations – starting from other equilibria.<sup>2</sup>

The novelty of this paper is the introduction of social competition. At each time, agents are randomly matched to play a stag hunt game, from which they earn payoffs. The ranking of agents in terms of payoffs is then used for the assignment of rewards due to social competition:

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<sup>1</sup>See Young (2015) for an overview of other mechanisms that can sustain conventions.

<sup>2</sup>While mistakes are typically assumed to affect strategy choice, more recently observational mistakes have also been considered as a source of noise for stochastic stability analysis (Sawa, 2021).

only the agents who are high in the ranking gain rewards. The fraction of agents who will benefit from rewards can be interpreted as an inverse measure of the harshness of social competition.

Social competition is widespread in social and economic settings. Status seeking is a prominent instance of social competition, often driven by scarcity of resources such as prestige and power. Social competition typically leads to a sort of tournament game where individuals compete (for the rewards of the social competition) by engaging in costly activities ([Hopkins and Kornienko, 2004](#)). So, social competition often generates pervasive externalities that can affect deeply economic behavior (see [Truyts, 2010](#), and references therein). Such externalities can lead to severe market inefficiencies ([Frank, 2008](#); [Aronsson and Johansson-Stenman, 2010](#)), and even shape preferences for public policy measures possibly aimed at correcting those inefficient outcomes (see [Gallice, 2018](#), and references therein). At the same time, social competition can help internalize previously existing externalities, thus being potentially beneficial ([Hopkins, 2012](#); [Bhaskar and Hopkins, 2016](#)). The effects of social competition on social coordination represent a channel that has not been investigated so far by the literature and that, based on the results in this paper, should be taken into consideration in more applied work, both theoretical and empirical, in the attempt to identify the relationship between inequality of rewards and economic performance (see, on this point, [Hopkins and Kornienko, 2010](#) and [Bilancini and Boncinelli, 2020b](#)).

To fix ideas on the relevance of our contribution, consider the following case of academic collaboration. Researchers meet at seminars, conferences, and faculty meetings, exchange emails and have online conversations. Sometimes, opportunities arise to work together for a paper or a project. Often, producing a good research output requires more than one researcher collaborating together, each contributing with specific skills and competences. Also, while effort is time-consuming at the individual level, the joint effort of all collaborators in a paper or project is required in order to yield a good output. A researcher who exerts

no effort in the joint activity saves time, which can be used to work on a single-author paper or project of passable quality (less than good). Given that only the best papers end up being published and the best projects being financed, a policy-relevant question is: how selective should the publication and funding processes be to promote effortful collaboration by researchers? This paper provides a simple evolutionary model which allows to shed light on this issue.

For a more abstract and more general example guide one can consider team production as the first-stage interaction, where strong complementarities impose that the individual costly investment is productive only if the partner also invests, and competition for social status as the second-stage interaction, where a higher status is obtained with a higher rank in the income distribution resulting from the first stage. Social status is pursued because it grants larger social support and earlier access to opportunities due to cultural or institutional factors. Here the question is how tough the social competition should be to favor individual investment in team production.

In this paper we investigate the relationship between the stochastically stable convention and the reward and harshness of social competition. We find that, when social competition is harsh (i.e., the fraction of winners is below one half), a high enough reward selects the payoff-dominant convention as stochastically stable. When instead competition is mild (i.e., the fraction of winners is above one half), a high enough reward selects the maximin convention as stochastically stable.

The intuition of our results is as follows. When competition is very harsh, the only way to obtain the social reward is by taking the risk and being lucky, i.e., choosing the payoff-dominant action and being matched with someone choosing the payoff-dominant action. This makes the payoff-dominant action more attractive relatively to the maximin action (which, if chosen, leads to no reward for sure). When competition is very mild, the only way *not* to obtain the social reward is by taking the risk and being unlucky, i.e., choosing the

92 payoff-dominant action and being matched with someone choosing the maximin action. This  
93 makes the payoff-dominant action less attractive relatively to the maximin action (which, if  
94 chosen, grants the reward).

95 The paper is organized as follows. Section 2 reviews the related literature, distinguishing  
96 between contributions to the evolution of conventions and those to the modeling of social  
97 competition as a tournament. Section 3 introduces the model, which is then analyzed in  
98 Section 4 in the absence of mistakes, and in Section 5 when mistakes are added with the aim  
99 of identifying the long-run equilibrium. Section 6 concludes with some final remarks.

## 100 2 Related Literature

101 The innovative feature of our model is the addition of social competition in the form of a  
102 tournament to an otherwise standard evolutionary analysis of social coordination in the stag  
103 hunt game. Below we briefly review the main findings regarding the evolution of conventions  
104 in the stag hunt (for a thorough review of evolutionary models and coordination games see  
105 [Newton, 2018](#)) and the modeling of social competition as a tournament ([Truyts, 2010](#)).

### 106 2.1 Evolution of conventions

107 The stag hunt game captures the tension that arises between a payoff-dominant action, which  
108 pays a higher payoff if the associated equilibrium is actually played, and a risk-dominant  
109 action, which performs better if out-of-equilibrium play happens ([Harsanyi and Selten, 1988](#)).  
110 Which of these equilibria is more likely to emerge in the long run has been a matter of study  
111 in evolutionary game theory ([Young, 1993](#); [Kandori et al., 1993](#)).

112 It is known since [Kandori et al. \(1993\)](#) that the risk-dominant convention is the long-  
113 run equilibrium in a stochastic evolutionary model where agents interact randomly with  
114 each other and update behavior by following a myopic best response rule. Starting from

this, a variety of contributions to the literature have explored the robustness of this finding. The general understanding is that the risk-dominant convention emerges in the long run whenever the interaction structure is assumed to be exogenously given – both in case of global interaction models (see, e.g., [Kandori et al., 1993](#), [Kandori and Rob, 1995](#), [Young, 1993](#)) and in case of local interaction models (see, e.g., [Blume, 1993, 1995](#), [Ellison, 1993, 2000](#), [Alós-Ferrer and Weidenholzer, 2007](#) and [Jiang and Weidenholzer, 2016](#)) – while the payoff-dominant convention is more likely to emerge under an endogenous interaction structure.<sup>3</sup>

In models considering endogenous network formation, where agents choose with whom to interact ([Goyal and Vega-Redondo, 2005](#); [Jackson and Watts, 2002](#); [Staudigl and Weidenholzer, 2014](#)), the payoff-dominant convention is shown to emerge in the long run if the single interaction is sufficiently costly or the total number of interactions per agent is sufficiently constrained. An alternative to network formation is that agents can choose where to interact – not with whom – selecting a location among a number of locations available and then interacting randomly with agents choosing the same location ([Oechssler, 1997](#); [Ely, 2002](#); [Bhaskar and Vega-Redondo, 2004](#)). The possibility to “vote by their feet” helps agents to coordinate on the payoff-dominant action.

The co-existence of both payoff-dominant and risk-dominant conventions can also be obtained as long run equilibrium. This is the case in models with location choice when locations are subject to a capacity constraint, thus impeding the possibility that all agents stay in the same location ([Anwar, 2002](#); [Blume and Temzelides, 2003](#); [Pin et al., 2017](#)). [Bilancini and Boncinelli \(2018\)](#) obtain coexistence of conventions also with network formation and constrained interaction, in a model where agents have different types, types are only locally observable, and type mismatches are costly.<sup>4</sup>

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<sup>3</sup>[Peski \(2010\)](#) provides a general framework for local interaction models with an exogenous interaction structure. See also [Weidenholzer \(2010\)](#) for a survey on local interaction models focusing on social coordination.

<sup>4</sup>[Carvalho \(2016\)](#) shows that the co-existence of conventions can emerge also under global random inter-

The literature on the evolution of conventions in the stag hunt game has also explored the role of the error model in selecting the long-run equilibrium. In particular, [Bilancini and Boncinelli \(2020a\)](#) study a model where interactions among agents are globally random but exhibit a degree of resilience over time, finding that the risk-dominant convention is the prominent outcome when mistakes are uniform or payoff-dependent, meaning that error probabilities converge to zero at an exponential rate that is, respectively, uniform across states or increasing in the payoff loss of the mistake. When mistakes are instead condition dependent, i.e., error probabilities converge to zero at an exponential rate that is higher the lower the experience payoff, the payoff-dominant convention emerges in the long run when interactions are sufficiently resilient over time, while the maximin convention can emerge even if it is not risk-dominant when interactions are quite volatile.

Other error models considered in the literature are intentional mistakes, where only actions that can lead to higher payoffs can be chosen by mistake ([Naidu et al., 2010](#)), and coalitional mistakes, where mistakes that can be rationalized as profitable deviations for a coalition of individuals are assumed to be more likely than other mistakes. ([Newton, 2012](#)). Under both these error models, the Pareto-efficient convention naturally rises as long-run equilibrium, because all agents are better off in such convention relative the the risk-dominant convention.

In comparison to the above literature, our paper explores a different dimension to assess which convention will eventually emerge in the long run. More precisely, we consider the case in which the resources that are gained by playing a stag hunt game are then used for social competition, which we model as a simple tournament where prizes are assigned to those highest ranked in the possession of resources.

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action, provided that the population is divided into cultural groups that have distinct taboos (some actions are never played voluntarily) and different cultural preferences (for the non-taboo actions).

## 2.2 Social competition as a tournament

Tournament-like social competition gives rise to pervasive externalities, possibly leading to inefficiencies. [Hopkins and Kornienko \(2004\)](#) model social competition as a tournament of conspicuous consumption, finding that this gives rise to inefficiencies that are mitigated by income inequality. [Hopkins and Kornienko \(2010\)](#) show that the opposite holds in terms of the inequality of the rewards of the social competition. [Bilancini and Boncinelli \(2020c\)](#) show that when social competition is repeated over time and the rewards of one tournament are the endowments of the subsequent tournament, then the inefficiencies are maximal for an intermediate level of inequality.

In some cases, however, social competition in the form of a tournament can help to internalize other externalities. [Hopkins \(2012\)](#) studies a model where first workers signal their productivity to firms by means of education and then firms and workers are matched assortatively in a matching tournament; in this case social competition is shown to foster inefficiencies in case of sticky wages and to reduce inefficiencies in case of flexible wages. [Bhaskar and Hopkins \(2016\)](#) investigate the effects of social competition on pre-marital investment when the return is stochastic, finding that, generically, investments exceed the Pareto-efficient level, but if the sexes are symmetric in all respects then efficiency is attained.

Social competition introduced as a tournament can affect the attitude towards risk. [Dijk et al. \(2014\)](#) investigate the effects of social competition introduced by tournament incentives in an experiment on portfolio choices, finding that decision-makers become more risk-taker and that this is due mainly to non-monetary incentives. This latter fact is confirmed by experimental evidence in the field ([Tran and Zeckhauser, 2012](#)) and supported by studies in psychology ([Boyce et al., 2010](#)). Also, experimental evidence suggests that tournament incentives may increase effort but also disreputable behavior ([Charness et al., 2014](#)).

In this paper we find that the presence of social competition in the form of a tournament can be beneficial or detrimental, depending on the harshness of the social competition.



	$A$	$B$
$A$	$a$	$c$
$B$	$d$	$b$

Figure 1: A stag hunt game.

### 3 Model

The set of agents is  $N$  of cardinality  $n$ , with  $n$  even. Time is discrete and denoted by  $t = 0, 1, \dots$ . At each  $t$ , first agents are randomly matched in pairs to play a stag hunt game (as in Robson and Vega-Redondo, 1996), and then they participate in a social competition where prizes are assigned depending on the payoffs earned in the stag hunt interaction.

The stag hunt game is represented in Figure 1, where  $b$  is the largest payoff, and  $d$  is the smallest one. This implies that two coordination equilibria indeed exist, with  $B$  being the payoff-dominant action, and  $A$  the maximin action. We make no assumption about which action is risk-dominant:  $A$  is risk-dominant if  $a + c > b + d$ , while  $B$  is risk-dominant if  $a + c < b + d$ .

Social competition works as a tournament where each winner gets a prize equal to  $x$ , which adds to the payoff earned from the stag hunt interaction. We call  $x$  the *reward of social competition*. Winners of the social competition are determined as the  $k$  individuals with highest payoff after the stag hunt interaction. We define  $\alpha = k/n$ , and we call it the *harshness of social competition*. Any payoff tie is randomly broken, so that, there are no ties in the ranking with the result that, if necessary, winners are randomly determined among tied individuals.

The state of the system, which we denote by  $s \in \{0, 1, \dots, n\}$ , is the number of agents playing  $B$ .

Given that every player interacts with just one random opponent in any given period, the precise expressions of the expected payoffs of  $A$  players and  $B$  players at a generic state  $s$  involves complicated combinatorial calculations to compute the expected prize from social competition. Nevertheless, it is quite easy to write approximating expressions when the population size becomes larger and larger. We denote by  $\Pi(z|m, n-1)$  the payoff of an agent playing  $z \in \{A, B\}$  at a state where  $m$  out of the  $n-1$  other agents are playing  $B$ . We have that:

$$\begin{aligned} \lim_{\substack{m, n \rightarrow \infty \\ m/n \rightarrow q}} \Pi(A|m, n-1) &= \underbrace{(1-q)a + qc}_{\text{payoff in interaction}} + x \underbrace{\left[ \min \left\{ \max \left\{ \frac{\alpha - q^2}{1-q}, 0 \right\}, 1 \right\} \right]}_{\text{payoff in social competition}}, \quad (1) \\ \lim_{\substack{m, n \rightarrow \infty \\ m/n \rightarrow q}} \Pi(B|m, n-1) &= \underbrace{(1-q)d + qb}_{\text{payoff in interaction}} + \dots \\ &\dots + \underbrace{x \left[ q \min \left\{ \frac{\alpha}{q^2}, 1 \right\} + (1-q) \max \left\{ \frac{\alpha - q^2 - (1-q)}{(1-q)q}, 0 \right\} \right]}_{\text{payoff in social competition}}. \quad (2) \end{aligned}$$

In both (1) and (2), the first term is the expected payoff coming from the stag hunt interaction, while the second term is the expected payoff due to social competition, which is given by the reward  $x$  multiplied by the winning probability. To see why those are the winning probabilities, consider the ranking of payoffs resulting from the stag hunt interaction: we have at the top stag agents interacting with stag agents, whose fraction is  $q^2$ ; then all hare agents, whose fraction is  $1-q$  (of whom  $q$  interact with stag agents and  $1-q$  interact with hare agents, neglecting the order between them since they come consecutively); and at the bottom stag agents interacting with hare agents, whose fraction is  $(1-q)q$ . Since there are only  $k$  rewards to be assigned, the winning probability of the agent ranked  $k+1$ -th or beyond drops to 0; of course, in any case the probability cannot exceed 1.

## 4 Unperturbed dynamics

The revision protocol for agents' decision is independent inertia: at each  $t$ , each agent  $i \in N$  has an independent probability  $\gamma \in (0, 1)$  to receive an opportunity to revise his strategy.

When given a revision opportunity, agents myopically best-reply to the current state  $s$ , and randomize over best replies if there are more than one.

**LEMMA 1** (Recurrent classes). *The recurrent classes of the unperturbed dynamics are  $\{0\}$  and  $\{n\}$ .*

*Proof.* Consider first state 0. An agent who receives a revision opportunity has an expected payoff equal to  $a + \frac{k}{n}x$  if he chooses  $A$ , and equal to  $d$  if he chooses  $B$ . Since  $a + \frac{k}{n}x > d$ , state 0 is an absorbing state.

Consider now state  $n$ . An agent who receives a revision opportunity has an expected payoff equal to  $c$  if he chooses  $A$ , and equal to  $d$  if he chooses  $B$ . Since  $b + \frac{k}{n}x > c$ , state  $n$  is an absorbing state.

Consider finally a generic state  $s \notin \{0, n\}$ . Suppose at such state there are  $k$  players choosing  $z \in \{A, B\}$  (note:  $z \in \{s, n - s\}$ ). We now show that, if  $z$  is best reply for some player at state  $s$ , then the dynamic system can move with positive probability in one time period from state  $s$  to state  $s'$ , whereas  $k + 1$  players choose  $z$  and there exists a player for whom  $z$  is best reply. Indeed, if  $z$  is best reply for some player at state  $s$ , then there exists a player who is playing  $z' \in \{A, B\}$  with  $z' \neq z$ , since  $s \notin \{0, n\}$ , for whom  $z$  is best reply at state  $s$ . Such player can be the only one to receive a revision opportunity; in such a case, the player switches from  $z'$  to  $z$ , so that with positive probability the system passes from state  $s$  to a state – call it  $s'$  – with  $k + 1$  players choosing  $z$ . Note that the player who has switched from  $z'$  to  $z$  has the same incentive to play  $z$  at state  $s'$  as at state  $s$ ; therefore,  $z$  is still best reply for him. We have thus shown that there cannot exist recurrent classes other than  $\{0\}$  and  $\{n\}$ .  $\square$

Lemma 1 tells us that, as in the standard stag hunt interaction, there are only two recurrent classes: all agents play  $A$  (i.e.,  $s = 0$ ), and all agents play  $B$  (i.e.,  $s = n$ ). In other words, the introduction of social competition has no effect on recurrent classes.

We refer to state 0 as *maximin convention*, and to state  $n$  as *payoff-dominant convention*. We use the term *risk-dominant convention* to denote either state 0, when  $a + c > b + d$ , or state  $n$ , when  $a + c < b + d$ .

## 5 Perturbed dynamics

In the spirit of Young (1993) and Kandori et al. (1993), we add agents' mistakes to the model described in the previous section. In particular, we adopt the uniform-mistake model, and we denote by  $\epsilon$  the probability that an agent who has received a revision opportunity makes a mistake, i.e., selects the action that is not best response. Due to the introduction of mistakes, we obtain a Markov chain that is irreducible, i.e., has a unique recurrent class. This in turn implies that there exists a unique invariant distribution that describes the fraction of time spent on each state in the long run, irrespectively of the initial state.

As  $\epsilon$  goes to zero, mistakes become rarer and rarer, and the invariant distribution converges to the so-called stochastically stable distribution. We say that a convention is *stochastically stable* if its states have positive probability in the stochastically stable distribution. We rely on the techniques developed by Young (1993, 2001), which allow to characterize stochastically stable conventions in terms of minimum stochastic potential. In our model, the *stochastic potential* of a convention is the minimum total number of mistakes over paths of states starting from the other convention and reaching such convention. A convention is stochastically stable if its stochastic potential is smaller than or equal to the stochastic potential of the other convention.

Our main results are formally stated in the following propositions.

272 **PROPOSITION 1** (Stochastically stable convention - harsh competition). *Consider  $\alpha < 0.5$*   
 273 *and  $n$  sufficiently large. If the maximin convention is risk dominant, then there exists a*  
 274 *threshold  $\hat{x}$  such that: (i) if  $x < \hat{x}$  then the maximin convention is uniquely stochastically*  
 275 *stable, and (ii) if  $x > \hat{x}$  then the payoff-dominant convention is uniquely stochastically stable.*  
 276 *If the payoff dominant convention is risk dominant, then it is uniquely stochastically stable.*

*Proof.* We denote by  $\Pi(x|s, y)$  the expected payoff of choosing  $z \in \{A, B\}$  for an agent who is currently choosing  $z' \in \{A, B\}$  at state  $s$ . Consider the state where  $n/2$  players choose  $B$ . We consider an approximating expression for large  $n$  of the difference in the expected payoffs of playing  $B$  and playing  $A$  for an agent who is currently playing  $A$  (the same approximation applies for an agent who is currently playing  $B$ ). In particular, we evaluate (1) and (2) at  $q = 0.5$  and, taking the difference, we obtain:

$$\lim_{\substack{m, n \rightarrow \infty \\ m/n \rightarrow 0.5}} \Pi(B|m, n-1) - \Pi(A|m, n-1) = \underbrace{\frac{(b-a) - (c-d)}{2}}_{\text{payoff difference in interaction}} + \underbrace{\beta(\alpha)x}_{\text{payoff difference in social competition}}, \quad (3)$$

where

$$\beta(\alpha) := \begin{cases} 2\alpha & \text{if } \alpha \in (0, 0.25), \\ 1 - 2\alpha & \text{if } \alpha \in [0.25, 0.5). \end{cases}$$

277 We note that  $[(b-a) - (c-d)]/2$  is positive if the payoff-dominant convention is risk dominant,  
 278 while it is negative if the maximin convention is risk dominant. Since  $\beta(\alpha)$  is constant in  $x$   
 279 and always positive when  $\alpha < 0.5$ , the quantity in (3) is strictly and unboundedly growing  
 280 in  $x$ . This means that there exists a threshold, call it  $\hat{x}$ , such that the quantity in (3) is  
 281 positive if  $x > \hat{x}$ , and has the same sign of  $[(b-a) - (c-d)]/2$  if  $x < \hat{x}$ .

282 Therefore, when  $n$  is large, all players find it optimal to play  $B$  at state  $n/2$  if  $x > \hat{x}$ .  
 283 Based on the reasoning in the third paragraph of the proof of Lemma 1, the system can  
 284 move from state  $n/2$  to state  $n$  in the unperturbed dynamics. This means that the minimum  
 285 number of mistakes to move from state 0 to state  $n$  is at most  $n/2$ . By an analogous  
 286 reasoning, the minimum number of mistakes to move from state  $n$  to state 0 is larger than

287  $n/2$ , so that we can conclude that state  $n$  is the only stochastically stable convention if  
 288  $x > \hat{x}$ .

289 We can proceed along the same line of reasoning when  $x > \hat{x}$ , concluding that state 0 is  
 290 the only stochastically stable convention in such a case.  $\square$

291 **PROPOSITION 2** (Stochastically stable convention - mild competition). *Consider  $\alpha > 0.5$   
 292 and  $n$  sufficiently large. If the payoff-dominant convention is risk dominant, then there ex-  
 293 ists a threshold  $\hat{x}$  such that: (i) if  $x < \hat{x}$  then the payoff-dominant convention is uniquely  
 294 stochastically stable, and (ii) if  $x > \hat{x}$  then the risk-dominant convention is uniquely stochas-  
 295 tically stable. If the maximin dominant convention is risk dominant, then it is uniquely  
 296 stochastically stable.*

*Proof.* The argument is the same as in the proof of Proposition 2, once we extend the  
 definition of  $\beta(\alpha)$  to the interval  $(0.5, 1)$  (starting again from (1) and (2) and taking their  
 difference for  $q = 0.5$ ), as follows:

$$\beta(\alpha) := \begin{cases} 1 - 2\alpha & \text{if } \alpha \in (0.5, 0.75), \\ 2\alpha - 2 & \text{if } \alpha \in [0.75, 1). \end{cases}$$

297 In particular, we note that  $\beta(\alpha)$  is constant in  $x$  and always negative for  $\alpha \in (0.5, 1)$ .  $\square$

298 The sketch of the proofs of Proposition 1 and Proposition 2 is as follows. Given that there  
 299 exist only two conventions, establishing which convention is stochastically stable amounts  
 300 to determining which of them has the largest basin of attraction. To do so, it is enough to  
 301 determine which basin of attraction contains the state where the population is equally split  
 302 between the two actions. Given that agents are myopic best responders, this in turn depends  
 303 on which action is best reply at such an intermediate state. When the social competition is  
 304 harsh, action stag gets an advantage over action hare because it allows to obtain the highest  
 305 possible payoff, which occurs in case of coordination on stag, maximizing the likelihood  
 306 of getting one the few prizes. When the social competition is mild, instead, hare gets an

307 advantage over stag because it allows to avoid the lowest possible payoff, which occurs in  
 308 case of miscoordination with stag facing hare and which maximizes the likelihood of not  
 309 obtaining a prize even if there are many. In other words, the harshness of competition is  
 310 key for stochastic stability in this setup because it affects the relative benefits and costs of  
 311 miscoordination. Figure 2 illustrates the expected rewards from social competition of playing  
 312 hare ( $A$ ) and stag ( $B$ ) as a function of its harshness ( $\alpha$ ), when half of the population plays  
 313 stag ( $q = 0.5$ ). Note that  $\beta(\alpha)$  in the proofs is measured by the vertical distance between  
 314 the blue line and the red line.

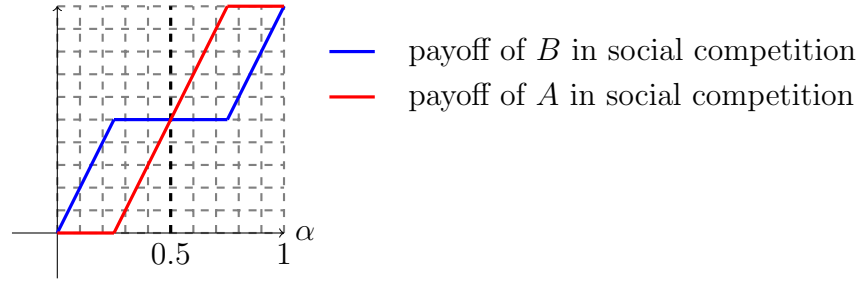


Figure 2: Action  $B$  has a payoff advantage in social competition over action  $A$  when  $\alpha < 0.5$ , and a payoff disadvantage when  $\alpha > 0.5$ .

## 315 6 Concluding remarks

316 In this paper we have explored the effect of social competition on the long-run convention  
 317 emerging in a large population of individuals who are myopic best responders subject to  
 318 a negligible amount of uniform mistakes. We have found that social competition may be  
 319 beneficial if it is harsh, i.e., a minority of the population ends up obtaining the prize of  
 320 competition, in in that it leads to the emergence of the payoff-dominant convention; instead,  
 321 social competition may be detrimental if it is mild, i.e., that a majority of the population  
 322 ends up obtaining the prize of competition, in that it leads to the emergence of the maximin  
 323 convention.

Proposition 1 and Proposition 2 can be seen as providing a robustness result with respect to previous findings in the literature. Indeed, it is well known that stochastic stability selects the risk-dominant convention in a stochastic evolutionary model where agents interact randomly with each other and update behavior by following a myopic best response rule. The addition of social competition, as we do in our model, does not change the selection operated via stochastic stability when the reward of social competition is small enough. At the same time, however, risk dominance loses any role as a predictor of the stochastically stable convention when the reward of social competition is high enough. What matters in that case is the harshness of social competition: if social competition is harsh, then the payoff-dominant convention is selected by stochastic stability; if, instead, social competition is mild, then the maximin convention is selected by stochastic stability.

A question that we can reasonably ask in our setting is which specific degree of harsh competition requires the minimum overall payment to promote the Pareto-dominant convention. More formally, we look for the values of  $\alpha$  that minimize  $\alpha \hat{x}(\alpha)$ . As illustrated in Figure 3, every  $\alpha \in (0, 0.25]$  allows to minimize the overall expenditure for prizes of the social competition.

As a matter of interpretation, we can conclude that efficiency requires social competition to be quite high, i.e.,  $\alpha$  must not exceed 0.25. At the same time, there is no additional benefit for making competition extreme, i.e., lowering  $\alpha$  below 0.25.

Our model makes use of several specific assumptions, which role is briefly discussed hereafter.

Our results are derived under the assumption that agents revise their strategy according to myopic best reply. Another plausible behavioral rule is imitation, which typically favors the long-run selection of the payoff-dominant convention. Robson and Vega-Redondo (1996) show that if revising agents adopt the strategy which yields the highest realized payoff, then the payoff-dominant convention is stochastically stable even if matching is completely



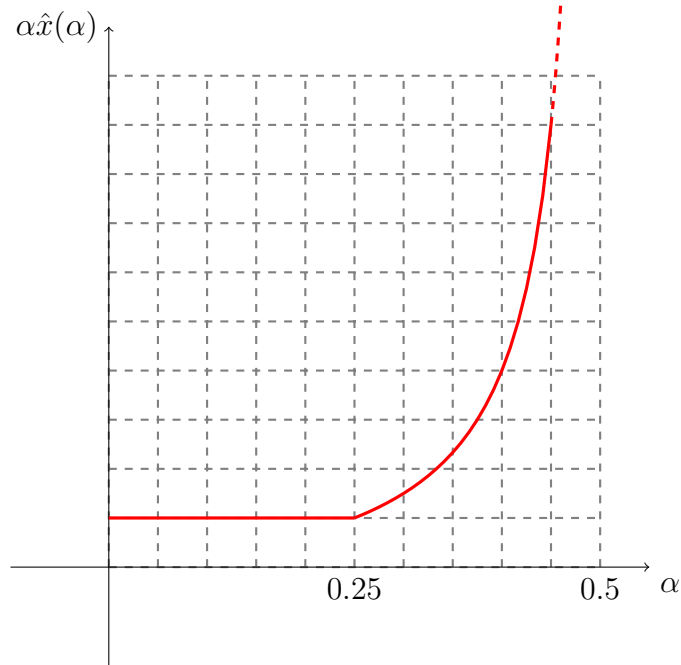


Figure 3: The plot depicts, for the case in which the maximin convention is risk dominant, the minimum expenditure for prizes ensuring that the Pareto-dominant convention is stochastically stable, as a function of the harshness of the social competition. The minimum value of such an expenditure is equal to  $[(c - d) - (b - a)]/2$ . When  $\alpha > 0.25$ , and up to  $\alpha = 0.5$ , the expenditure increases by a factor  $\alpha/(1 - 2\alpha)$ .

random. [Alós-Ferrer and Weidenholzer \(2008\)](#) show that this is the case even if interaction is not random but takes place with given neighbors, provided it is sufficiently local. We stress that, while the assumption of myopic best reply brings a contrast between our findings and the standard result on the long-run emergence of the risk-dominant convention, no role is left for social competition in case of imitation of the highest realized payoff. This is so because prizes in the second stage are given to those who get the highest payoff in the stag hunt game, thus preserving the existing ranking in terms of payoffs.

Another important assumption regards the matching in the first stage. We assume that agents are randomly matched in pairs and that a single interaction determines the payoff which in turn is used to assign the prizes in the second stage. The expected payoff for

the first stage is equivalent to the one obtained with round robin tournament. However, assuming a round robin tournament in the first stage would lead to the emergence of the risk dominant convention for any harshness of competition and prizes in the second stage. This is so because with the round robin tournament the risk-dominant action always pays more when half of the population adopts it.

An assumption which can be relaxed without affecting the quality of our results regards the assignment of prizes. More prize tiers can be introduced, assuming that prizes increase as we approach the highest positions in the ranking of payoffs. Also, while we opted for a deterministic rule for the assignment of prizes, it can be assumed to be probabilistic, with the probability of getting a higher prize increasing in the payoff obtained at the first stage. In all such cases, what really drives results is the average prize for quartiles, because this is what determines differences in the expected payoff of strategies when half of the population adopts stag and half of it adopts hare (which in turn determines which basin of attraction is the largest).

We conclude by pointing out to a characteristic of the model in this paper which may be of methodological interest. It seems plain to think of people playing many different games in their daily life interactions. Yet, games are typically analyzed in isolation one from the other. This approach is by no means a restriction if we assume that behaviors are independent across games. But there are reasons why isolation may not hold. In this paper, in particular, we exploit the idea that the payoff in a game may represent the endowment when agents start to play a following game. In cases like this, behaviors are better understood when studied in a meta-game comprised of a sequence of base games.

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